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Jurnal	:	Journal Mathematics Teaching-Research Journal (MTRJ)
Penulis	:	Dini Andiani, Elah Nurlaelah, and Darhim

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DINI ANDIANI < diniandiani@upi.edu >
to mtrj.submissions@gmail.com ▼

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Dini Andiani
Universitas Pendidikan Indonesia
Jl. Dr. Setiabudhi No. 229
Bandung 40154
Indonesia

Dr. Bronislaw Czarnocha, Chief Editor
Professor
Hostos Community College, CUNY
Bronx, NY 10451, USA
bczarnocha@hostos.cuny.edu
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September 26th, 2024

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I am pleased to submit an original research manuscript entitled “ Learning Trajectory of Trigonometric Ratios Applying Realistic Mathematics Education” to be considered for publication in the Journal Mathematics Teaching-Research Journal (MTRJ).

Trigonometry is still regarded as a challenging subject in secondary education. Consequently, creating a learning trajectory that addresses this issue is essential. This publication delineates a study on formulating a learning trajectory for trigonometric ratios employing a realistic mathematics teaching approach. The results of this study demonstrate that the learning trajectory for trigonometric ratios comprises six activities. The six activities increase students' comprehension of trigonometric ratios. These findings will inspire education practitioners to teach trigonometric ratios so that learning outcomes are achieved.

Owing to the result of research in the teaching field, this manuscript is appropriate for publication in Journal Mathematics Teaching-Research Journal (MTRJ).

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Bronx, NY 10451, USA

bczarnocha@hostos.cuny.edu

Dr. Malgorzata Marciniak, Managing Editor

Associate Professor

LaGuardia Community College, CUNY

Long Island City, NY 11101, USA

mmarciniak@lagcc.cuny.edu

Dr. Ivan Retamoso, *The Problem Corner* Editor and Webmaster

Assistant Professor

Borough of Manhattan Community College, CUNY

Manhattan, NY 10007, USA

iretamoso@bmcc.cuny.edu

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Hostos Community College, CUNY

Bronx, NY 10451, USA

bczarnocha@hostos.cuny.edu

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Associate Professor

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Mathematics Department

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Dr. Bronislaw Czarnocha
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Mathematics Department
Hostos CC,
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Learning Trajectory of Trigonometric Ratios Applying Realistic Mathematics Education

Dini Andiani ^{1,2}, Darhim ¹, Elah Nurlaelah ¹

¹ Department of Mathematics Education, Universitas Pendidikan Indonesia,

Bandung, Indonesia, ² Department of Mathematics, Universitas Bale

Bandung, Indonesia

diniandiani@upi.edu, diniandiani367@gmail.com,

darhim0355@gmail.com

Abstract: Trigonometry material is still considered complex material at the secondary level. It is necessary to design an appropriate trigonometric ratios learning trajectory so that students can quickly build the concept of trigonometric ratios. This study aims to design a learning trajectory for trigonometric ratio material through Realistic Mathematics Education (RME). This study uses a research design involving 10th-grade high school students in Bandung City in the 2023-2024 academic year. The research data was taken through data collection using written test instruments, student worksheets, observations, interviews, and documentation. The results of the study showed that the trigonometric ratios learning trajectory consisted of raffia stretching activities from the table base to the student's head, measuring the student's height and shadow, experimenting with making three right triangles from paper, observing a picture of a calendar in the form of an equilateral triangular prism, observing a bed or bed, and observing a picture of a skier going down an iceberg. Students can quickly build the concept of trigonometric ratios from several activities provided. In addition, various learning flows from various activities make it easier for students to build knowledge about trigonometric ratios and solve problems presented contextually.

Keywords: Learning Trajectory, Trigonometric Ratios, Realistic Mathematics Education

INTRODUCTION

Trigonometry material is vital in everyday applications such as statistics, surveying, architecture, and other engineering fields (Weber, 2005). Building knowledge about trigonometry takes work (Blackett & Tall, 1991; Demir et al., 2012; Kendal & Stacey, 1997; Ngu & Phan, 2020; Sugianto et al., 2023). Students need to learn several things related to one concept and another to understand the concept of trigonometry. Kamber's research (2017) stated that students struggle to understand trigonometry through the unit circle

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(Kamber & Takaci, 2017). Students need help distinguishing trigonometry through right triangles, the unit circle, or the relationship between the two (Maknun et al., 2019). Students fail to understand the concept of trigonometric ratios (Weber, 2005) even though trigonometric ratios are essential in understanding trigonometry (Nordlander, 2022).

Referring to the Independent Curriculum implemented in Indonesia, trigonometric ratio is a grade 10 material or element which is phase E. Learning outcomes (CP) in phase E are that students involve trigonometric ratios and their applications to solve problems involving right triangles (Badan et al., 2022).

Given the outlined challenges in acquiring knowledge about trigonometric ratios, it is imperative to develop a learning trajectory that facilitates the development of students' understanding in this area. In accordance with (Simon, 1993) definition of hypothetical learning trajectory (HLT), Clements (2011) utilizes the notion of learning trajectory to characterize students' cognitive processes and acquisition of knowledge on a specific mathematical subject. It also encompasses the sequence of assumptions that guide their thinking as they progress through a set of learning activities aimed at generating the intended mental processes or actions. According to Clements (2011), this cognitive process guides students in developing a certain degree of thinking in order to attain specified learning objectives for the subject matter (Clements, 2011).

The Realistic Mathematics Education (RME) approach is strategically developed to effectively support the achievement of the intended learning trajectory. The instructional approach known as Realistic Mathematics Education (RME) is based on the philosophy of Hans Freudenthal, who views mathematics as a creative endeavor of humanity (Freudenthal, 1973; Gravemeijer & Terwel, 2000; Treffers, 1987; Van den Heuvel-Panhuizen & Drijvers, 2020). Realistic Mathematics Education enables students to engage in the learning process within the context of real-life situations and their everyday experiences (Yuanita et al., 2018). A study conducted by Aiyub also shown that learning designs that incorporate real or contextual issue designs can enhance students' algebraic thinking (Aiyub et al., 2024).

Students build understanding through informal processes. Students observe, see, and understand the environment in which they are located (Meika, 2018). Realistic Mathematics Education encompasses three key concepts: directed reinvention and progressive mathematical processes, didactic phenomenology, and model construct development (Gravemeijer, 1994). Within the teaching and learning process, these three concepts are further developed into six distinct features of RME: 1) The principle of activity, 2) The principle of reality, 3) The principle of level, 4) The principle of intertwinement, 5) The principle of interactivity, and 6) The principle of guidance (Panhuizen & Drijvers, 2014).

Under the title "Realistic Mathematics Education" in the Encyclopedia of Mathematics Education, Panhuijen elucidates the six concepts. Under the RME approach, the activity principle entails that students assume an active role in the learning process. This assertion is based on Freudenthal's theoretical understanding of mathematics as a human endeavor. The reality principle posits that students have the ability to use mathematics in order to address practical issues, and the process of learning should commence with tangible problems that can be expressed mathematically. The level principle posits that students progress through several levels of comprehension in order to grasp the connection between ideas and procedures in mathematics. The interconnection refers to the phenomenon where the students, while acquiring mathematical information, are able to employ a range of tools and methods to construct a comprehensive process of integrating different concepts. The interaction principle posits that the process of learning mathematics is not only an individual endeavor, but rather a social and reflective one, therefore facilitating a deeper comprehension of the subject. The guidance principle in mathematics education entails the concept of "guided reinvention," as proposed by Freudenthal. Under these circumstances, the learning

process should motivate students to adapt in order to enhance their comprehension. It is imperative that the learning program aligns with a cohesive long-term learning trajectory (Panhuizen & Drijvers, 2014).

Method

The employed research methodology is design research as described by Gravemeijer and Van Eerde in 2009 (Gravemeijer & Van Eerde, 2009). This research methodology is focused on local instruction theory (LIT) to enhance the quality of learning by fostering effective collaboration between researchers and teachers (Gravemeijer & Van Eerde, 2009). Design research will enable the attainment of the goals of the research paradigm in the development of sequential activities and the enhancement of learning quality. The primary objective of this work is to develop a comparative trigonometry learning trajectory by utilizing the RME approach. The study was carried out in two cyclical phases, each following the design research stages outlined by Gravemeijer (2004): 1) preparatory design, 2) teaching experiment, and 3) retrospective analysis. The outcomes of the initial cycle activities that have been executed and the subsequent analytical review have contributed valuable insights (Gravemeijer, 2004).

During the first stage of the design process, the researcher personally observed the school and conducted direct interviews with the teachers and the students. Furthermore, the researcher gathered, perused, and comprehended several source materials pertaining to the challenges faced by students and teachers in grasping the notion of trigonometric ratios. Furthermore, the researcher examined many sources that elucidated the learning process activities based on the Realistic Mathematics Education (RME) methodology. A sequence of researcher actions served as a point of reference for the researcher in developing the Hypothetical Learning Trajectory (HLT). The Hypothetical Learning Trajectory (HLT) comprises three elements: learning objectives that will be focused on during the learning process, learning activities, and hypotheses that will forecast the cultivation of students' cognitive abilities in the learning process (Simon, 1993). The learning process hypothesis serves as the fundamental basis for constructing learning activities. This document is a compilation of learning activities designed by the researcher.

Learning objectives	Learning Activities	Student Thinking Conjecture
Students can identify the position of the right angle, opposite side, and hypotenuse in a right triangle.	Stretch the raffia from the table's base to the top of the head to form a right triangle.	A group of students can model the activity into a right triangle. One of the students was able to sketch the right triangle that was formed. Students can identify the position of right angles, front sides, hypotenuses, and side sides in sketches formed with various answers without knowing the reasons. Students need to be corrected in identifying the position of right angles.
Students can apply the definition of trigonometric ratios $\sin \theta$, $\cos \theta$, and $\tan \theta$ to	Measuring students' height and shadow	Students can draw sketches formed from the models presented. The definitions of trigonometric ratios $\tan \theta$, $\sin \theta$, and $\cos \theta$ can be used by students to the model of measuring the heights of both students and their shadows.

Learning objectives	Learning Activities	Student Thinking Conjecture
contextual problems using the concept of ratios in right triangles.		Students can determine the position θ of different angles to obtain different trigonometric ratio values.
Students can understand the relationship between the concept of similarity and trigonometric ratios.	Experiment to make three right triangles from paper	Students can make three similar right triangles from paper. Students can make sketches from paper formed into three similar right triangles. Students can identify the shortest and longest sides of each triangle formed. Students can determine the ratio between the shortest side and the longest side of each right triangle. Students can relate the concept of similarity with trigonometric ratios.
Students can determine the value of trigonometric ratios at particular angles (0° , 30° , 45° , 60° , and 90°).	Observations on the calendar image in the form of an equilateral triangular prism.	Students can sketch a calendar model as an equilateral triangular prism. Students can construct a right triangle in which one of the angles corresponds to a specifically defined angle. Students can determine the value of trigonometric ratios at particular angles (0°, 30°, 45°, 60°, and 90°).
The application of trigonometric ratios enables students to ascertain the length of sides in a right triangle that are not known.	Bed or bed observation	Students can make sketches or drawings of bed models. Students can identify the position of right angles, oblique angles, side angles, and front side positions in sketches. Students can apply trigonometric ratios to determine the length of unknown sides in problems presented related to beds.
Students can solve contextual problems related to right triangles using trigonometric ratios.	Observation of the image of a skier descending an iceberg	Students can identify the position of the angles, opposite sides and hypotenuses in a right triangle formed as a sketch of a picture of a skier descending an iceberg. Students can apply the definition of trigonometric ratios to skier problems. Students can solve the problems given in the case of skiers.

Table 1: Hypothetical Learning Trajectory in Teaching Experiment 1

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Within cycle 1, the research participants consisted of 25 10th-grade students who attended a high school located in Bandung City. In cycle 2, the research participants consisted of 25 10th-grade students from a different high school within the same municipal area. The HLT Design was first implemented in teaching experiment cycle 1 to assess the efficacy of the prepared learning activity design in investigating students' cognitive patterns. Moreover, the results of teaching experiment 1 indicated an enhancement in HLT. A refined Hypothetical Learning Trajectory (HLT) was implemented in phase 2 of the teaching experiment. The data collecting protocols encompass classroom observations, videos documenting learning activities, student workbooks, and recordings of interviews conducted with students.

Phase 1 of the retrospective analysis involved the collection and analysis of all the data by the researcher. An analysis was conducted to evaluate the outcomes of including the learning trajectory in teaching experiment 1 with the original HLT design. Moreover, the researcher investigated and examined the learning process activities included to determine the level of students' comprehension of the idea of trigonometric ratios. An analysis was conducted on the outcomes of the HLT enhancements implemented in teaching experiment 2, by comparing the initial HLT with the execution of the learning process.

The researcher selected 6 participant subjects to conduct interviews. The six participants had various abilities; two had high, medium, and low abilities. In addition, interviews were also conducted with the mathematics teacher of trigonometric ratios at the school that taught the students.

Results

The learning trajectory design discussed in this paper is a sequence of student activities aimed at comprehending the subject of trigonometric ratios through the Realistic Mathematics Education approach. The trigonometric ratios learning trajectory comprises six learning activities: elongating a raffia from the table's base to the student's head, measuring the student's height and shadow, creating three right triangles from paper, visually examining a calendar image in the shape of an equilateral triangular prism, observing a bed or bed, and observing an image of a skier descending an iceberg.

Activity 1: Observation of “Stretching the Raffia from the Base of the Table to the Top of the Student's Head”

The primary aim of activity 1 is to enable students to accurately determine the location of the right angle, the opposing side, and the hypotenuse within a right triangle. During this exercise, the teacher instructs students to showcase their use of contextual equipment found in the classroom, specifically a raffia, a table base, and one of their peers. The students are segregated into multiple groups. Each group comprises from 4 to 5 students. A single student from each group grasps one end of the raffia, while the other end is fastened to the bottom of the table foundation.

Furthermore, the other students in the group observed the shape between the student's height, the floor, and the raffia. The observation process is shown in Figure 1. Furthermore, by describing the teaching materials, the teacher directs students to sketch the shapes in the demonstration.

Each group sketched their demonstration and represented the information in a picture or symbol. One of the sketch results is shown in Figure 2. Students sketched the table legs, the student's height, and the raffia stretched to form a right triangle. They represented the student's height in an illustration of the student, and the raffia was represented as a line.

After sketching in demonstration activity 1, students began to identify the position of the right angle, the position of the angle θ , the position of the hypotenuse, the front side, and the side. Different groups of students exhibit variations in their results in identifying the position of the angle θ and the front and side.

In this case, students are asked to explain the reasons. Some groups can explain their reasons. For those who cannot explain, other groups are allowed to present their reasons so that the understanding they gain

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helps others build the same understanding.

To obtain findings through teacher guidance, the teacher provides guided reinvention for groups that do not yet understand the reasons for the position of the corners θ and the position of the front and side sides as in the following conversation.

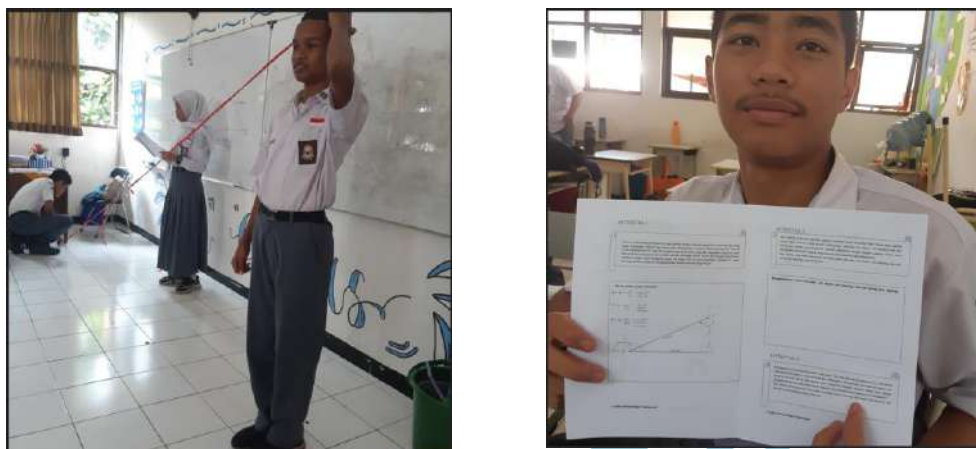


Figure 1: Activity 1 "Stretching a Raffia from the Base of the Table to the Top of the Student's Head"

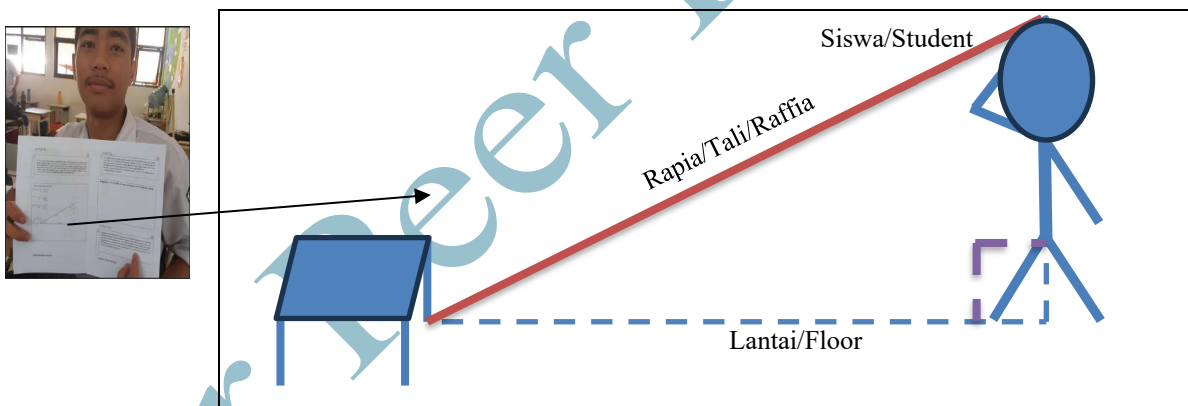


Figure 2: Example of Student Sketches in Activity 1

- Teacher : "Okay, S-1, please explain what you drew."
- S-1 : "Okay, ma'am, if you look at the instructions in the worksheet, we are asked to practice what is written in activity 1, and then we share roles; S-2 ties the raffia raffia to the base of the bottom table, S-3 holds the end of the raffia that is the result of extending S-2's raffia, S-4 reads what is on the worksheet, and I make the sketch."
- Teacher : "Okay, have you considered whether your sketch will be a right triangle?"
- S-1 : "At first, I did not, Ma'am, but when you said, "Look, if you draw it, is a right angle formed?" I remembered."
- Teacher : "Okay, how do you determine the corner points?"
- S-1 : "The benchmarks are at the end of the table legs, Leg S-3, and head S-3, Ma'am."

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- Teacher : *"Why is that, S-1?"*
- S-1 : *"Yes, that is what it looks like when drawn, ma'am."*
- Teacher : *"Okay, the benchmarks that S-1 mentioned earlier make a move from the rapid raffia, right?"*
- S-1 : *"What do you mean, ma'am?"*
- Teacher : *"Mom, look, you represent the rapier raffia, the floor, and the height of S-2 with a line, right?"*
- S-1 : *"That is right, ma'am."*
- Teacher : *"Well, when you make that line, there is a phase where you turn; that is called a benchmark."*
- S-1 : *"Oh yes, that is right, ma'am, that is right."*
- Teacher : *"S-1, try to determine the right angle, slanted side, front side, and side in the sketch you have made!"*
- Student : *"This one is a right angle, this one is the hypotenuse, this one is the front side, and this one is the side side." (pointing to the picture).*
- Teacher : *"How do you determine a right angle, S-1?"*
- Student : *"There is a line like this and like this, ma'am." (pointing at the picture).*
- Teacher : *"Now, try to observe; what degree angle do the two lines form, or what angle is it called then?"*
- Student : *"Oh yes, ma'am, right angles."*
- Teacher : *"How do you determine the hypotenuse?"*
- Student : *"The side in front of the right angle, ma'am."*
- Teacher : *"How do you determine the front side?"*
- Student : *"This one, ma'am? Yes, the one in front of this corner (pointing to one of the corners of the picture)."*
- Teacher : *"Okay, how do you determine the side?"*
- Student : *"The one on the front side, ma'am."*

In this activity, students have determined the expected position even though it is not specific because it still depends on the position of the angle θ . This identification procedure enhances pupils' ability to determine the precise location of the angles and sides within a right triangle. Upon completing the task, the teacher and students jointly draw conclusions and engage in reflections regarding the process of determining the position of the right angle, the hypotenuse, and the angles θ that impact the identification of the front side and side of a right triangle.

Activity 2: Observation "Measurement of Students' Height and Shadow"

The aim of activity 2 is to enable students to utilize the definition of trigonometric ratios $\sin \theta$, $\cos \theta$, and $\tan \theta$ in practical situations by employing the notion of comparison in right triangles. As part of this exercise, students are required to transform issues involving comparison in right triangles into tasks that may be directly practiced in groups. The teacher instructs a single student from a group to face away from the lamp until a shadow forms in front of the said student. Moreover, the teacher instructs students to illustrate the encountered shape in this exercise.

A student in his/her group is assigned to calculate the height of the student whose height will be measured and the height of the shadow that occurs. This activity is carried out alternately so that all group members have their height and shadow height measured. The measurement data is shown in a tabular format on the student's designated worksheet. Figure 4 is an example of student work in representing it in table form.

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Furthermore, the teacher determines the angle θ as the angle formed between the shadow and the oblique line. In this exercise, the teacher instructs the group to calculate the ratio of the student's height to the shadow height using the available data. This task aims to help students recall the process of determining the value of trigonometric ratios by drawing the resulting shapes.

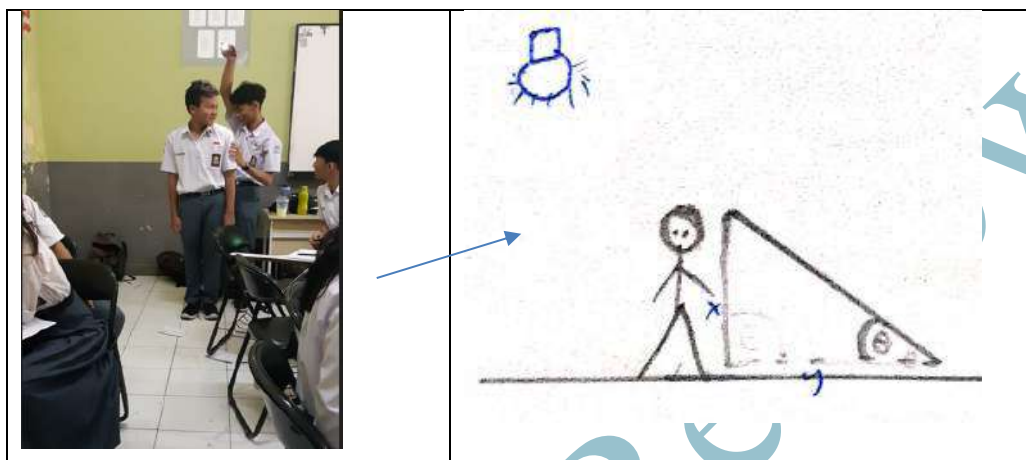


Figure 3: Activity 2 and Example of Student Sketches in Activity 2

	Kamu	Nama temanmu	Nama temanmu	Nama temanmu	Nama temanmu
	<u>Nabila</u>	<u>Bunga</u>	<u>Naufal M</u>	<u>Jahsyi</u>	<u>Nalvin</u>
Tinggi badan	155	153	173	169	161
Tinggi bayangan	165	163	183	179	171

	Your name	Your Classmate's name	Your Classmate's name	Your Classmate's name	Your Classmate's name
	<u>Nabila</u>	<u>Bunga</u>	<u>Naufal M</u>	<u>Jahsyi</u>	<u>Nalvin</u>
Height	155	153	173	169	161
Shadow Height	165	163	183	179	171

Figure 4: Example of a table of results measuring height and shadow in activity 2.

The objective of this project is for students to accurately calculate the trigonometric ratio values of $\sin \theta$, $\cos \theta$, and $\tan \theta$ in every triangle sketch they create (Figure 5). In each activity, the teacher monitors students through visits to each group and guides students in solving problems. Each student can use their way as an alternative, both formally and informally. The teacher facilitates students in revisiting the knowledge they have acquired about determining trigonometric ratio values by providing scaffolding.

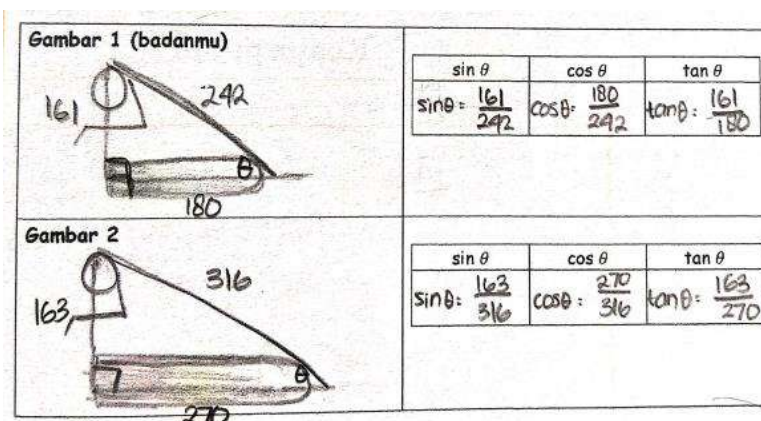


Figure 5: Example of Sketch and Application of Trigonometric Ratios Values in Activity 2

One of the students (S-5), as a representative of one of the group members, participated in presenting activity two and explained how the values of $\sin \theta$, $\cos \theta$, and $\tan \theta$ were obtained. After sketching the experiment, this group identified the right angle to determine the other positions. The following is a dialogue explaining the concept of trigonometric ratios described by the students.

- Teacher : "Okay, S-5. Explain how your group determines each trigonometric ratio value requested in the table."
- S-5 : "Okay, Ma'am, first we sketch according to our height in the group, then, from the sketch formed, we first determine the right angle, draw the symbol on the sketch, then determine the angle θ"
- Teacher : "Okay, do you mean identifying the position of a right angle?"
- S-5 : "Yes, ma'am."
- Teacher : "What do you mean by determining the angle?"
- S-5 : "So that we can determine the front and side, ma'am."
- Teacher : "Okay, what is the next process?"
- S-5 : "Earlier, after determining the angle position θ , we then wrote down each of our heights according to what was measured earlier, and wrote down the height of the shadow."
- Teacher : "Okay, how do you determine the hypotenuse?"
- S-5 : "Using the Pythagorean theorem, ma'am, with the help of a calculator."
- Teacher : "Okay, how do you calculate $\sin \theta$, $\cos \theta$, and $\tan \theta$?"
- S-5 : "Well, because the value of $\sin \theta$ is the front per hypotenuse, look at the picture (while pointing to picture 1 in UBA 2 Activity 2); the front of the angle θ is the front side, which is 161, and the hypotenuse is 242 so the $\sin \theta$ is 161 per 242."
- Teacher : "How to calculate $\cos \theta$ and $\tan \theta$?"
- S-5 : "Same here, ma'am, just a different position according to what was learned previously: $\cos \theta$ means side per hypotenuse, and $\tan \theta$ means front per side."

The conversation above helped all students understand the concept of trigonometric ratios correctly. At the end of the activity, one student reflected on the learning that day and concluded how to apply trigonometric ratios correctly.

Activity 3: Experiment with "Making Three Right Triangles from Paper"

The teaching aim of exercise 3 is to enable students to comprehend the correlation between the notion of

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resemblance and trigonometric ratios. The students are divided into multiple groups. Each group comprises from 4 to 5 students.

In this activity, the teacher asks each group to prepare the necessary tools, scissors, paper, rulers, and stationery through student worksheets. Furthermore, each group makes a picture through the experiment of cutting rectangular paper so that two equal and similar right triangles are obtained. Students can determine the similarity in three equal and similar right triangles. The teacher guides students in finding the equal and similar sides of three equal and similar right triangles.

Students use several alternative methods to find the concept of similarity, namely diagrams and teaching aids. Examples of activities that occur as in Figure 6 and 7. The results of the presentation in one group are the results of group discussions using diagrams, while the other group uses concrete teaching aid models made by the group. The results of both alternatives in each group find a way to build the concept of similarity and trigonometric ratios in right triangles.

Students are guided to find formal patterns of similarity in right triangles through guided reinvention through directed questions. Students are guided to find a formal way to use Geometry theorems by comparing three similar right triangles. To facilitate a gradual comprehension of the notion of similarity, participants are encouraged to identify the shortest and longest sides of each right triangle under comparison.

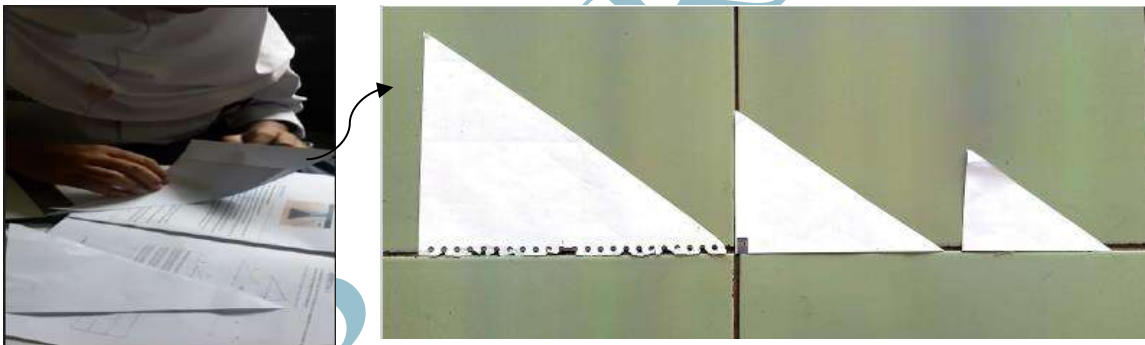


Figure 6. Activities and Examples of Student Responses using Teaching Aids in Activity 3.

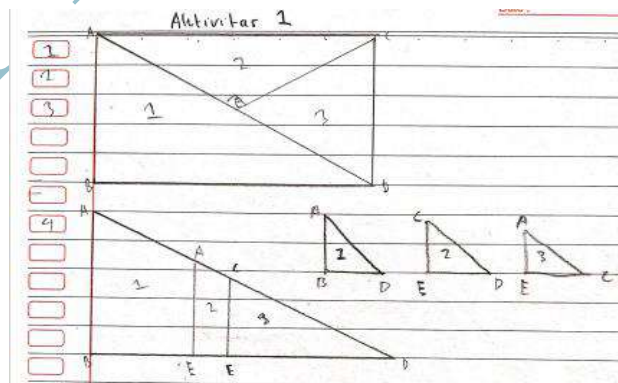


Figure 7. Example of Student Responses using Diagrams in Activity 3.

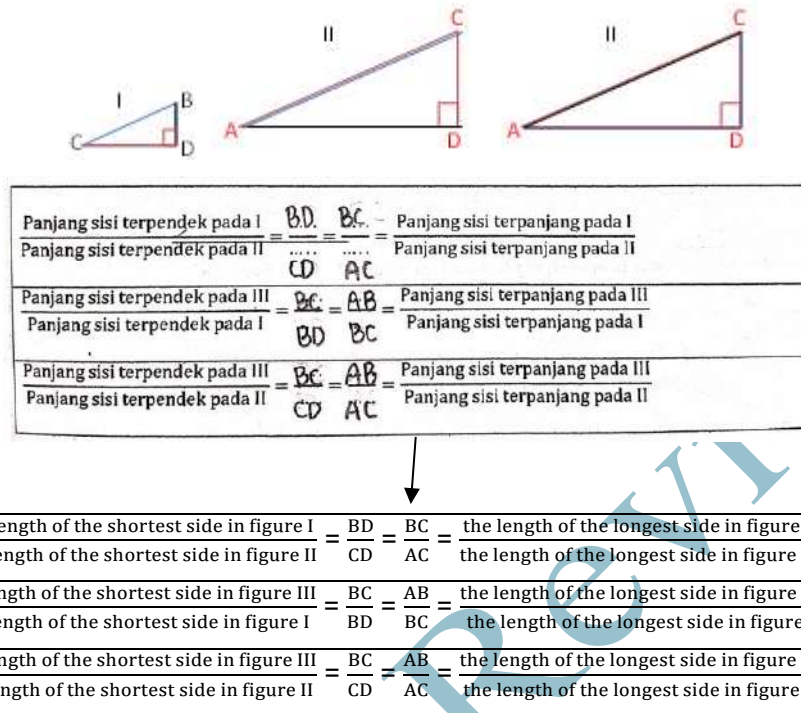


Figure 8. Example of Student Responses in Using the Concept of Comparison in Activity 3.

The teacher directs students in determining the location of the shortest and longest sides of each triangle, enabling them to articulate the resulting comparative thinking patterns. One example of a group response is presented in Figure 8.

In this exercise, the teacher presents questions to facilitate students' comprehension of the analogy between the notion of similarity and trigonometric ratios. Figure 9 displays the provided questions. In this inquiry, the teacher presents contextual inquiries formatted like a garbage can including a logo. The logo is situated on a base of an equilateral triangle of 6 centimetres in side length. The teacher asks students to calculate the logo's height, sketch the triangle formed, and explain the concept used in solving the given problem.



The trash can logo featured below is designed as an equilateral triangle. The dimension of the side measures 6 centimeters. What is the estimated vertical dimension of the logo? Begin by creating a preliminary outline of the triangle. Which concept are you employing? Are there any other concepts available apart from the one you have employed? What is the procedure?

Logo tempat sampah berikut berbentuk segitiga sama sisi dengan panjang sisi 6 cm. Berapa kira-kira tinggi logo tersebut? Buatlah sketsa segitiga terlebih dahulu. Konsep apa kah yang kamu gunakan? Adakah alternatif konsep selain yang sudah kamu gunakan? Bagaimana caranya?

Figure 9. Example of Questions on the Relationship between the Concept of Similarity and Trigonometric Ratios in Activity 3.

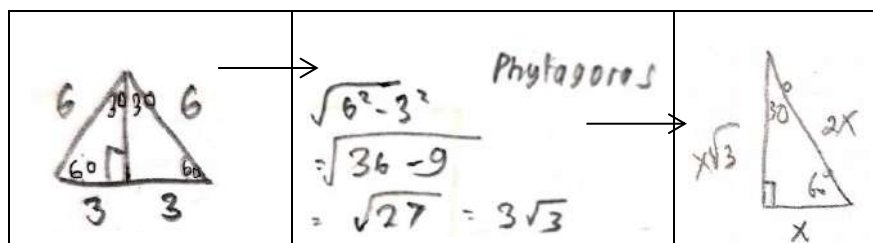


Figure 10. Example of Student Responses to Contextual Questions in Activity 3.

In Figure 10, one group sketches the shape formed and then calculates the height of the logo using the Pythagorean Theorem. They then use the concept of similarity to check the results they obtained.

At the conclusion of the activity, the teacher and students synthesize the idea under study concerning the correlation between similarity and trigonometric ratios.

Activity 4: Observation of "Calendar Image in the form of an Equilateral Triangular Prism"

The learning objective of activity 4 is that students can determine the magnitude of the trigonometric ratio at particular angles (0° , 30° , 45° , 60° , and 90°). For this exercise, the teacher organizes students into groups of 4-5. Using the student worksheet, the teacher instructs each group of students to visually analyze a calendar image as an equilateral triangular prism, as depicted in Figure 11. Subsequently, the teacher directs the students to examine the face of the calendar in order to derive an equilateral triangle. Students are instructed to construct a perfectly symmetrical triangle measuring 2 units in length and annotate each corner. Subsequently, the teacher instructs the students to draw a vertical line from one of the highest points of the triangle to the center of the side directly in front of the triangle, resulting in two equal sections of the triangle.

The students are instructed to redraw a portion of the triangular again, note the length of each side, and record the known angles. Utilising their acquired knowledge, students employ the Pythagorean Theorem to determine the length of the side that is not known. Subsequently, students are required to ascertain the numerical values of the trigonometric ratios $\sin \theta$, $\cos \theta$, and $\tan \theta$ for the particular angles 30° and 60° . Students compile the findings into a tabular format. At the conclusion of the question, the teacher makes an assumption about the length of the shortest side of the triangle, denoted as x . Students are then tasked with determining the lengths of the remaining sides while still utilizing the variable x .

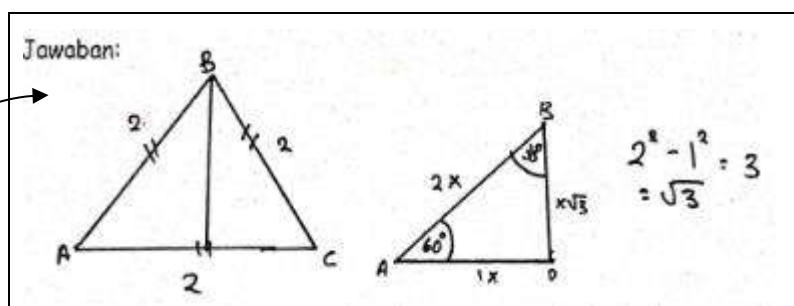


Figure 11. Calendar Image and Example of Student Responses in Making Sketches in Activity 4

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By employing guided innovation with variables x , students are direct to ascertain the comparison of the lengths of the sides of the triangle they have constructed. The student effort in activity 4 of creating a calendar sketch from the front is exemplified in Figure 11. Figure 12 illustrates student replies in activity 4 when answering the problem of calculating the value of trigonometric ratios at particular angles of 30° and 60° in a right triangle.

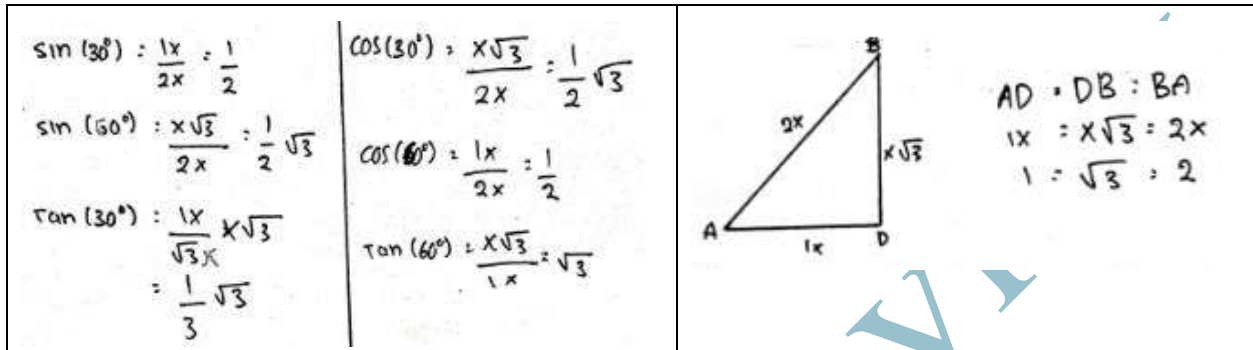


Figure 12. Example of Student Responses regarding memorable and unique 30° angles 60° in Activity 4

In the student worksheet activity 4, the teacher also guides students to determine the magnitude of the trigonometric ratio values at the particular angles of 45° in a right triangle. In this case, students are asked to draw an isosceles right triangle with each side measuring 1 unit. After that, students label each angle. Using the knowledge they have built, students determine the hypotenuse length using the Pythagorean Theorem. After that, students determine the trigonometric ratio values $\sin \theta$, $\cos \theta$, and $\tan \theta$ for the particular angle 45° . Figure 13 shows one of the student responses in their group.

Jawaban:

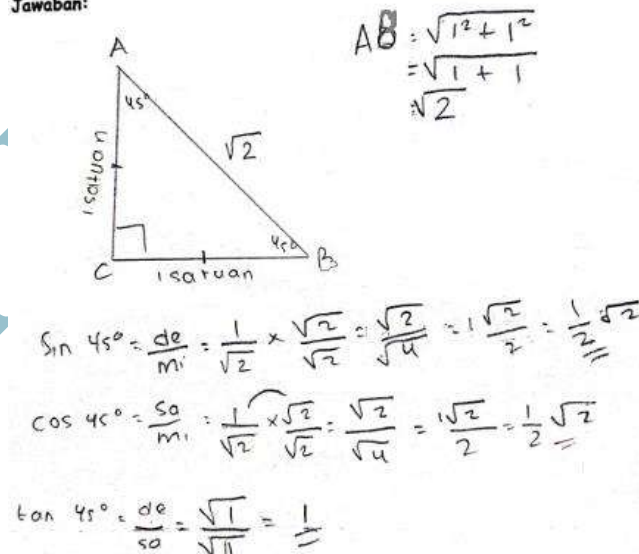


Figure 13. Example of Student Responses Regarding Particular angles 45° in Activity 4

In the initial stage, students sketch an isosceles triangle with a length of 1 unit. Students mark one line on each side that has the same length. In this case, students can already represent according to the question's

intent. Furthermore, students are accustomed to representing the information obtained in a sketch. Figure 13 displays students' labels indicating the angle size of 45 degrees for all angles except the right angle. Subsequently, the image illustrates the process by which students get the hypotenuse value by the use of the Pythagorean Theorem. By the conclusion of the exercise, students ascertain the numerical value of the trigonometric ratio at the particular angles of 45 degrees and 0 degrees in a right triangle. Lastly, students display the outcomes for numerous specific angles in a tabular format, as depicted in Figure 13.

	Rumus	30°	45°	60°
sin	$\sin \theta = \frac{\text{sisl depan}}{\text{sisl miring}}$	$\frac{1}{2}$	$\frac{1}{2} \sqrt{2}$	$\frac{1}{2} \sqrt{3}$
cos	$\cos \theta = \frac{\text{sisl samping}}{\text{sisl miring}}$	$\frac{1}{2} \sqrt{3}$	$\frac{1}{2} \sqrt{2}$	$\frac{1}{2}$
tan	$\tan \theta = \frac{\text{sisl depan}}{\text{sisl samping}}$	$\frac{1}{3} \sqrt{3}$	1	$\sqrt{3}$

Figure 14. Example of Student Responses Represented in a Table in Activity 4

By the conclusion of the exercise, the teacher and students reached a general conclusion on the determination of the value of trigonometric ratios at particular angles.

Activity 5: Observation of "Bed"

The purpose of exercise 5 is to enable students to use trigonometric ratios in order to ascertain the length of dimensions of unknown sides in a right triangle. For this exercise, the teacher instructs students to be organized into many groups. Each group comprises from 4 to 5 students. The instructor distributes a worksheet to the students that includes instructions for action 5. Each group scrutinizes the image (refer to Figure 15). Figure 16 depicts a triangular figure encompassing the stairs, bed, and floor. Students are instructed to perform the horizontal mathematization method by creating a diagram that represents the triangle produced by the three elements.



Figure 15. Activities and Questions in Activity 5

Furthermore, students are asked to identify whether the shape formed is a right triangle. The teacher

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determines the angle size θ at one corner of the triangle and the distance from the bed to the stairs. Students are asked to represent the known data in a sketch, and the teacher instructs the students to determine the displacement between the floor and the terminus of the stairs that constitute a right triangle by employing the tangent ratio.

From activity 5, the findings of each group are poured into a sketch, as shown in Figure 16.

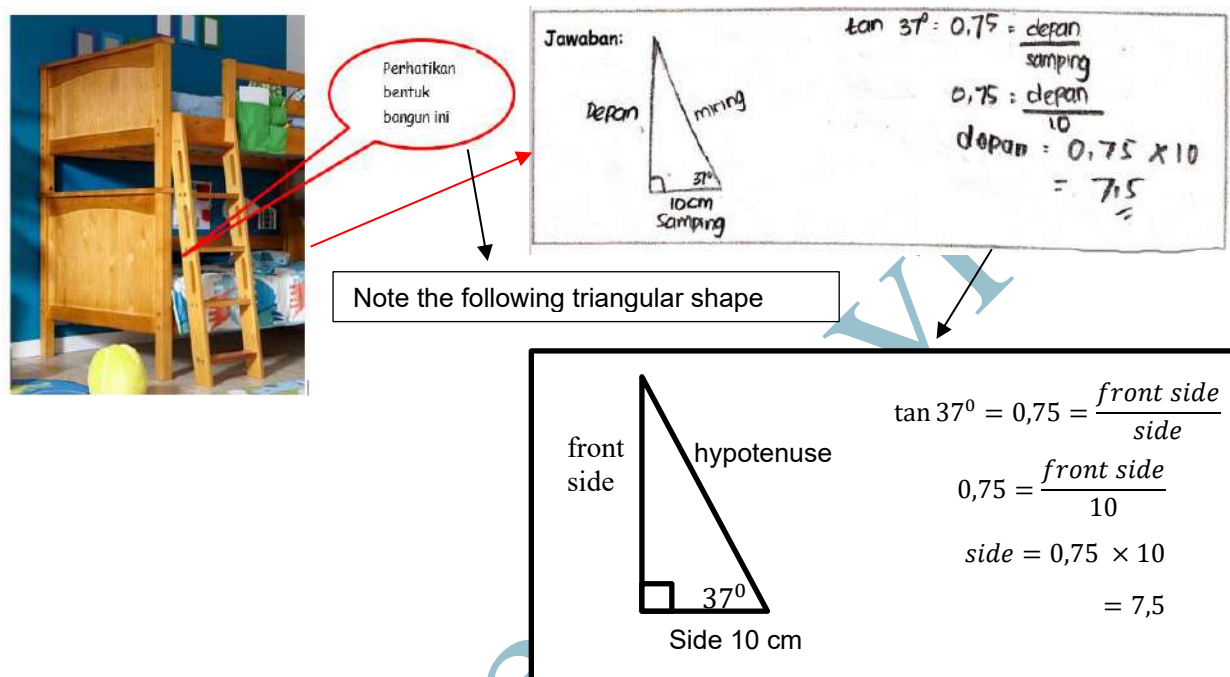


Figure 16. Questions and Student Responses Representing Data into Sketches in Activity 5

Through teacher guidance, students pour out their understanding by sketching a right triangle between the bed, floor, and stairs. At the beginning of the worksheet, students are given questions to identify whether the sketch formed is a right triangle. Students are asked to give a right angle sign if the triangle formed is a right triangle. Furthermore, the teacher provides information through the worksheet that the angle formed between the stairs and the floor is 37° and the distance from the bed to the stairs is 10 cm. Students are asked to calculate the distance from the floor to the end of the stairs that form a right triangle using the tangent ratio.

Following the completion of the given problems by all groups, one group will deliver the outcomes of their group discussion to the entire class. When employing trigonometric ratios to calculate the length of the unknown side of a right triangle, students articulate their understanding of the concept. At the conclusion of the activity, the teacher poses questions to assess the extent to which student's knowledge has been developed, as shown in the dialogue below.

Teacher : "Okay, how do you find the length of the unknown side in a right triangle?"

S-6 : "First, draw a sketch formed from the existing problem, then determine what information is contained in the problem, whether the value of the front side, hypotenuse, or side, even whether the angle value is also known. Next, from the information we get, we see

what principles fulfill the information collected so that we know the principle of trigonometric ratios that we use."

Teacher : "Can you give an example?"

S-6 : "For example, based on the bed problem earlier, after we sketch a right triangle, we obtain information that the angle formed between the stairs and the floor is 37° and the distance from the bed to the stairs is 10 cm. That means we label the angle as large 37° on the sketch and label 10 cm on the sketch for the side between the bed and the stairs. Then, we refer to the sketch to determine the information we obtain, namely the side, the size of the angle, and the tangent value 37° . This information means we use the trigonometric ratios of tangent, which is the comparison between the value of the front side and the value of the side.

Activity 6: Observation of "Picture of a Skier Descending an Iceberg"

The aim of task 6 is to enable students to accurately solve contextual situations involving right triangles by applying trigonometric ratios. This activity divides students into several groups consisting of 4-5 students. The teacher provides student worksheets as learning materials for activity 6. Each group observes the images presented (see Figure 17).



Figure 17. Questions in Activity 6

The student worksheet in this activity presents open-ended questions. Each group or student has the opportunity to have a different value. This activity encourages students to freely determine the angle's value, the opposite side's value, and the hypotenuse. This question is taken from a Geometry book by Larson (Larson et al., 2007). In this question, students are asked to observe a picture of a skier going down an iceberg. Through the worksheet, the teacher asks students to determine the angle size according to their wishes. In addition, students determine the height of the iceberg themselves. Furthermore, from the data and values they obtain, students determine the track length traveled by the iceberg skier. The teacher allows students to ask the teacher if they need a trigonometric ratio value from a particular angle value.

In activity 6, students express their ability to think logically through questions. The teacher guides students in determining values so that they can think logically. Referring to the values they present, students must determine for themselves the principle of trigonometric ratio values that they must use to resolve the issue. Figure 18 shows an example of student responses in activity 6. The results of the students' responses show that the student assumes the size of the angle requested in the question with the value 75° . The height of the iceberg is 90 units. From the value given by the student, he calculates the length of the track traveled by the skier using the comparison value $\sin \theta$ so that the desired track length value is obtained.

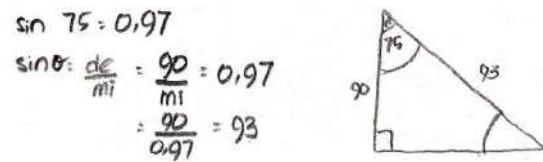


Figure 18. Example of Student Responses to Activity 6

From several findings from each group, one group presents the results of their discussion and simultaneously concludes the learning outcomes.

DISCUSSION

Learning activities using the Realistic Mathematics Education approach regarding trigonometric ratios followed by students consist of six activities, namely raffia stretching activities, student height measurement activities, activities to make three right triangles from paper, activities to observe calendar images, activities to observe bed images, and activities to observe skier images. Each activity has its purpose and is interrelated and continuous. The goals of each action are as outlined below: Firstly, stretching the raffia allows students to determine the location of the angles and sides of a right triangle. Secondly, measuring the height of students and their shadows allows students to apply the concept of trigonometric ratios. 3) Creating a teaching tool including three right triangles is intended to facilitate students' comprehension of the correlation between the notion of resemblance and trigonometric ratios. 4) The purpose of observing the calendar image is to permit students to ascertain the numerical value of trigonometric ratios at specific angles. The objective of viewing the bed image is to facilitate students in applying trigonometric ratios to ascertain the length of the desired side in a right triangle. Similarly, observing the skier image seeks to empower students in solving contextual difficulties by employing the concept of trigonometric ratios. Every worksheet provided incorporates the specific features of the Realistic Mathematics Education pedagogy in relation to trigonometric ratios. These findings corroborate Subekti's assertion that student worksheets will enhance the learning process (Subekti & Prahmana, 2021).

The implementation of the Realistic Mathematics Education approach in different learning process activities can facilitate students' comprehension of mathematical skills and enable teachers to effectively assist students towards attaining learning objectives (Ekowati & Nenohai, 2017). The reason for this is that the mathematics learning process, which follows the Realistic Mathematics Education approach, leverages the surrounding world to encourage students to generate ideas and allow them to develop their knowledge (Nirawati et al., 2021).

According to research conducted by Sukendra (2020), a sequence of learning process activities utilizing the RME approach significantly facilitates students' comprehension of the idea of trigonometric ratios (Sukendra, 2020). Thus, the deliberate learning trajectory promotes students to construct their knowledge, enhance their current skills, and effectively address challenges across many subject areas (Prahmana et al., 2017; Nursyahidah et al., 2020;; Wijaya et al., 2021; Kuncoro et al., 2023).

CONCLUSIONS

The objective of this work is to develop a learning trajectory for trigonometric ratios by utilizing the RME approach. The findings of this study suggest that implementing the RME approach in designing a learning trajectory for trigonometric ratios can enhance students' mathematical proficiency in comprehending the concepts of trigonometric ratios. The learning trajectory designed using the RME approach consists of six

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activities. Through a series of continuous and ongoing activities in the six activities, students find it easier to build their understanding in understanding the concept of trigonometric ratios. Students build their understanding starting from their ability to identify angle and side positions, apply the definition of trigonometric ratios to contextual problems, understand the relationship between similarity and trigonometric ratios, determine the magnitude of trigonometric ratios values at particular angles, apply trigonometric ratios, and solve contextual problems using trigonometric ratios.

This research is expected to encourage other researchers to design learning trajectories using the same topic but different approaches. The researcher hopes that the results of this design can contribute to the researchers' contribution to building a generation according to the nation's expectations.

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2. Bukti Konfirmasi Review dan Hasil Review Pertama (16 Februari 2025)



Hello colleagues;

We are interested in your submission, however to be publishable with us the paper **needs to undergo some revisions**, specified in the attached reviews. Do not forget to blur the faces of students in their photos. Together with resubmission, send a filled up Matrix of corrections.

Looking forward to it.

Best
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Chief Editor

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Learning Trajectory of Trigonometric Ratios Applying Realistic Mathematics Education

Dini Andiani ^{1,2}, Darhim ¹, Elah Nurlaelah ¹

¹ Department of Mathematics Education, Universitas Pendidikan Indonesia,
Bandung, Indonesia, ² Department of Mathematics, Universitas Bale
Bandung, Indonesia

diniandiani@upi.edu, diniandiani367@gmail.com,

darhim0355@gmail.com

Abstract: Trigonometry material is still considered complex material at the secondary level. It is necessary to design an appropriate trigonometric ratios learning trajectory so that students can quickly build the concept of trigonometric ratios. This study aims to design a learning trajectory for trigonometric ratio material through Realistic Mathematics Education (RME). This study uses a research design involving 10th-grade high school students in Bandung City in the 2023-2024 academic year. The research data was taken through data collection using written test instruments, student worksheets, observations, interviews, and documentation. The results of the study showed that the trigonometric ratios learning trajectory consisted of raffia stretching activities from the table base to the student's head, measuring the student's height and shadow, experimenting with making three right triangles from paper, observing a picture of a calendar in the form of an equilateral triangular prism, observing a bed or bed, and observing a picture of a skier going down an iceberg. Students can quickly build the concept of trigonometric ratios from several activities provided. In addition, various learning flows from various activities make it easier for students to build knowledge about trigonometric ratios and solve problems presented contextually.

Keywords: Learning Trajectory, Trigonometric Ratios, Realistic Mathematics Education

INTRODUCTION

Trigonometry material is vital in everyday applications such as statistics, surveying, architecture, and other engineering fields (Weber, 2005). Building knowledge about trigonometry takes work (Blackett & Tall, 1991; Demir et al., 2012; Kendal & Stacey, 1997; Ngu & Phan, 2020; Sugianto et al., 2023). Students need to learn several things related to one concept and another to understand the concept of trigonometry. Kamber's research (2017) stated that students struggle to understand trigonometry through the unit circle

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(Kamber & Takaci, 2017). Students need help distinguishing trigonometry through right triangles, the unit circle, or the relationship between the two (Maknun et al., 2019). Students fail to understand the concept of trigonometric ratios (Weber, 2005) even though trigonometric ratios are essential in understanding trigonometry (Nordlander, 2022).

Referring to the Independent Curriculum implemented in Indonesia, trigonometric ratio is a grade 10 material or element which is phase E. Learning outcomes (CP) in phase E are that students involve trigonometric ratios and their applications to solve problems involving right triangles (Badan et al., 2022).

Given the outlined challenges in acquiring knowledge about trigonometric ratios, it is imperative to develop a learning trajectory that facilitates the development of students' understanding in this area. In accordance with (Simon, 1993) definition of hypothetical learning trajectory (HLT), Clements (2011) utilizes the notion of learning trajectory to characterize students' cognitive processes and acquisition of knowledge on a specific mathematical subject. It also encompasses the sequence of assumptions that guide their thinking as they progress through a set of learning activities aimed at generating the intended mental processes or actions. According to Clements (2011), this cognitive process guides students in developing a certain degree of thinking in order to attain specified learning objectives for the subject matter (Clements, 2011).

The Realistic Mathematics Education (RME) approach is strategically developed to effectively support the achievement of the intended learning trajectory. The instructional approach known as Realistic Mathematics Education (RME) is based on the philosophy of Hans Freudenthal, who views mathematics as a creative endeavor of humanity (Freudenthal, 1973; Gravemeijer & Terwel, 2000; Treffers, 1987; Van den Heuvel-Panhuizen & Drijvers, 2020). Realistic Mathematics Education enables students to engage in the learning process within the context of real-life situations and their everyday experiences (Yuanita et al., 2018). A study conducted by Aiyub also shown that learning designs that incorporate real or contextual issue designs can enhance students' algebraic thinking (Aiyub et al., 2024).

Students build understanding through informal processes. Students observe, see, and understand the environment in which they are located (Meika, 2018). Realistic Mathematics Education encompasses three key concepts: directed reinvention and progressive mathematical processes, didactic phenomenology, and model construct development (Gravemeijer, 1994). Within the teaching and learning process, these three concepts are further developed into six distinct features of RME: 1) The principle of activity, 2) The principle of reality, 3) The principle of level, 4) The principle of intertwinement, 5) The principle of interactivity, and 6) The principle of guidance (Panhuizen & Drijvers, 2014).

Under the title "Realistic Mathematics Education" in the Encyclopedia of Mathematics Education, Panhuijen elucidates the six concepts. Under the RME approach, the activity principle entails that students assume an active role in the learning process. This assertion is based on Freudenthal's theoretical understanding of mathematics as a human endeavor. The reality principle posits that students have the ability to use mathematics in order to address practical issues, and the process of learning should commence with tangible problems that can be expressed mathematically. The level principle posits that students progress through several levels of comprehension in order to grasp the connection between ideas and procedures in mathematics. The interconnection refers to the phenomenon where the students, while acquiring mathematical information, are able to employ a range of tools and methods to construct a comprehensive process of integrating different concepts. The interaction principle posits that the process of learning mathematics is not only an individual endeavor, but rather a social and reflective one, therefore facilitating a deeper comprehension of the subject. The guidance principle in mathematics education entails the concept of "guided reinvention," as proposed by Freudenthal. Under these circumstances, the learning

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process should motivate students to adapt in order to enhance their comprehension. It is imperative that the learning program aligns with a cohesive long-term learning trajectory (Panhuizen & Drijvers, 2014).

Method

The employed research methodology is design research as described by Gravemeijer and Van Eerde in 2009 (Gravemeijer & Van Eerde, 2009). This research methodology is focused on local instruction theory (LIT) to enhance the quality of learning by fostering effective collaboration between researchers and teachers (Gravemeijer & Van Eerde, 2009). Design research will enable the attainment of the goals of the research paradigm in the development of sequential activities and the enhancement of learning quality. The primary objective of this work is to develop a comparative trigonometry learning trajectory by utilizing the RME approach. The study was carried out in two cyclical phases, each following the design research stages outlined by Gravemeijer (2004): 1) preparatory design, 2) teaching experiment, and 3) retrospective analysis. The outcomes of the initial cycle activities that have been executed and the subsequent analytical review have contributed valuable insights (Gravemeijer, 2004).

During the first stage of the design process, the researcher personally observed the school and conducted direct interviews with the teachers and the students. Furthermore, the researcher gathered, perused, and comprehended several source materials pertaining to the challenges faced by students and teachers in grasping the notion of trigonometric ratios. Furthermore, the researcher examined many sources that elucidated the learning process activities based on the Realistic Mathematics Education (RME) methodology. A sequence of researcher actions served as a point of reference for the researcher in developing the Hypothetical Learning Trajectory (HLT). The Hypothetical Learning Trajectory (HLT) comprises three elements: learning objectives that will be focused on during the learning process, learning activities, and hypotheses that will forecast the cultivation of students' cognitive abilities in the learning process (Simon, 1993). The learning process hypothesis serves as the fundamental basis for constructing learning activities. This document is a compilation of learning activities designed by the researcher.

Learning objectives	Learning Activities	Student Thinking Conjecture
Students can identify the position of the right angle, opposite side, and hypotenuse in a right triangle.	Stretch the raffia from the table's base to the top of the head to form a right triangle.	A group of students can model the activity into a right triangle. One of the students was able to sketch the right triangle that was formed. Students can identify the position of right angles, front sides, hypotenuses, and side sides in sketches formed with various answers without knowing the reasons. Students need to be corrected in identifying the position of right angles.
Students can apply the definition of trigonometric ratios $\sin \theta$, $\cos \theta$, and $\tan \theta$ to	Measuring students' height and shadow	Students can draw sketches formed from the models presented. The definitions of trigonometric ratios $\tan \theta$, $\sin \theta$, and $\cos \theta$ can be used by students to the model of measuring the heights of both students and their shadows.

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Learning objectives	Learning Activities	Student Thinking Conjecture
contextual problems using the concept of ratios in right triangles.		Students can determine the position θ of different angles to obtain different trigonometric ratio values.
Students can understand the relationship between the concept of similarity and trigonometric ratios.	Experiment to make three right triangles from paper	Students can make three similar right triangles from paper. Students can make sketches from paper formed into three similar right triangles. Students can identify the shortest and longest sides of each triangle formed. Students can determine the ratio between the shortest side and the longest side of each right triangle. Students can relate the concept of similarity with trigonometric ratios.
Students can determine the value of trigonometric ratios at particular angles (0° , 30° , 45° , 60° , and 90°).	Observations on the calendar image in the form of an equilateral triangular prism.	Students can sketch a calendar model as an equilateral triangular prism. Students can construct a right triangle in which one of the angles corresponds to a specifically defined angle. Students can determine the value of trigonometric ratios at particular angles (0°, 30°, 45°, 60°, and 90°).
The application of trigonometric ratios enables students to ascertain the length of sides in a right triangle that are not known.	Bed or bed observation	Students can make sketches or drawings of bed models. Students can identify the position of right angles, oblique angles, side angles, and front side positions in sketches. Students can apply trigonometric ratios to determine the length of unknown sides in problems presented related to beds.
Students can solve contextual problems related to right triangles using trigonometric ratios.	Observation of the image of a skier descending an iceberg	Students can identify the position of the angles, opposite sides and hypotenuses in a right triangle formed as a sketch of a picture of a skier descending an iceberg. Students can apply the definition of trigonometric ratios to skier problems. Students can solve the problems given in the case of skiers.

Table 1: Hypothetical Learning Trajectory in Teaching Experiment 1

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Within cycle 1, the research participants consisted of 25 10th-grade students who attended a high school located in Bandung City. In cycle 2, the research participants consisted of 25 10th-grade students from a different high school within the same municipal area. The HLT Design was first implemented in teaching experiment cycle 1 to assess the efficacy of the prepared learning activity design in investigating students' cognitive patterns. Moreover, the results of teaching experiment 1 indicated an enhancement in HLT. A refined Hypothetical Learning Trajectory (HLT) was implemented in phase 2 of the teaching experiment. The data collecting protocols encompass classroom observations, videos documenting learning activities, student workbooks, and recordings of interviews conducted with students.

Phase 1 of the retrospective analysis involved the collection and analysis of all the data by the researcher. An analysis was conducted to evaluate the outcomes of including the learning trajectory in teaching experiment 1 with the original HLT design. Moreover, the researcher investigated and examined the learning process activities included to determine the level of students' comprehension of the idea of trigonometric ratios. An analysis was conducted on the outcomes of the HLT enhancements implemented in teaching experiment 2, by comparing the initial HLT with the execution of the learning process.

The researcher selected 6 participant subjects to conduct interviews. The six participants had various abilities; two had high, medium, and low abilities. In addition, interviews were also conducted with the mathematics teacher of trigonometric ratios at the school that taught the students.

Results

The learning trajectory design discussed in this paper is a sequence of student activities aimed at comprehending the subject of trigonometric ratios through the Realistic Mathematics Education approach. The trigonometric ratios learning trajectory comprises six learning activities: elongating a raffia from the table's base to the student's head, measuring the student's height and shadow, creating three right triangles from paper, visually examining a calendar image in the shape of an equilateral triangular prism, observing a bed or bed, and observing an image of a skier descending an iceberg.

Activity 1: Observation of "Stretching the Raffia from the Base of the Table to the Top of the Student's Head"

The primary aim of activity 1 is to enable students to accurately determine the location of the right angle, the opposing side, and the hypotenuse within a right triangle. During this exercise, the teacher instructs students to showcase their use of contextual equipment found in the classroom, specifically a raffia, a table base, and one of their peers. The students are segregated into multiple groups. Each group comprises from 4 to 5 students. A single student from each group grasps one end of the raffia, while the other end is fastened to the bottom of the table foundation.

Furthermore, the other students in the group observed the shape between the student's height, the floor, and the raffia. The observation process is shown in Figure 1. Furthermore, by describing the teaching materials, the teacher directs students to sketch the shapes in the demonstration.

Each group sketched their demonstration and represented the information in a picture or symbol. One of the sketch results is shown in Figure 2. Students sketched the table legs, the student's height, and the raffia stretched to form a right triangle. They represented the student's height in an illustration of the student, and the raffia was represented as a line.

After sketching in demonstration activity 1, students began to identify the position of the right angle, the position of the angle θ , the position of the hypotenuse, the front side, and the side. Different groups of students exhibit variations in their results in identifying the position of the angle θ and the front and side.

In this case, students are asked to explain the reasons. Some groups can explain their reasons. For those who cannot explain, other groups are allowed to present their reasons so that the understanding they gain

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Commented [bc1]: What did the enhancement of HLT consisted in, precisely, please

helps others build the same understanding.

To obtain findings through teacher guidance, the teacher provides guided reinvention for groups that do not yet understand the reasons for the position of the corners θ and the position of the front and side sides as in the following conversation.

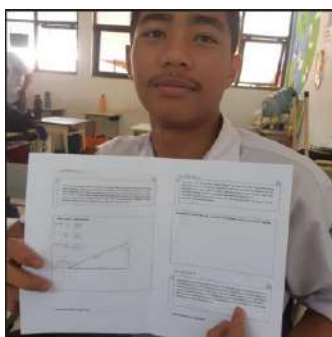


Figure 1: Activity 1 "Stretching a Raffia from the Base of the Table to the Top of the Student's Head"

Commented [bc2]: Faces of students have to be blurred

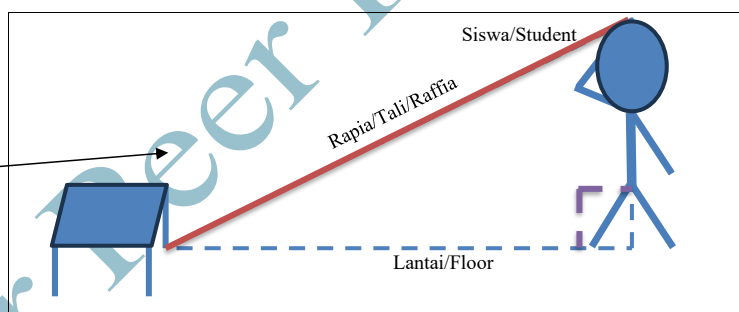
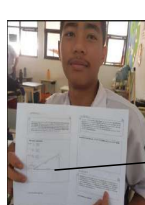


Figure 2: Example of Student Sketches in Activity 1

- Teacher : "Okay, S-1, please explain what you drew."
S-1 : "Okay, ma'am, if you look at the instructions in the worksheet, we are asked to practice what is written in activity 1, and then we share roles; S-2 ties the raffia raffia to the base of the bottom table, S-3 holds the end of the raffia that is the result of extending S-2's raffia, S-4 reads what is on the worksheet, and I make the sketch."
Teacher : "Okay, have you considered whether your sketch will be a right triangle?"
S-1 : "At first, I did not, Ma'am, but when you said, "Look, if you draw it, is a right angle formed?" I remembered."
Teacher : "Okay, how do you determine the corner points?"
S-1 : "The benchmarks are at the end of the table legs, Leg S-3, and head S-3, Ma'am."

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- Teacher : "Why is that, S-1?"
- S-1 : "Yes, that is what it looks like when drawn, ma'am."
- Teacher : "Okay, the benchmarks that S-1 mentioned earlier make a move from the rapid raffia, right?"
- S-1 : "What do you mean, ma'am?"
- Teacher : "Mom, look, you represent the rapier raffia, the floor, and the height of S-2 with a line, right?"
- S-1 : "That is right, ma'am."
- Teacher : "Well, when you make that line, there is a phase where you turn; that is called a benchmark."
- S-1 : "Oh yes, that is right, ma'am, that is right."
- Teacher : "S-1, try to determine the right angle, slanted side, front side, and side in the sketch you have made!"
- Student : "This one is a right angle, this one is the hypotenuse, this one is the front side, and this one is the side side." (pointing to the picture).
- Teacher : "How do you determine a right angle, S-1?"
- Student : "There is a line like this and like this, ma'am." (pointing at the picture).
- Teacher : "Now, try to observe; what degree angle do the two lines form, or what angle is it called then?"
- Student : "Oh yes, ma'am, right angles."
- Teacher : "How do you determine the hypotenuse?"
- Student : "The side in front of the right angle, ma'am."
- Teacher : "How do you determine the front side?"
- Student : "This one, ma'am? Yes, the one in front of this corner (pointing to one of the corners of the picture)".
- Teacher : "Okay, how do you determine the side?"
- Student : "The one on the front side, ma'am."

In this activity, students have determined the expected position even though it is not specific because it still depends on the position of the angle θ . This identification procedure enhances pupils' ability to determine the precise location of the angles and sides within a right triangle. Upon completing the task, the teacher and students jointly draw conclusions and engage in reflections regarding the process of determining the position of the right angle, the hypotenuse, and the angles θ that impact the identification of the front side and side of a right triangle.

Activity 2: Observation "Measurement of Students' Height and Shadow"

The aim of activity 2 is to enable students to utilize the definition of trigonometric ratios $\sin \theta$, $\cos \theta$, and $\tan \theta$ in practical situations by employing the notion of comparison in right triangles. As part of this exercise, students are required to transform issues involving comparison in right triangles into tasks that may be directly practiced in groups. The teacher instructs a single student from a group to face away from the lamp until a shadow forms in front of the said student. Moreover, the teacher instructs students to illustrate the encountered shape in this exercise.

A student in his/her group is assigned to calculate the height of the student whose height will be measured and the height of the shadow that occurs. This activity is carried out alternately so that all group members have their height and shadow height measured. The measurement data is shown in a tabular format on the student's designated worksheet. Figure 4 is an example of student work in representing it in table form.

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Furthermore, the teacher determines the angle θ as the angle formed between the shadow and the oblique line. In this exercise, the teacher instructs the group to calculate the ratio of the student's height to the shadow height using the available data. This task aims to help students recall the process of determining the value of trigonometric ratios by drawing the resulting shapes.

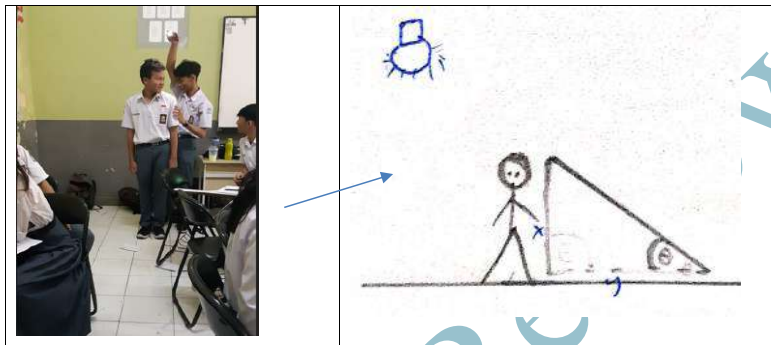


Figure 3: Activity 2 and Example of Student Sketches in Activity 2

	Kamu	Nama temanmu	Nama temanmu	Nama temanmu	Nama temanmu
	<u>Nabila</u>	<u>Bunga</u>	<u>Naufal M</u>	<u>Jahsyi</u>	<u>Nalvin</u>
Tinggi badan	155	153	173	169	161
Tinggi bayangan	165	163	183	179	171

	Your name	Your Classmate's name	Your Classmate's name	Your Classmate's name	Your Classmate's name
	<u>Nabila</u>	<u>Bunga</u>	<u>Naufal M</u>	<u>Jahsyi</u>	<u>Nalvin</u>
Height	155	153	173	169	161
Shadow Height	165	163	183	179	171

Figure 4: Example of a table of results measuring height and shadow in activity 2.

The objective of this project is for students to accurately calculate the trigonometric ratio values of $\sin \theta$, $\cos \theta$, and $\tan \theta$ in every triangle sketch they create (Figure 5). In each activity, the teacher monitors students through visits to each group and guides students in solving problems. Each student can use their way as an alternative, both formally and informally. The teacher facilitates students in revisiting the knowledge they have acquired about determining trigonometric ratio values by providing scaffolding.

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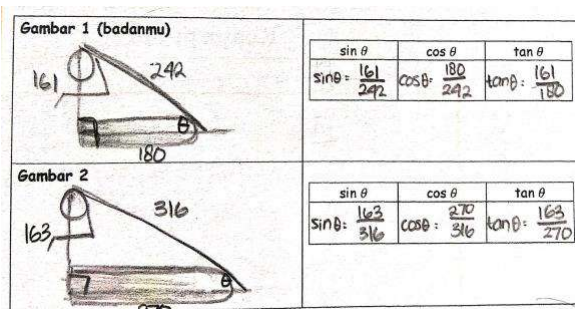


Figure 5: Example of Sketch and Application of Trigonometric Ratios Values in Activity 2

One of the students (S-5), as a representative of one of the group members, participated in presenting activity two and explained how the values of $\sin \theta$, $\cos \theta$, and $\tan \theta$ were obtained. After sketching the experiment, this group identified the right angle to determine the other positions. The following is a dialogue explaining the concept of trigonometric ratios described by the students.

Teacher : "Okay, S-5. Explain how your group determines each trigonometric ratio value requested in the table."

S-5 : "Okay, Ma'am, first we sketch according to our height in the group, then, from the sketch formed, we first determine the right angle, draw the symbol on the sketch, then determine the angle θ"

Teacher : "Okay, do you mean identifying the position of a right angle?"

S-5 : "Yes, ma'am."

Teacher : " θ What do you mean by determining the angle?"

S-5 : "So that we can determine the front and side, ma'am."

Teacher : "Okay, what is the next process?"

S-5 : "Earlier, after determining the angle position θ , we then wrote down each of our heights according to what was measured earlier, and wrote down the height of the shadow."

Teacher : "Okay, how do you determine the hypotenuse?"

S-5 : "Using the Pythagorean theorem, ma'am, with the help of a calculator."

Teacher : "Okay, how do you calculate $\sin \theta$, $\cos \theta$, and $\tan \theta$?"

S-5 : "Well, because the value of $\sin \theta$ is the front per hypotenuse, look at the picture (while pointing to picture 1 in UBA 2 Activity 2); the front of the angle θ is the front side, which is 161, and the hypotenuse is 242 so the $\sin \theta$ is 161 per 242."

Teacher : "How to calculate $\cos \theta$ and $\tan \theta$?"

S-5 : "Same here, ma'am, just a different position according to what was learned previously: $\cos \theta$ means side per hypotenuse, and $\tan \theta$ means front per side."

The conversation above helped all students understand the concept of trigonometric ratios correctly. At the end of the activity, one student reflected on the learning that day and concluded how to apply trigonometric ratios correctly.

Activity 3: Experiment with "Making Three Right Triangles from Paper"

The teaching aim of exercise 3 is to enable students to comprehend the correlation between the notion of

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resemblance and trigonometric ratios. The students are divided into multiple groups. Each group comprises from 4 to 5 students.

In this activity, the teacher asks each group to prepare the necessary tools, scissors, paper, rulers, and stationery through student worksheets. Furthermore, each group makes a picture through the experiment of cutting rectangular paper so that two equal and similar right triangles are obtained. Students can determine the similarity in three equal and similar right triangles. The teacher guides students in finding the equal and similar sides of three equal and similar right triangles.

Students use several alternative methods to find the concept of similarity, namely diagrams and teaching aids. Examples of activities that occur as in Figure 6 and 7. The results of the presentation in one group are the results of group discussions using diagrams, while the other group uses concrete teaching aid models made by the group. The results of both alternatives in each group find a way to build the concept of similarity and trigonometric ratios in right triangles.

Students are guided to find formal patterns of similarity in right triangles through guided reinvention through directed questions. Students are guided to find a formal way to use Geometry theorems by comparing three similar right triangles. To facilitate a gradual comprehension of the notion of similarity, participants are encouraged to identify the shortest and longest sides of each right triangle under comparison.

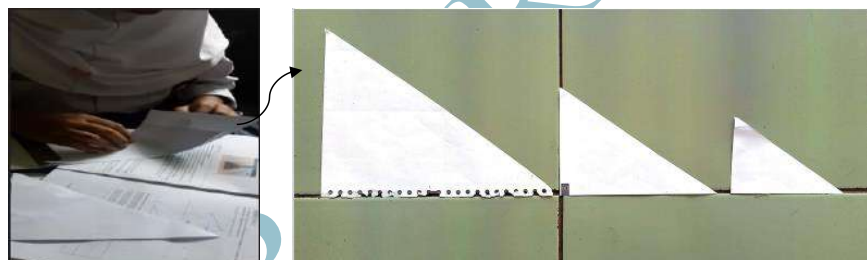


Figure 6. Activities and Examples of Student Responses using Teaching Aids in Activity 3.

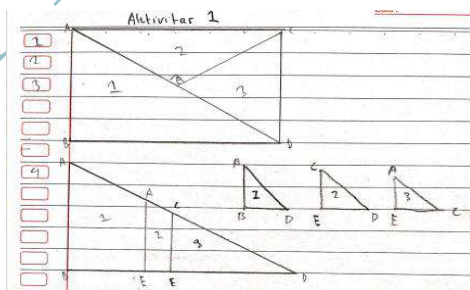


Figure 7. Example of Student Responses using Diagrams in Activity 3.

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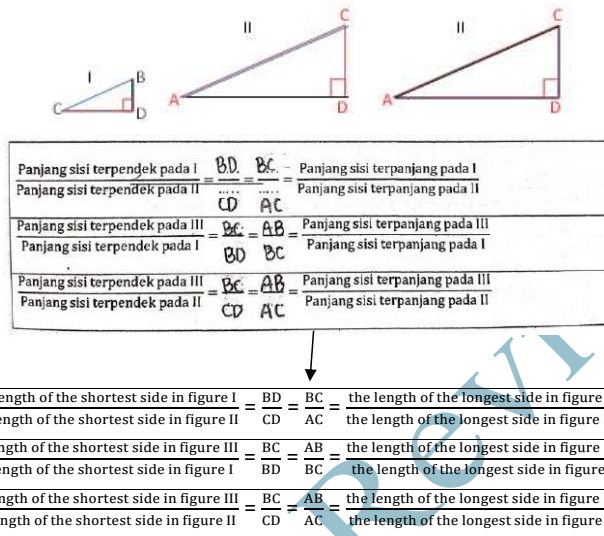


Figure 8. Example of Student Responses in Using the Concept of Comparison in Activity 3.

The teacher directs students in determining the location of the shortest and longest sides of each triangle, enabling them to articulate the resulting comparative thinking patterns. One example of a group response is presented in Figure 8.

In this exercise, the teacher presents questions to facilitate students' comprehension of the analogy between the notion of similarity and trigonometric ratios. Figure 9 displays the provided questions. In this inquiry, the teacher presents contextual inquiries formatted like a garbage can including a logo. The logo is situated on a base of an equilateral triangle of 6 centimetres in side length. The teacher asks students to calculate the logo's height, sketch the triangle formed, and explain the concept used in solving the given problem.



The trash can logo featured below is designed as an equilateral triangle. The dimension of the side measures 6 centimeters. What is the estimated vertical dimension of the logo? Begin by creating a preliminary outline of the triangle. Which concept are you employing? Are there any other concepts available apart from the one you have employed? What is the procedure?

Logo tempat sampah berikut berbentuk segitiga sama sisi dengan panjang sisi 6 cm. Berapa kira-kira tinggi logo tersebut? Buatlah sketsa segitiga terlebih dahulu. Konsep apa kah yang kamu gunakan? Adakah alternatif konsep selain yang sudah kamu gunakan? Bagaimana caranya?

Figure 9. Example of Questions on the Relationship between the Concept of Similarity and Trigonometric Ratios in Activity 3.

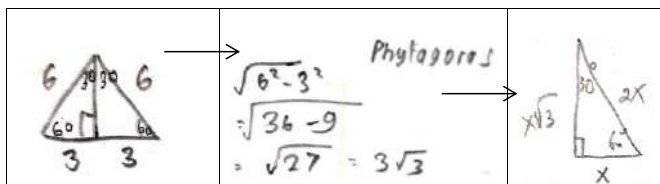


Figure 10. Example of Student Responses to Contextual Questions in Activity 3.

In Figure 10, one group sketches the shape formed and then calculates the height of the logo using the Pythagorean Theorem. They then use the concept of similarity to check the results they obtained.

At the conclusion of the activity, the teacher and students synthesize the idea under study concerning the correlation between similarity and trigonometric ratios.

Activity 4: Observation of "Calendar Image in the form of an Equilateral Triangular Prism"

The learning objective of activity 4 is that students can determine the magnitude of the trigonometric ratio at particular angles (0° , 30° , 45° , 60° , and 90°). For this exercise, the teacher organizes students into groups of 4-5. Using the student worksheet, the teacher instructs each group of students to visually analyze a calendar image as an equilateral triangular prism, as depicted in Figure 11. Subsequently, the teacher directs the students to examine the face of the calendar in order to derive an equilateral triangle. Students are instructed to construct a perfectly symmetrical triangle measuring 2 units in length and annotate each corner. Subsequently, the teacher instructs the students to draw a vertical line from one of the highest points of the triangle to the center of the side directly in front of the triangle, resulting in two equal sections of the triangle.

The students are instructed to redraw a portion of the triangular again, note the length of each side, and record the known angles. Utilising their acquired knowledge, students employ the Pythagorean Theorem to determine the length of the side that is not known. Subsequently, students are required to ascertain the numerical values of the trigonometric ratios $\sin \theta$, $\cos \theta$, and $\tan \theta$ for the particular angles 30° and 60° . Students compile the findings into a tabular format. At the conclusion of the question, the teacher makes an assumption about the length of the shortest side of the triangle, denoted as x . Students are then tasked with determining the lengths of the remaining sides while still utilizing the variable x .

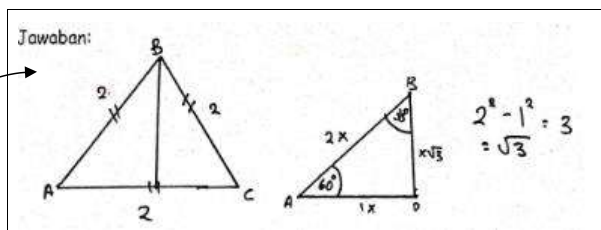


Figure 11. Calendar Image and Example of Student Responses in Making Sketches in Activity 4

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By employing guided innovation with variables x , students are direct to ascertain the comparison of the lengths of the sides of the triangle they have constructed. The student effort in activity 4 of creating a calendar sketch from the front is exemplified in Figure 11. Figure 12 illustrates student replies in activity 4 when answering the problem of calculating the value of trigonometric ratios at particular angles of 30° and 60° in a right triangle.

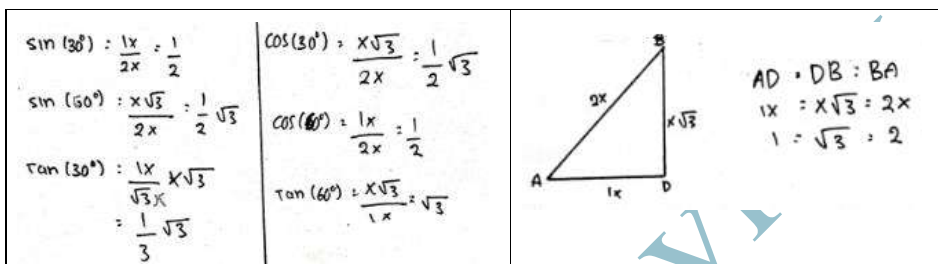


Figure 12. Example of Student Responses regarding memorable and unique 30° angles 60° in Activity 4

In the student worksheet activity 4, the teacher also guides students to determine the magnitude of the trigonometric ratio values at the particular angles of 45° in a right triangle. In this case, students are asked to draw an isosceles right triangle with each side measuring 1 unit. After that, students label each angle. Using the knowledge they have built, students determine the hypotenuse length using the Pythagorean Theorem. After that, students determine the trigonometric ratio values $\sin \theta$, $\cos \theta$, and $\tan \theta$ for the particular angle 45° . Figure 13 shows one of the student responses in their group.

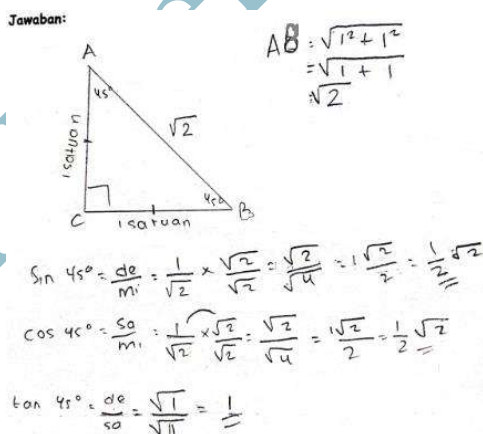


Figure 13. Example of Student Responses Regarding Particular angles 45° in Activity 4

In the initial stage, students sketch an isosceles triangle with a length of 1 unit. Students mark one line on each side that has the same length. In this case, students can already represent according to the question's

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intent. Furthermore, students are accustomed to representing the information obtained in a sketch. Figure 13 displays students' labels indicating the angle size of 45 degrees for all angles except the right angle. Subsequently, the image illustrates the process by which students get the hypotenuse value by the use of the Pythagorean Theorem. By the conclusion of the exercise, students ascertain the numerical value of the trigonometric ratio at the particular angles of 45 degrees and 0 degrees in a right triangle. Lastly, students display the outcomes for numerous specific angles in a tabular format, as depicted in Figure 13.

	Rumus	30°	45°	60°
sin	sin θ = $\frac{\text{sisi depan}}{\text{sisi miring}}$	$\frac{1}{2}$	$\frac{1}{2} \sqrt{2}$	$\frac{1}{2} \sqrt{3}$
cos	cos θ = $\frac{\text{sisi samping}}{\text{sisi miring}}$	$\frac{1}{2} \sqrt{3}$	$\frac{1}{2} \sqrt{2}$	$\frac{1}{2}$
tan	tan θ = $\frac{\text{sisi depan}}{\text{sisi samping}}$	$\frac{1}{3} \sqrt{3}$	1	$\sqrt{3}$

Figure 14. Example of Student Responses Represented in a Table in Activity 4

By the conclusion of the exercise, the teacher and students reached a general conclusion on the determination of the value of trigonometric ratios at particular angles.

Activity 5: Observation of "Bed"

The purpose of exercise 5 is to enable students to use trigonometric ratios in order to ascertain the length of dimensions of unknown sides in a right triangle. For this exercise, the teacher instructs students to be organized into many groups. Each group comprises from 4 to 5 students. The instructor distributes a worksheet to the students that includes instructions for action 5. Each group scrutinizes the image (refer to Figure 15). Figure 16 depicts a triangular figure encompassing the stairs, bed, and floor. Students are instructed to perform the horizontal mathematization method by creating a diagram that represents the triangle produced by the three elements.



Figure 15. Activities and Questions in Activity 5

Furthermore, students are asked to identify whether the shape formed is a right triangle. The teacher

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determines the angle size θ at one corner of the triangle and the distance from the bed to the stairs. Students are asked to represent the known data in a sketch, and the teacher instructs the students to determine the displacement between the floor and the terminus of the stairs that constitute a right triangle by employing the tangent ratio.

From activity 5, the findings of each group are poured into a sketch, as shown in Figure 16.

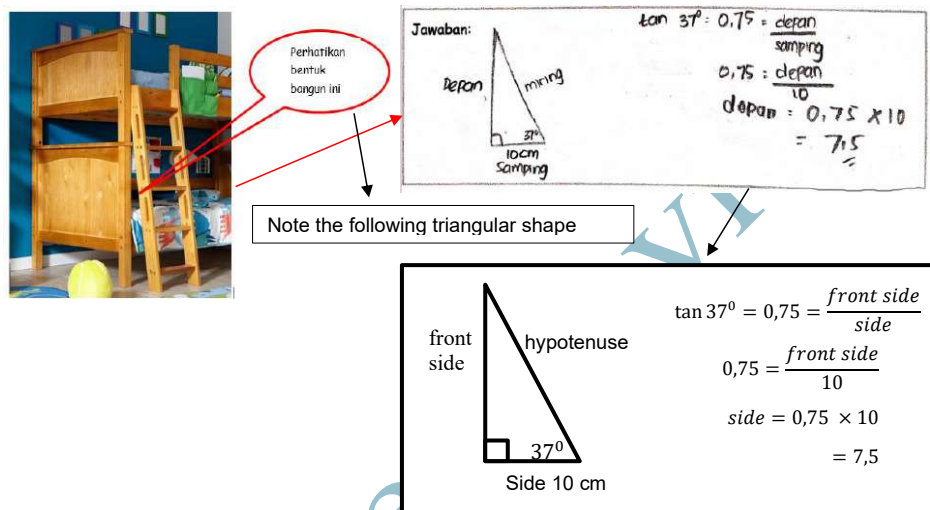


Figure 16. Questions and Student Responses Representing Data into Sketches in Activity 5

Through teacher guidance, students pour out their understanding by sketching a right triangle between the bed, floor, and stairs. At the beginning of the worksheet, students are given questions to identify whether the sketch formed is a right triangle. Students are asked to give a right angle sign if the triangle formed is a right triangle. Furthermore, the teacher provides information through the worksheet that the angle formed between the stairs and the floor is 37° and the distance from the bed to the stairs is 10 cm. Students are asked to calculate the distance from the floor to the end of the stairs that form a right triangle using the tangent ratio.

Following the completion of the given problems by all groups, one group will deliver the outcomes of their group discussion to the entire class. When employing trigonometric ratios to calculate the length of the unknown side of a right triangle, students articulate their understanding of the concept. At the conclusion of the activity, the teacher poses questions to assess the extent to which student's knowledge has been developed, as shown in the dialogue below.

- Teacher : "Okay, how do you find the length of the unknown side in a right triangle?"
- S-6 : "First, draw a sketch formed from the existing problem, then determine what information is contained in the problem, whether the value of the front side, hypotenuse, or side, even whether the angle value is also known. Next, from the information we get, we see

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what principles fulfill the information collected so that we know the principle of trigonometric ratios that we use."

Teacher : "Can you give an example?"

S-6 : "For example, based on the bed problem earlier, after we sketch a right triangle, we obtain information that the angle formed between the stairs and the floor is 37° and the distance from the bed to the stairs is 10 cm. That means we label the angle as large 37° on the sketch and label 10 cm on the sketch for the side between the bed and the stairs. Then, we refer to the sketch to determine the information we obtain, namely the side, the size of the angle, and the tangent value 37° . This information means we use the trigonometric ratios of tangent, which is the comparison between the value of the front side and the value of the side.

Activity 6: Observation of "Picture of a Skier Descending an Iceberg"

The aim of task 6 is to enable students to accurately solve contextual situations involving right triangles by applying trigonometric ratios. This activity divides students into several groups consisting of 4-5 students. The teacher provides student worksheets as learning materials for activity 6. Each group observes the images presented (see Figure 17).



Figure 17. Questions in Activity 6

The student worksheet in this activity presents open-ended questions. Each group or student has the opportunity to have a different value. This activity encourages students to freely determine the angle's value, the opposite side's value, and the hypotenuse. This question is taken from a Geometry book by Larson (Larson et al., 2007). In this question, students are asked to observe a picture of a skier going down an iceberg. Through the worksheet, the teacher asks students to determine the angle size according to their wishes. In addition, students determine the height of the iceberg themselves. Furthermore, from the data and values they obtain, students determine the track length traveled by the iceberg skier. The teacher allows students to ask the teacher if they need a trigonometric ratio value from a particular angle value.

In activity 6, students express their ability to think logically through questions. The teacher guides students in determining values so that they can think logically. Referring to the values they present, students must determine for themselves the principle of trigonometric ratio values that they must use to resolve the issue. Figure 18 shows an example of student responses in activity 6. The results of the students' responses show that the student assumes the size of the angle requested in the question with the value 75° . The height of the iceberg is 90 units. From the value given by the student, he calculates the length of the track traveled by the skier using the comparison value $\sin \theta$ so that the desired track length value is obtained.

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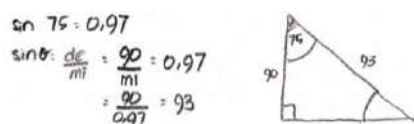


Figure 18. Example of Student Responses to Activity 6

From several findings from each group, one group presents the results of their discussion and simultaneously concludes the learning outcomes.

DISCUSSION

Learning activities using the Realistic Mathematics Education approach regarding trigonometric ratios followed by students consist of six activities, namely raffia stretching activities, student height measurement activities, activities to make three right triangles from paper, activities to observe calendar images, activities to observe bed images, and activities to observe skier images. Each activity has its purpose and is interrelated and continuous. The goals of each action are as outlined below: Firstly, stretching the raffia allows students to determine the location of the angles and sides of a right triangle. Secondly, measuring the height of students and their shadows allows students to apply the concept of trigonometric ratios. 3) Creating a teaching tool including three right triangles is intended to facilitate students' comprehension of the correlation between the notion of resemblance and trigonometric ratios. 4) The purpose of observing the calendar image is to permit students to ascertain the numerical value of trigonometric ratios at specific angles. The objective of viewing the bed image is to facilitate students in applying trigonometric ratios to ascertain the length of the desired side in a right triangle. Similarly, observing the skier image seeks to empower students in solving contextual difficulties by employing the concept of trigonometric ratios. Every worksheet provided incorporates the specific features of the Realistic Mathematics Education pedagogy in relation to trigonometric ratios. These findings corroborate Subekti's assertion that student worksheets will enhance the learning process (Subekti & Prahmana, 2021).

The implementation of the Realistic Mathematics Education approach in different learning process activities can facilitate students' comprehension of mathematical skills and enable teachers to effectively assist students towards attaining learning objectives (Ekowati & Nenohai, 2017). The reason for this is that the mathematics learning process, which follows the Realistic Mathematics Education approach, leverages the surrounding world to encourage students to generate ideas and allow them to develop their knowledge (Nirawati et al., 2021).

According to research conducted by Sukendra (2020), a sequence of learning process activities utilizing the RME approach significantly facilitates students' comprehension of the idea of trigonometric ratios (Sukendra, 2020). Thus, the deliberate learning trajectory promotes students to construct their knowledge, enhance their current skills, and effectively address challenges across many subject areas (Prahmana et al., 2017; Nursyahidah et al., 2020;; Wijaya et al., 2021; Kuncoro et al., 2023).

CONCLUSIONS

The objective of this work is to develop a learning trajectory for trigonometric ratios by utilizing the RME approach. The findings of this study suggest that implementing the RME approach in designing a learning trajectory for trigonometric ratios can enhance students' mathematical proficiency in comprehending the concepts of trigonometric ratios. The learning trajectory designed using the RME approach consists of six

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Commented [bc3]: In order to assess the improvement you need to show us how students understood the trig c oncepts before the intervention. Otherwise how do you know they improved?

activities. Through a series of continuous and ongoing activities in the six activities, students find it easier to build their understanding in understanding the concept of trigonometric ratios. Students build their understanding starting from their ability to identify angle and side positions, apply the definition of trigonometric ratios to contextual problems, understand the relationship between similarity and trigonometric ratios, determine the magnitude of trigonometric ratios values at particular angles, apply trigonometric ratios, and solve contextual problems using trigonometric ratios.

This research is expected to encourage other researchers to design learning trajectories using the same topic but different approaches. The researcher hopes that the results of this design can contribute to the researchers' contribution to building a generation according to the nation's expectations.

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REVIEW FORM

Article Title : Learning Trajectory of Trigonometric Ratios Applying Realistic Mathematics Education

Author's Name : Dini Andiani, Darhim, Elah Nurlaelah

❖ Recommendation to the Editor (Please mark "X" for appropriate option)
☐ Excellent, accept the submission without revisions (5) ☐ Good, accept the submission after minor revisions (4) No further reviews are required ☒ Likely acceptable after major revisions (3) Further reviews are required. ☐ Possibly acceptable after serious revisions (2) Multiple further reviews may be required ☐ Decline the submission (1) Reject without resubmission

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❖ Evaluation (Please evaluate various aspects of the manuscript by grades 1-5)	
5=excellent, 4=good and needs minor improvements, 3= average and needs major improvements , 2=below average and needs serious improvements, 1=unacceptable	
Items	Grade brief description
<u>Contribution to the existing knowledge</u> What is the paper about? Does it propose a new idea, a new approach? How does it fit in the MTRJ mission in terms of classroom involvement and research involvement? Who is the possible audience?	The paper focuses on designing a learning trajectory for teaching trigonometric ratios to 10th-grade students in Bandung, Indonesia, using the Realistic Mathematics Education (RME) approach. It proposes a structured and practical methodology that incorporates hands-on activities and real-life contexts to enhance students' understanding of trigonometry. The paper aligns with the Mathematics Teacher Research Journal (MTRJ) mission by promoting classroom involvement through interactive and student-centered learning experiences.
<u>Readability, language, and formatting</u> Does the paper need proofreading and help with English? Is the formatting correct (tables are numbered, figures are labeled and mentioned in the text, etc.)? Are all pictures clear and readable?	The paper good in English. But, there is need clarity of figures and tables, a detailed review of the document is necessary.

<u>Motivation or justification</u> Why is the didactic proposal made? The research problem or other studies of similar characteristics are presented.	The didactic proposal is made to address specific learning obstacles that students face when understanding trigonometric ratios. The research highlights the need for a structured learning trajectory that utilizes the RME approach to enhance students' comprehension and engagement with the material. Previous studies have indicated that employing the RME approach can significantly facilitate students' understanding of mathematical concepts, including trigonometric ratios. Additionally, the proposal is informed by existing literature that emphasizes the importance of contextual and hands-on learning experiences in mathematics education, which can help students construct their knowledge more effectively. By developing a HLT, the research aims to create a framework that not only addresses these learning challenges but also aligns with best practices in mathematics teaching
<u>Adequacy of literature review</u> Motivation for the investigations and placing the work in the existing knowledge	Overall, the motivation for the investigations stems from the observed challenges that students face in understanding trigonometric ratios, which are critical for their mathematical development. Previous studies have indicated that traditional teaching methods often fail to engage students effectively, leading to superficial comprehension of mathematical concepts
<u>Soundness of methodology</u> How does the proposal improve students' learning experience? Is the methodology, the procedures, the source of the data collection, and analysis clearly stated (sample size, characteristics, qualitative or quantitative or mixed, etc.)? Evidence of the development of the proposal in the classroom together with all materials and practical guidelines	the methodology is sound, clearly articulated, and supported by evidence from classroom implementation, making it a valuable contribution to the field of mathematics education.

necessary for repeating the experiment by other researchers. Are students' answers and teachers' comments included in the submission?	
<u>Results and Conclusions</u> Clear connection among the goals, the literature review and the conclusions. Analysis of an overall perspective of the development of the activity. Difficulties during the implementation and possible remedies.	The goals of the didactic proposal are explicitly aligned with the findings from the literature review, which emphasizes the importance of contextual and hands-on learning in mathematics education. The proposal aims to enhance students' understanding of trigonometric ratios through the RME approach, which is supported by previous research indicating that such methodologies significantly improve comprehension and engagement. The conclusions drawn from the study affirm that the implementation of the RME approach effectively addresses the identified learning challenges, thereby achieving the stated goals of improving students' mathematical proficiency
Do the References include MTRJ papers? Are the pre-test and the post-test included in the appendices?	Including MTRJ papers
❖ Strengths Overall, these strengths contribute to the paper's potential impact on mathematics education, particularly in enhancing the teaching and learning of trigonometric ratios.	
❖ Weaknesses While the paper discusses student engagement and understanding, it lacks a detailed analysis of specific misconceptions that students may have regarding trigonometric ratios. Addressing these misconceptions is crucial for effective teaching and learning.	
❖ Suggestions to the Author/s Incorporate Qualitative Data: To know in detail, please see comments on manuscripts	

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Learning Trajectory of Trigonometric Ratios Applying Realistic Mathematics Education

Dini Andiani ^{1,2}, Darhim ¹, Elah Nurlaelah ¹

¹ Department of Mathematics Education, Universitas Pendidikan Indonesia,
Bandung, Indonesia, ² Department of Mathematics, Universitas Bale
Bandung, Indonesia

diniandiani@upi.edu, diniandiani367@gmail.com,

darhim0355@gmail.com

Abstract: Trigonometry material is still considered complex material at the secondary level. It is necessary to design an appropriate trigonometric ratios learning trajectory so that students can quickly build the concept of trigonometric ratios. This study aims to design a learning trajectory for trigonometric ratio material through Realistic Mathematics Education (RME). This study uses a research design involving 10th-grade high school students in Bandung City in the 2023-2024 academic year. The research data was taken through data collection using written test instruments, student worksheets, observations, interviews, and documentation. The results of the study showed that the trigonometric ratios learning trajectory consisted of raffia stretching activities from the table base to the student's head, measuring the student's height and shadow, experimenting with making three right triangles from paper, observing a picture of a calendar in the form of an equilateral triangular prism, observing a bed or bed, and observing a picture of a skier going down an iceberg. Students can quickly build the concept of trigonometric ratios from several activities provided. In addition, various learning flows from various activities make it easier for students to build knowledge about trigonometric ratios and solve problems presented contextually.

Keywords: Learning Trajectory, Trigonometric Ratios, Realistic Mathematics Education

INTRODUCTION

Trigonometry material is vital in everyday applications such as statistics, surveying, architecture, and other engineering fields (Weber, 2005). Building knowledge about trigonometry takes work (Blackett & Tall, 1991; Demir et al., 2012; Kendal & Stacey, 1997; Ngu & Phan, 2020; Sugianto et al., 2023). Students need to learn several things related to one concept and another to understand the concept of trigonometry. Kamber's research (2017) stated that students struggle to understand trigonometry through the unit circle

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(Kamber & Takaci, 2017). Students need help distinguishing trigonometry through right triangles, the unit circle, or the relationship between the two (Maknun et al., 2019). Students fail to understand the concept of trigonometric ratios (Weber, 2005) even though trigonometric ratios are essential in understanding trigonometry (Nordlander, 2022).

Referring to the Independent Curriculum implemented in Indonesia, trigonometric ratio is a grade 10 material or element which is phase E. Learning outcomes (CP) in phase E are that students involve trigonometric ratios and their applications to solve problems involving right triangles (Badan et al., 2022).

Given the outlined challenges in acquiring knowledge about trigonometric ratios, it is imperative to develop a learning trajectory that facilitates the development of students' understanding in this area. In accordance with (Simon, 1993) definition of hypothetical learning trajectory (HLT), Clements (2011) utilizes the notion of learning trajectory to characterize students' cognitive processes and acquisition of knowledge on a specific mathematical subject. It also encompasses the sequence of assumptions that guide their thinking as they progress through a set of learning activities aimed at generating the intended mental processes or actions. According to Clements (2011), this cognitive process guides students in developing a certain degree of thinking in order to attain specified learning objectives for the subject matter (Clements, 2011).

The Realistic Mathematics Education (RME) approach is strategically developed to effectively support the achievement of the intended learning trajectory. The instructional approach known as Realistic Mathematics Education (RME) is based on the philosophy of Hans Freudenthal, who views mathematics as a creative endeavor of humanity (Freudenthal, 1973; Gravemeijer & Terwel, 2000; Treffers, 1987; Van den Heuvel-Panhuizen & Drijvers, 2020). Realistic Mathematics Education enables students to engage in the learning process within the context of real-life situations and their everyday experiences (Yuanita et al., 2018). A study conducted by Aiyub also shown that learning designs that incorporate real or contextual issue designs can enhance students' algebraic thinking (Aiyub et al., 2024).

Students build understanding through informal processes. Students observe, see, and understand the environment in which they are located (Meika, 2018). Realistic Mathematics Education encompasses three key concepts: directed reinvention and progressive mathematical processes, didactic phenomenology, and model construct development (Gravemeijer, 1994). Within the teaching and learning process, these three concepts are further developed into six distinct features of RME: 1) The principle of activity, 2) The principle of reality, 3) The principle of level, 4) The principle of intertwinement, 5) The principle of interactivity, and 6) The principle of guidance (Panhuizen & Drijvers, 2014).

Under the title "Realistic Mathematics Education" in the Encyclopedia of Mathematics Education, Panhuijen elucidates the six concepts. Under the RME approach, the activity principle entails that students assume an active role in the learning process. This assertion is based on Freudenthal's theoretical understanding of mathematics as a human endeavor. The reality principle posits that students have the ability to use mathematics in order to address practical issues, and the process of learning should commence with tangible problems that can be expressed mathematically. The level principle posits that students progress through several levels of comprehension in order to grasp the connection between ideas and procedures in mathematics. The interconnection refers to the phenomenon where the students, while acquiring mathematical information, are able to employ a range of tools and methods to construct a comprehensive process of integrating different concepts. The interaction principle posits that the process of learning mathematics is not only an individual endeavor, but rather a social and reflective one, therefore facilitating a deeper comprehension of the subject. The guidance principle in mathematics education entails the concept of "guided reinvention," as proposed by Freudenthal. Under these circumstances, the learning

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process should motivate students to adapt in order to enhance their comprehension. It is imperative that the learning program aligns with a cohesive long-term learning trajectory (Panhuizen & Drijvers, 2014).

Method

The employed research methodology is design research as described by Gravemeijer and Van Eerde in 2009 (Gravemeijer & Van Eerde, 2009). This research methodology is focused on local instruction theory (LIT) to enhance the quality of learning by fostering effective collaboration between researchers and teachers (Gravemeijer & Van Eerde, 2009). Design research will enable the attainment of the goals of the research paradigm in the development of sequential activities and the enhancement of learning quality. The primary objective of this work is to develop a comparative trigonometry learning trajectory by utilizing the RME approach. The study was carried out in two cyclical phases, each following the design research stages outlined by Gravemeijer (2004): 1) preparatory design, 2) teaching experiment, and 3) retrospective analysis. The outcomes of the initial cycle activities that have been executed and the subsequent analytical review have contributed valuable insights (Gravemeijer, 2004).

During the first stage of the design process, the researcher personally observed the school and conducted direct interviews with the teachers and the students. Furthermore, the researcher gathered, perused, and comprehended several source materials pertaining to the challenges faced by students and teachers in grasping the notion of trigonometric ratios. Furthermore, the researcher examined many sources that elucidated the learning process activities based on the Realistic Mathematics Education (RME) methodology. A sequence of researcher actions served as a point of reference for the researcher in developing the Hypothetical Learning Trajectory (HLT). The Hypothetical Learning Trajectory (HLT) comprises three elements: learning objectives that will be focused on during the learning process, learning activities, and hypotheses that will forecast the cultivation of students' cognitive abilities in the learning process (Simon, 1993). The learning process hypothesis serves as the fundamental basis for constructing learning activities. This document is a compilation of learning activities designed by the researcher.

Learning objectives	Learning Activities	Student Thinking Conjecture
Students can identify the position of the right angle, opposite side, and hypotenuse in a right triangle.	Stretch the raffia from the table's base to the top of the head to form a right triangle.	A group of students can model the activity into a right triangle. One of the students was able to sketch the right triangle that was formed. Students can identify the position of right angles, front sides, hypotenuses, and side sides in sketches formed with various answers without knowing the reasons. Students need to be corrected in identifying the position of right angles.
Students can apply the definition of trigonometric ratios $\sin \theta$, $\cos \theta$, and $\tan \theta$ to	Measuring students' height and shadow	Students can draw sketches formed from the models presented. The definitions of trigonometric ratios $\tan \theta$, $\sin \theta$, and $\cos \theta$ can be used by students to the model of measuring the heights of both students and their shadows.

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Learning objectives	Learning Activities	Student Thinking Conjecture
contextual problems using the concept of ratios in right triangles.		Students can determine the position θ of different angles to obtain different trigonometric ratio values.
Students can understand the relationship between the concept of similarity and trigonometric ratios.	Experiment to make three right triangles from paper	Students can make three similar right triangles from paper. Students can make sketches from paper formed into three similar right triangles. Students can identify the shortest and longest sides of each triangle formed. Students can determine the ratio between the shortest side and the longest side of each right triangle. Students can relate the concept of similarity with trigonometric ratios.
Students can determine the value of trigonometric ratios at particular angles (0° , 30° , 45° , 60° , and 90°).	Observations on the calendar image in the form of an equilateral triangular prism.	Students can sketch a calendar model as an equilateral triangular prism. Students can construct a right triangle in which one of the angles corresponds to a specifically defined angle. Students can determine the value of trigonometric ratios at particular angles (0°, 30°, 45°, 60°, and 90°).
The application of trigonometric ratios enables students to ascertain the length of sides in a right triangle that are not known.	Bed or bed observation	Students can make sketches or drawings of bed models. Students can identify the position of right angles, oblique angles, side angles, and front side positions in sketches. Students can apply trigonometric ratios to determine the length of unknown sides in problems presented related to beds.
Students can solve contextual problems related to right triangles using trigonometric ratios.	Observation of the image of a skier descending an iceberg	Students can identify the position of the angles, opposite sides and hypotenuses in a right triangle formed as a sketch of a picture of a skier descending an iceberg. Students can apply the definition of trigonometric ratios to skier problems. Students can solve the problems given in the case of skiers.

Table 1: Hypothetical Learning Trajectory in Teaching Experiment 1

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Within cycle 1, the research participants consisted of 25 10th-grade students who attended a high school located in Bandung City. In cycle 2, the research participants consisted of 25 10th-grade students from a different high school within the same municipal area. The HLT Design was first implemented in teaching experiment cycle 1 to assess the efficacy of the prepared learning activity design in investigating students' cognitive patterns. Moreover, the results of teaching experiment 1 indicated an enhancement in HLT. A refined Hypothetical Learning Trajectory (HLT) was implemented in phase 2 of the teaching experiment. The data collecting protocols encompass classroom observations, videos documenting learning activities, student workbooks, and recordings of interviews conducted with students.

Phase 1 of the retrospective analysis involved the collection and analysis of all the data by the researcher. An analysis was conducted to evaluate the outcomes of including the learning trajectory in teaching experiment 1 with the original HLT design. Moreover, the researcher investigated and examined the learning process activities included to determine the level of students' comprehension of the idea of trigonometric ratios. An analysis was conducted on the outcomes of the HLT enhancements implemented in teaching experiment 2, by comparing the initial HLT with the execution of the learning process.

The researcher selected 6 participant subjects to conduct interviews. The six participants had various abilities; two had high, medium, and low abilities. In addition, interviews were also conducted with the mathematics teacher of trigonometric ratios at the school that taught the students.

Results

The learning trajectory design discussed in this paper is a sequence of student activities aimed at comprehending the subject of trigonometric ratios through the Realistic Mathematics Education approach. The trigonometric ratios learning trajectory comprises six learning activities: elongating a raffia from the table's base to the student's head, measuring the student's height and shadow, creating three right triangles from paper, visually examining a calendar image in the shape of an equilateral triangular prism, observing a bed or bed, and observing an image of a skier descending an iceberg.

Activity 1: Observation of "Stretching the Raffia from the Base of the Table to the Top of the Student's Head"

The primary aim of activity 1 is to enable students to accurately determine the location of the right angle, the opposing side, and the hypotenuse within a right triangle. During this exercise, the teacher instructs students to showcase their use of contextual equipment found in the classroom, specifically a raffia, a table base, and one of their peers. The students are segregated into multiple groups. Each group comprises from 4 to 5 students. A single student from each group grasps one end of the raffia, while the other end is fastened to the bottom of the table foundation.

Furthermore, the other students in the group observed the shape between the student's height, the floor, and the raffia. The observation process is shown in Figure 1. Furthermore, by describing the teaching materials, the teacher directs students to sketch the shapes in the demonstration.

Each group sketched their demonstration and represented the information in a picture or symbol. One of the sketch results is shown in Figure 2. Students sketched the table legs, the student's height, and the raffia stretched to form a right triangle. They represented the student's height in an illustration of the student, and the raffia was represented as a line.

After sketching in demonstration activity 1, students began to identify the position of the right angle, the position of the angle θ , the position of the hypotenuse, the front side, and the side. Different groups of students exhibit variations in their results in identifying the position of the angle θ and the front and side.

In this case, students are asked to explain the reasons. Some groups can explain their reasons. For those who cannot explain, other groups are allowed to present their reasons so that the understanding they gain

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Commented [bc1]: What did the enhancement of HLT consist in, precisely, please

helps others build the same understanding.

To obtain findings through teacher guidance, the teacher provides guided reinvention for groups that do not yet understand the reasons for the position of the corners θ and the position of the front and side sides as in the following conversation.

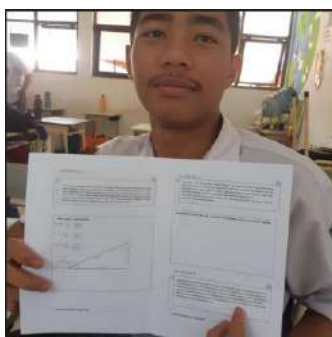


Figure 1: Activity 1 "Stretching a Raffia from the Base of the Table to the Top of the Student's Head"

Commented [bc2]: Faces of students have to be blurred

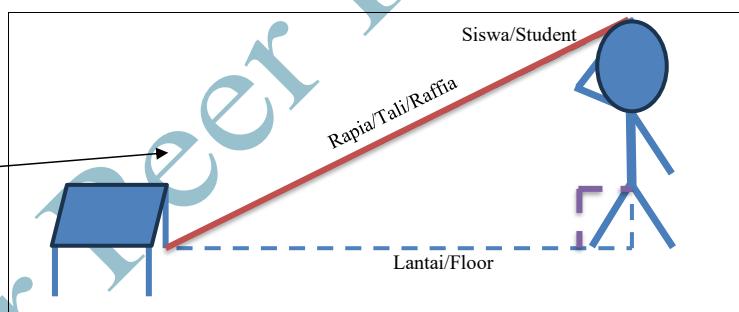
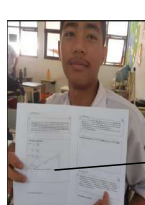


Figure 2: Example of Student Sketches in Activity 1

- Teacher : "Okay, S-1, please explain what you drew."
- S-1 : "Okay, ma'am, if you look at the instructions in the worksheet, we are asked to practice what is written in activity 1, and then we share roles; S-2 ties the raffia raffia to the base of the bottom table, S-3 holds the end of the raffia that is the result of extending S-2's raffia, S-4 reads what is on the worksheet, and I make the sketch."
- Teacher : "Okay, have you considered whether your sketch will be a right triangle?"
- S-1 : "At first, I did not, Ma'am, but when you said, "Look, if you draw it, is a right angle formed?" I remembered."
- Teacher : "Okay, how do you determine the corner points?"
- S-1 : "The benchmarks are at the end of the table legs, Leg S-3, and head S-3, Ma'am."

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- Teacher : "Why is that, S-1?"
- S-1 : "Yes, that is what it looks like when drawn, ma'am."
- Teacher : "Okay, the benchmarks that S-1 mentioned earlier make a move from the rapid raffia, right?"
- S-1 : "What do you mean, ma'am?"
- Teacher : "Mom, look, you represent the rapier raffia, the floor, and the height of S-2 with a line, right?"
- S-1 : "That is right, ma'am."
- Teacher : "Well, when you make that line, there is a phase where you turn; that is called a benchmark."
- S-1 : "Oh yes, that is right, ma'am, that is right."
- Teacher : "S-1, try to determine the right angle, slanted side, front side, and side in the sketch you have made!"
- Student : "This one is a right angle, this one is the hypotenuse, this one is the front side, and this one is the side side." (pointing to the picture).
- Teacher : "How do you determine a right angle, S-1?"
- Student : "There is a line like this and like this, ma'am." (pointing at the picture).
- Teacher : "Now, try to observe; what degree angle do the two lines form, or what angle is it called then?"
- Student : "Oh yes, ma'am, right angles."
- Teacher : "How do you determine the hypotenuse?"
- Student : "The side in front of the right angle, ma'am."
- Teacher : "How do you determine the front side?"
- Student : "This one, ma'am? Yes, the one in front of this corner (pointing to one of the corners of the picture)".
- Teacher : "Okay, how do you determine the side?"
- Student : "The one on the front side, ma'am."

In this activity, students have determined the expected position even though it is not specific because it still depends on the position of the angle θ . This identification procedure enhances pupils' ability to determine the precise location of the angles and sides within a right triangle. Upon completing the task, the teacher and students jointly draw conclusions and engage in reflections regarding the process of determining the position of the right angle, the hypotenuse, and the angles θ that impact the identification of the front side and side of a right triangle.

Activity 2: Observation "Measurement of Students' Height and Shadow"

The aim of activity 2 is to enable students to utilize the definition of trigonometric ratios $\sin \theta$, $\cos \theta$, and $\tan \theta$ in practical situations by employing the notion of comparison in right triangles. As part of this exercise, students are required to transform issues involving comparison in right triangles into tasks that may be directly practiced in groups. The teacher instructs a single student from a group to face away from the lamp until a shadow forms in front of the said student. Moreover, the teacher instructs students to illustrate the encountered shape in this exercise.

A student in his/her group is assigned to calculate the height of the student whose height will be measured and the height of the shadow that occurs. This activity is carried out alternately so that all group members have their height and shadow height measured. The measurement data is shown in a tabular format on the student's designated worksheet. Figure 4 is an example of student work in representing it in table form.

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Furthermore, the teacher determines the angle θ as the angle formed between the shadow and the oblique line. In this exercise, the teacher instructs the group to calculate the ratio of the student's height to the shadow height using the available data. This task aims to help students recall the process of determining the value of trigonometric ratios by drawing the resulting shapes.

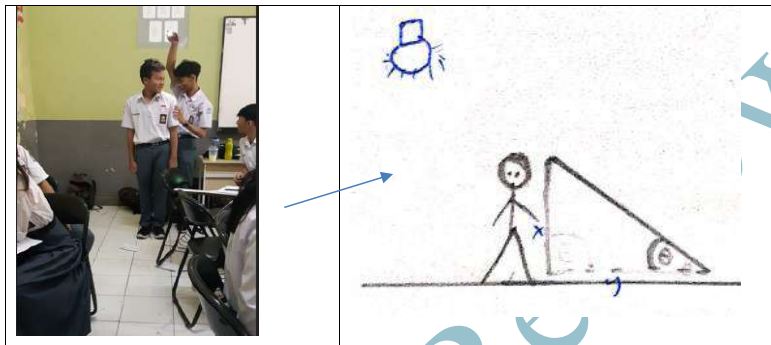


Figure 3: Activity 2 and Example of Student Sketches in Activity 2

	Kamu	Nama temanmu	Nama temanmu	Nama temanmu	Nama temanmu
	<u>Nabila</u>	<u>Bunga</u>	<u>Naufal M</u>	<u>Jahsyi</u>	<u>Nalvin</u>
Tinggi badan	155	153	173	169	161
Tinggi bayangan	165	163	183	179	171

	Your name	Your Classmate's name	Your Classmate's name	Your Classmate's name	Your Classmate's name
	<u>Nabila</u>	<u>Bunga</u>	<u>Naufal M</u>	<u>Jahsyi</u>	<u>Nalvin</u>
Height	155	153	173	169	161
Shadow Height	165	163	183	179	171

Figure 4: Example of a table of results measuring height and shadow in activity 2.

The objective of this project is for students to accurately calculate the trigonometric ratio values of $\sin \theta$, $\cos \theta$, and $\tan \theta$ in every triangle sketch they create (Figure 5). In each activity, the teacher monitors students through visits to each group and guides students in solving problems. Each student can use their way as an alternative, both formally and informally. The teacher facilitates students in revisiting the knowledge they have acquired about determining trigonometric ratio values by providing scaffolding.

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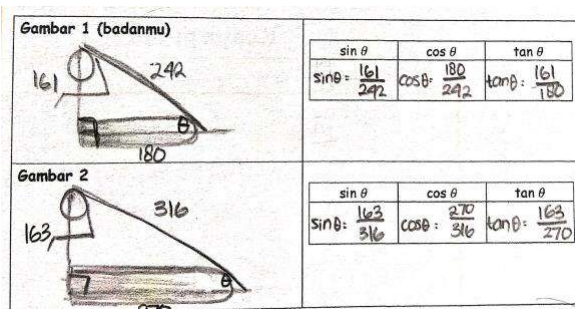


Figure 5: Example of Sketch and Application of Trigonometric Ratios Values in Activity 2

One of the students (S-5), as a representative of one of the group members, participated in presenting activity two and explained how the values of $\sin \theta$, $\cos \theta$, and $\tan \theta$ were obtained. After sketching the experiment, this group identified the right angle to determine the other positions. The following is a dialogue explaining the concept of trigonometric ratios described by the students.

Teacher : "Okay, S-5. Explain how your group determines each trigonometric ratio value requested in the table."

S-5 : "Okay, Ma'am, first we sketch according to our height in the group, then, from the sketch formed, we first determine the right angle, draw the symbol on the sketch, then determine the angle θ"

Teacher : "Okay, do you mean identifying the position of a right angle?"

S-5 : "Yes, ma'am."

Teacher : "What do you mean by determining the angle?"

S-5 : "So that we can determine the front and side, ma'am."

Teacher : "Okay, what is the next process?"

S-5 : "Earlier, after determining the angle position θ , we then wrote down each of our heights according to what was measured earlier, and wrote down the height of the shadow."

Teacher : "Okay, how do you determine the hypotenuse?"

S-5 : "Using the Pythagorean theorem, ma'am, with the help of a calculator."

Teacher : "Okay, how do you calculate $\sin \theta$, $\cos \theta$, and $\tan \theta$?"

S-5 : "Well, because the value of $\sin \theta$ is the front per hypotenuse, look at the picture (while pointing to picture 1 in UBA 2 Activity 2); the front of the angle θ is the front side, which is 161, and the hypotenuse is 242 so the $\sin \theta$ is 161 per 242."

Teacher : "How to calculate $\cos \theta$ and $\tan \theta$?"

S-5 : "Same here, ma'am, just a different position according to what was learned previously: $\cos \theta$ means side per hypotenuse, and $\tan \theta$ means front per side."

The conversation above helped all students understand the concept of trigonometric ratios correctly. At the end of the activity, one student reflected on the learning that day and concluded how to apply trigonometric ratios correctly.

Activity 3: Experiment with "Making Three Right Triangles from Paper"

The teaching aim of exercise 3 is to enable students to comprehend the correlation between the notion of

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resemblance and trigonometric ratios. The students are divided into multiple groups. Each group comprises from 4 to 5 students.

In this activity, the teacher asks each group to prepare the necessary tools, scissors, paper, rulers, and stationery through student worksheets. Furthermore, each group makes a picture through the experiment of cutting rectangular paper so that two equal and similar right triangles are obtained. Students can determine the similarity in three equal and similar right triangles. The teacher guides students in finding the equal and similar sides of three equal and similar right triangles.

Students use several alternative methods to find the concept of similarity, namely diagrams and teaching aids. Examples of activities that occur as in Figure 6 and 7. The results of the presentation in one group are the results of group discussions using diagrams, while the other group uses concrete teaching aid models made by the group. The results of both alternatives in each group find a way to build the concept of similarity and trigonometric ratios in right triangles.

Students are guided to find formal patterns of similarity in right triangles through guided reinvention through directed questions. Students are guided to find a formal way to use Geometry theorems by comparing three similar right triangles. To facilitate a gradual comprehension of the notion of similarity, participants are encouraged to identify the shortest and longest sides of each right triangle under comparison.

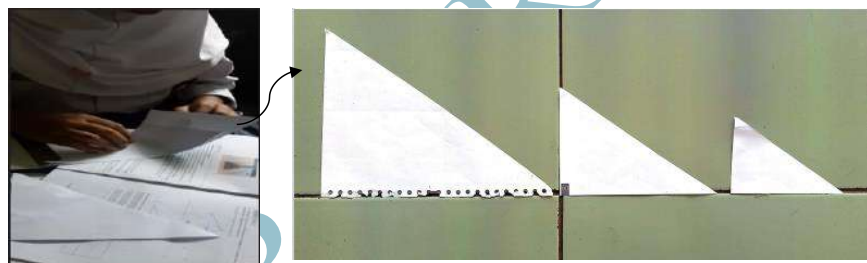


Figure 6. Activities and Examples of Student Responses using Teaching Aids in Activity 3.

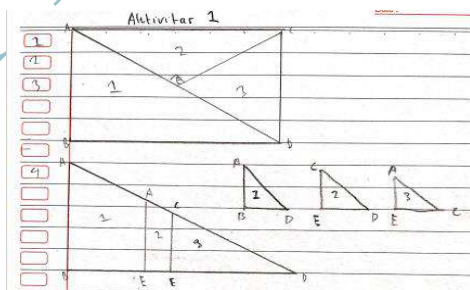


Figure 7. Example of Student Responses using Diagrams in Activity 3.

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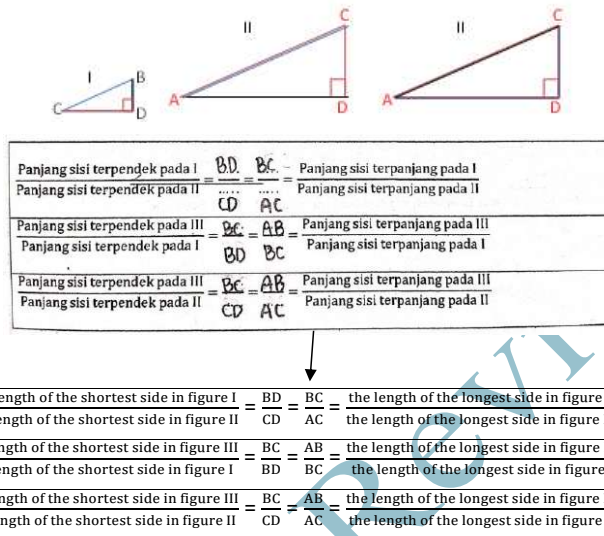


Figure 8. Example of Student Responses in Using the Concept of Comparison in Activity 3.

The teacher directs students in determining the location of the shortest and longest sides of each triangle, enabling them to articulate the resulting comparative thinking patterns. One example of a group response is presented in Figure 8.

In this exercise, the teacher presents questions to facilitate students' comprehension of the analogy between the notion of similarity and trigonometric ratios. Figure 9 displays the provided questions. In this inquiry, the teacher presents contextual inquiries formatted like a garbage can including a logo. The logo is situated on a base of an equilateral triangle of 6 centimetres in side length. The teacher asks students to calculate the logo's height, sketch the triangle formed, and explain the concept used in solving the given problem.



The trash can logo featured below is designed as an equilateral triangle. The dimension of the side measures 6 centimeters. What is the estimated vertical dimension of the logo? Begin by creating a preliminary outline of the triangle. Which concept are you employing? Are there any other concepts available apart from the one you have employed? What is the procedure?

Logo tempat sampah berikut berbentuk segitiga sama sisi dengan panjang sisi 6 cm. Berapa kira-kira tinggi logo tersebut? Buatlah sketsa segitiga terlebih dahulu. Konsep apa kah yang kamu gunakan? Adakah alternatif konsep selain yang sudah kamu gunakan? Bagaimana caranya?

Figure 9. Example of Questions on the Relationship between the Concept of Similarity and Trigonometric Ratios in Activity 3.

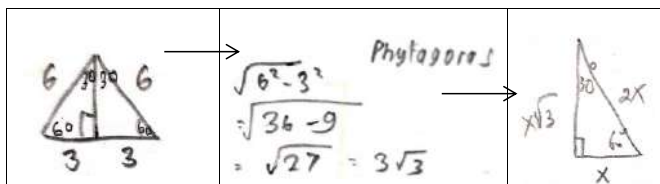


Figure 10. Example of Student Responses to Contextual Questions in Activity 3.

In Figure 10, one group sketches the shape formed and then calculates the height of the logo using the Pythagorean Theorem. They then use the concept of similarity to check the results they obtained.

At the conclusion of the activity, the teacher and students synthesize the idea under study concerning the correlation between similarity and trigonometric ratios.

Activity 4: Observation of "Calendar Image in the form of an Equilateral Triangular Prism"

The learning objective of activity 4 is that students can determine the magnitude of the trigonometric ratio at particular angles (0° , 30° , 45° , 60° , and 90°). For this exercise, the teacher organizes students into groups of 4-5. Using the student worksheet, the teacher instructs each group of students to visually analyze a calendar image as an equilateral triangular prism, as depicted in Figure 11. Subsequently, the teacher directs the students to examine the face of the calendar in order to derive an equilateral triangle. Students are instructed to construct a perfectly symmetrical triangle measuring 2 units in length and annotate each corner. Subsequently, the teacher instructs the students to draw a vertical line from one of the highest points of the triangle to the center of the side directly in front of the triangle, resulting in two equal sections of the triangle.

The students are instructed to redraw a portion of the triangular again, note the length of each side, and record the known angles. Utilising their acquired knowledge, students employ the Pythagorean Theorem to determine the length of the side that is not known. Subsequently, students are required to ascertain the numerical values of the trigonometric ratios $\sin \theta$, $\cos \theta$, and $\tan \theta$ for the particular angles 30° and 60° . Students compile the findings into a tabular format. At the conclusion of the question, the teacher makes an assumption about the length of the shortest side of the triangle, denoted as x . Students are then tasked with determining the lengths of the remaining sides while still utilizing the variable x .

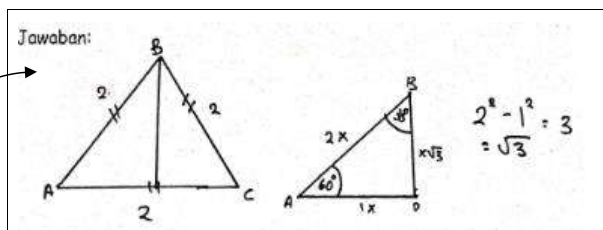


Figure 11. Calendar Image and Example of Student Responses in Making Sketches in Activity 4

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By employing guided innovation with variables x , students are direct to ascertain the comparison of the lengths of the sides of the triangle they have constructed. The student effort in activity 4 of creating a calendar sketch from the front is exemplified in Figure 11. Figure 12 illustrates student replies in activity 4 when answering the problem of calculating the value of trigonometric ratios at particular angles of 30° and 60° in a right triangle.

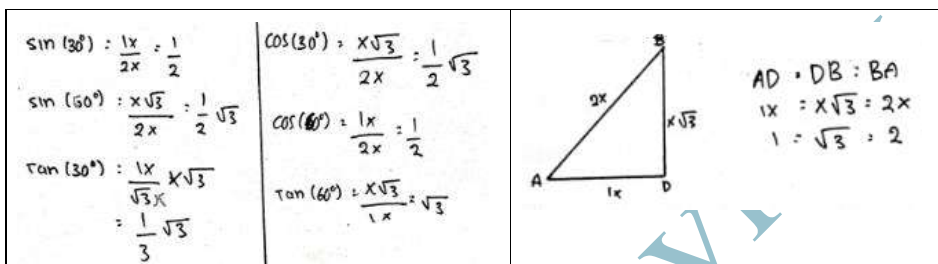


Figure 12. Example of Student Responses regarding memorable and unique 30° angles 60° in Activity 4

In the student worksheet activity 4, the teacher also guides students to determine the magnitude of the trigonometric ratio values at the particular angles of 45° in a right triangle. In this case, students are asked to draw an isosceles right triangle with each side measuring 1 unit. After that, students label each angle. Using the knowledge they have built, students determine the hypotenuse length using the Pythagorean Theorem. After that, students determine the trigonometric ratio values $\sin \theta$, $\cos \theta$, and $\tan \theta$ for the particular angle 45° . Figure 13 shows one of the student responses in their group.

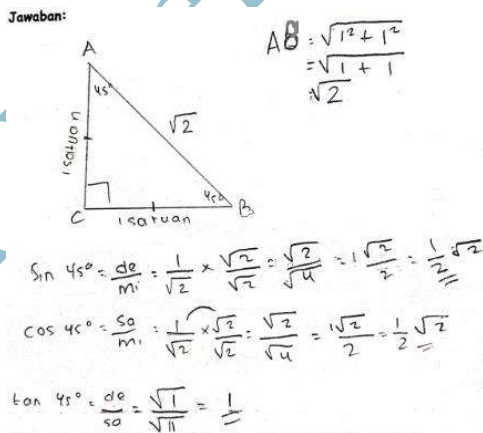


Figure 13. Example of Student Responses Regarding Particular angles 45° in Activity 4

In the initial stage, students sketch an isosceles triangle with a length of 1 unit. Students mark one line on each side that has the same length. In this case, students can already represent according to the question's

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intent. Furthermore, students are accustomed to representing the information obtained in a sketch. Figure 13 displays students' labels indicating the angle size of 45 degrees for all angles except the right angle. Subsequently, the image illustrates the process by which students get the hypotenuse value by the use of the Pythagorean Theorem. By the conclusion of the exercise, students ascertain the numerical value of the trigonometric ratio at the particular angles of 45 degrees and 0 degrees in a right triangle. Lastly, students display the outcomes for numerous specific angles in a tabular format, as depicted in Figure 13.

	Rumus	30°	45°	60°
sin	sin θ = $\frac{\text{sisi depan}}{\text{sisi miring}}$	$\frac{1}{2}$	$\frac{1}{2} \sqrt{2}$	$\frac{1}{2} \sqrt{3}$
cos	cos θ = $\frac{\text{sisi samping}}{\text{sisi miring}}$	$\frac{1}{2} \sqrt{3}$	$\frac{1}{2} \sqrt{2}$	$\frac{1}{2}$
tan	tan θ = $\frac{\text{sisi depan}}{\text{sisi samping}}$	$\frac{1}{3} \sqrt{3}$	1	$\sqrt{3}$

Figure 14. Example of Student Responses Represented in a Table in Activity 4

By the conclusion of the exercise, the teacher and students reached a general conclusion on the determination of the value of trigonometric ratios at particular angles.

Activity 5: Observation of "Bed"

The purpose of exercise 5 is to enable students to use trigonometric ratios in order to ascertain the length of dimensions of unknown sides in a right triangle. For this exercise, the teacher instructs students to be organized into many groups. Each group comprises from 4 to 5 students. The instructor distributes a worksheet to the students that includes instructions for action 5. Each group scrutinizes the image (refer to Figure 15). Figure 16 depicts a triangular figure encompassing the stairs, bed, and floor. Students are instructed to perform the horizontal mathematization method by creating a diagram that represents the triangle produced by the three elements.



Figure 15. Activities and Questions in Activity 5

Furthermore, students are asked to identify whether the shape formed is a right triangle. The teacher

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determines the angle size θ at one corner of the triangle and the distance from the bed to the stairs. Students are asked to represent the known data in a sketch, and the teacher instructs the students to determine the displacement between the floor and the terminus of the stairs that constitute a right triangle by employing the tangent ratio.

From activity 5, the findings of each group are poured into a sketch, as shown in Figure 16.

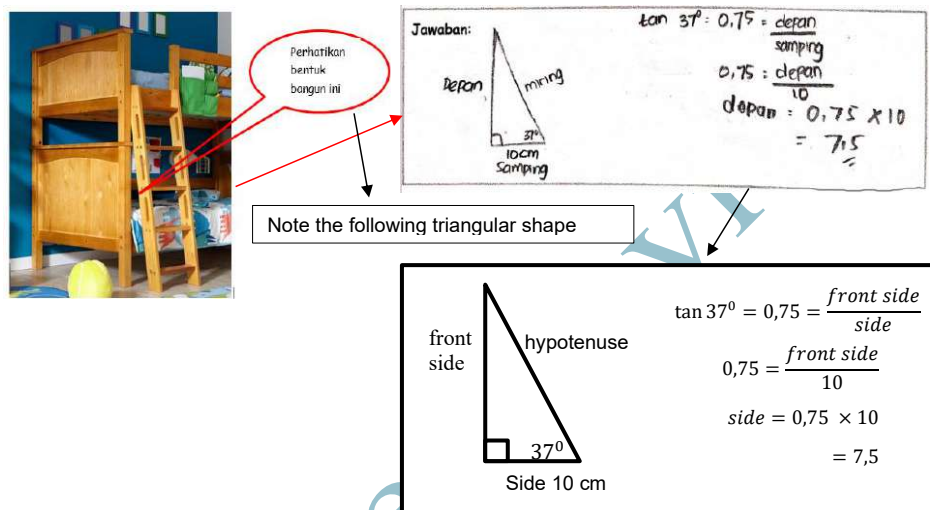


Figure 16. Questions and Student Responses Representing Data into Sketches in Activity 5

Through teacher guidance, students pour out their understanding by sketching a right triangle between the bed, floor, and stairs. At the beginning of the worksheet, students are given questions to identify whether the sketch formed is a right triangle. Students are asked to give a right angle sign if the triangle formed is a right triangle. Furthermore, the teacher provides information through the worksheet that the angle formed between the stairs and the floor is 37° and the distance from the bed to the stairs is 10 cm. Students are asked to calculate the distance from the floor to the end of the stairs that form a right triangle using the tangent ratio.

Following the completion of the given problems by all groups, one group will deliver the outcomes of their group discussion to the entire class. When employing trigonometric ratios to calculate the length of the unknown side of a right triangle, students articulate their understanding of the concept. At the conclusion of the activity, the teacher poses questions to assess the extent to which student's knowledge has been developed, as shown in the dialogue below.

- Teacher : "Okay, how do you find the length of the unknown side in a right triangle?"
- S-6 : "First, draw a sketch formed from the existing problem, then determine what information is contained in the problem, whether the value of the front side, hypotenuse, or side, even whether the angle value is also known. Next, from the information we get, we see

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what principles fulfill the information collected so that we know the principle of trigonometric ratios that we use."

Teacher : "Can you give an example?"

S-6 : "For example, based on the bed problem earlier, after we sketch a right triangle, we obtain information that the angle formed between the stairs and the floor is 37° and the distance from the bed to the stairs is 10 cm. That means we label the angle as large 37° on the sketch and label 10 cm on the sketch for the side between the bed and the stairs. Then, we refer to the sketch to determine the information we obtain, namely the side, the size of the angle, and the tangent value 37° . This information means we use the trigonometric ratios of tangent, which is the comparison between the value of the front side and the value of the side.

Activity 6: Observation of "Picture of a Skier Descending an Iceberg"

The aim of task 6 is to enable students to accurately solve contextual situations involving right triangles by applying trigonometric ratios. This activity divides students into several groups consisting of 4-5 students. The teacher provides student worksheets as learning materials for activity 6. Each group observes the images presented (see Figure 17).



Figure 17. Questions in Activity 6

The student worksheet in this activity presents open-ended questions. Each group or student has the opportunity to have a different value. This activity encourages students to freely determine the angle's value, the opposite side's value, and the hypotenuse. This question is taken from a Geometry book by Larson (Larson et al., 2007). In this question, students are asked to observe a picture of a skier going down an iceberg. Through the worksheet, the teacher asks students to determine the angle size according to their wishes. In addition, students determine the height of the iceberg themselves. Furthermore, from the data and values they obtain, students determine the track length traveled by the iceberg skier. The teacher allows students to ask the teacher if they need a trigonometric ratio value from a particular angle value.

In activity 6, students express their ability to think logically through questions. The teacher guides students in determining values so that they can think logically. Referring to the values they present, students must determine for themselves the principle of trigonometric ratio values that they must use to resolve the issue. Figure 18 shows an example of student responses in activity 6. The results of the students' responses show that the student assumes the size of the angle requested in the question with the value 75° . The height of the iceberg is 90 units. From the value given by the student, he calculates the length of the track traveled by the skier using the comparison value $\sin \theta$ so that the desired track length value is obtained.

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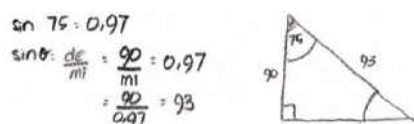


Figure 18. Example of Student Responses to Activity 6

From several findings from each group, one group presents the results of their discussion and simultaneously concludes the learning outcomes.

DISCUSSION

Learning activities using the Realistic Mathematics Education approach regarding trigonometric ratios followed by students consist of six activities, namely raffia stretching activities, student height measurement activities, activities to make three right triangles from paper, activities to observe calendar images, activities to observe bed images, and activities to observe skier images. Each activity has its purpose and is interrelated and continuous. The goals of each action are as outlined below: Firstly, stretching the raffia allows students to determine the location of the angles and sides of a right triangle. Secondly, measuring the height of students and their shadows allows students to apply the concept of trigonometric ratios. 3) Creating a teaching tool including three right triangles is intended to facilitate students' comprehension of the correlation between the notion of resemblance and trigonometric ratios. 4) The purpose of observing the calendar image is to permit students to ascertain the numerical value of trigonometric ratios at specific angles. The objective of viewing the bed image is to facilitate students in applying trigonometric ratios to ascertain the length of the desired side in a right triangle. Similarly, observing the skier image seeks to empower students in solving contextual difficulties by employing the concept of trigonometric ratios. Every worksheet provided incorporates the specific features of the Realistic Mathematics Education pedagogy in relation to trigonometric ratios. These findings corroborate Subekti's assertion that student worksheets will enhance the learning process (Subekti & Prahmana, 2021).

The implementation of the Realistic Mathematics Education approach in different learning process activities can facilitate students' comprehension of mathematical skills and enable teachers to effectively assist students towards attaining learning objectives (Ekowati & Nenohai, 2017). The reason for this is that the mathematics learning process, which follows the Realistic Mathematics Education approach, leverages the surrounding world to encourage students to generate ideas and allow them to develop their knowledge (Nirawati et al., 2021).

According to research conducted by Sukendra (2020), a sequence of learning process activities utilizing the RME approach significantly facilitates students' comprehension of the idea of trigonometric ratios (Sukendra, 2020). Thus, the deliberate learning trajectory promotes students to construct their knowledge, enhance their current skills, and effectively address challenges across many subject areas (Prahmana et al., 2017; Nursyahidah et al., 2020;; Wijaya et al., 2021; Kuncoro et al., 2023).

CONCLUSIONS

The objective of this work is to develop a learning trajectory for trigonometric ratios by utilizing the RME approach. The findings of this study suggest that implementing the RME approach in designing a learning trajectory for trigonometric ratios can enhance students' mathematical proficiency in comprehending the concepts of trigonometric ratios. The learning trajectory designed using the RME approach consists of six

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Commented [bc3]: In order to assess the improvement you need to show us how students understood the trig c oncepts before the intervention. Otherwise how do you know they improved?

activities. Through a series of continuous and ongoing activities in the six activities, students find it easier to build their understanding in understanding the concept of trigonometric ratios. Students build their understanding starting from their ability to identify angle and side positions, apply the definition of trigonometric ratios to contextual problems, understand the relationship between similarity and trigonometric ratios, determine the magnitude of trigonometric ratios values at particular angles, apply trigonometric ratios, and solve contextual problems using trigonometric ratios.

This research is expected to encourage other researchers to design learning trajectories using the same topic but different approaches. The researcher hopes that the results of this design can contribute to the researchers' contribution to building a generation according to the nation's expectations.

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Learning Trajectory of Trigonometric Ratios Applying Realistic Mathematics Education

Dini Andiani^{1,2}, Darhim¹, Elah Nurlaelah¹

¹ Department of Mathematics Education, Universitas Pendidikan Indonesia,

Bandung, Indonesia, ² Department of Mathematics, Universitas Bale

Bandung, Indonesia

diniandiani@upi.edu, diniandiani367@gmail.com,

darhim0355@gmail.com

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Abstract: Trigonometry material is still considered complex material at the secondary level. It is necessary to design an appropriate trigonometric ratios learning trajectory so that students can quickly build the concept of trigonometric ratios. This study aims to design a learning trajectory for trigonometric ratio material through Realistic Mathematics Education (RME). This study uses a research design involving 10th-grade high school students in Bandung City in the 2023-2024 academic year. The research data was taken through data collection using written test instruments, student worksheets, observations, interviews, and documentation. The results of the study showed that the trigonometric ratios learning trajectory consisted of raffia stretching activities from the table base to the student's head, measuring the student's height and shadow, experimenting with making three right triangles from paper, observing a picture of a calendar in the form of an equilateral triangular prism, observing a bed or bed, and observing a picture of a skier going down an iceberg. Students can quickly build the concept of trigonometric ratios from several activities provided. In addition, various learning flows from various activities make it easier for students to build knowledge about trigonometric ratios and solve problems presented contextually.

Commented [LS2]: Is this the trajectory in which students learn the trigonometric ratios concept you found?

Keywords: Learning Trajectory, Trigonometric Ratios, Realistic Mathematics Education

INTRODUCTION

Trigonometry material is vital in everyday applications such as statistics, surveying, architecture, and other engineering fields (Weber, 2005). Building knowledge about trigonometry takes work (Blackett & Tall, 1991; Demir et al., 2012; Kendal & Stacey, 1997; Ngu & Phan, 2020; Sugianto et al., 2023). Students need to learn several things related to one concept and another to understand the concept of trigonometry. Kamber's research (2017) stated that students struggle to understand trigonometry through the unit circle

Commented [LS3]: - Please check the MTRJ format for the paragraph alignment.

- In general, I did not find a specific and clear problem for students on the concept of trigonometric ratios in your article.

- There are no research questions, research gap, and also the novelty. I assume, it is because you didn't explore a specific and clear problem for students

- Proofreading from professional is needed

Commented [LS4]: Do you mean "One concept and another" in the domain of trigonometry concept or in the broader mathematics concept?

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(Kamber & Takaci, 2017). Students need help distinguishing trigonometry through right triangles, the unit circle, or the relationship between the two (Maknun et al., 2019). Students fail to understand the concept of trigonometric ratios (Weber, 2005) even though trigonometric ratios are essential in understanding trigonometry (Nordlander, 2022).

Commented [LS5]: You can explore this one as a problem

Referring to the Independent Curriculum implemented in Indonesia, trigonometric ratio is a grade 10 material or element which is phase E. Learning outcomes (CP) in phase E are that students involve trigonometric ratios and their applications to solve problems involving right triangles (Badan et al., 2022).

Commented [LS6]: -From my point of view, you don't need to talk about this, because it's too specific. just talk about the content at the junior high school level.
-Only one sentence in this paragraph.

Given the outlined challenges in acquiring knowledge about trigonometric ratios, it is imperative to develop a learning trajectory that facilitates the development of students' understanding in this area. In accordance with (Simon, 1993) definition of hypothetical learning trajectory (HLT), Clements (2011) utilizes the notion of learning trajectory to characterize students' cognitive processes and acquisition of knowledge on a specific mathematical subject. It also encompasses the sequence of assumptions that guide their thinking as they progress through a set of learning activities aimed at generating the intended mental processes or actions. According to Clements (2011), this cognitive process guides students in developing a certain degree of thinking in order to attain specified learning objectives for the subject matter (Clements, 2011).

The Realistic Mathematics Education (RME) approach is strategically developed to effectively support the achievement of the intended learning trajectory. The instructional approach known as Realistic Mathematics Education (RME) is based on the philosophy of Hans Freudenthal, who views mathematics as a creative endeavor of humanity (Freudenthal, 1973; Gravemeijer & Terwel, 2000; Treffers, 1987; Van den Heuvel-Panhuizen & Drijvers, 2020). Realistic Mathematics Education enables students to engage in the learning process within the context of real-life situations and their everyday experiences (Yuanita et al., 2018). A study conducted by Aiyub also shown that learning designs that incorporate real or contextual issue designs can enhance students' algebraic thinking (Aiyub et al., 2024).

Commented [LS7]: Why RME in accordance with the student's understanding trigonometry ratio?

Students build understanding through informal processes. Students observe, see, and understand the environment in which they are located (Meika, 2018). Realistic Mathematics Education encompasses three key concepts: directed reinvention and progressive mathematical processes, didactic phenomenology, and model construct development (Gravemeijer, 1994). Within the teaching and learning process, these three concepts are further developed into six distinct features of RME: 1) The principle of activity, 2) The principle of reality, 3) The principle of level, 4) The principle of intertwinement, 5) The principle of interactivity, and 6) The principle of guidance (Panhuizen & Drijvers, 2014).

Under the title "Realistic Mathematics Education" in the Encyclopedia of Mathematics Education, Panhuijen elucidates the six concepts. Under the RME approach, the activity principle entails that students assume an active role in the learning process. This assertion is based on Freudenthal's theoretical understanding of mathematics as a human endeavor. The reality principle posits that students have the ability to use mathematics in order to address practical issues, and the process of learning should commence with tangible problems that can be expressed mathematically. The level principle posits that students progress through several levels of comprehension in order to grasp the connection between ideas and procedures in mathematics. The interconnection refers to the phenomenon where the students, while acquiring mathematical information, are able to employ a range of tools and methods to construct a comprehensive process of integrating different concepts. The interaction principle posits that the process of learning mathematics is not only an individual endeavor, but rather a social and reflective one, therefore facilitating a deeper comprehension of the subject. The guidance principle in mathematics education entails the concept of "guided reinvention," as proposed by Freudenthal. Under these circumstances, the learning

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process should motivate students to adapt in order to enhance their comprehension. It is imperative that the learning program aligns with a cohesive long-term learning trajectory (Panhuizen & Drijvers, 2014).

Method

The employed research methodology is design research as described by Gravemeijer and Van Eerde in 2009 (Gravemeijer & Van Eerde, 2009). This research methodology is focused on **local instruction theory (LIT)** to enhance the quality of learning by fostering effective collaboration between researchers and teachers (Gravemeijer & Van Eerde, 2009). Design research will enable the attainment of the goals of the research paradigm in the development of sequential activities and the enhancement of learning quality. The primary objective of this work is to develop a comparative trigonometry learning trajectory by utilizing the RME approach. The study was carried out in two cyclical phases, each following the design research stages outlined by Gravemeijer (2004): 1) preparatory design, 2) teaching experiment, and 3) retrospective analysis. The outcomes of the initial cycle activities that have been executed and the subsequent analytical review have contributed valuable insights (Gravemeijer, 2004).

During the first stage of the design process, the researcher personally observed the school and conducted direct interviews with the teachers and the students. Furthermore, the researcher gathered, perused, and comprehended several source materials pertaining to the challenges faced by students and teachers in grasping the notion of **trigonometric ratios**. Furthermore, the researcher examined many sources that elucidated the learning process activities based on the Realistic Mathematics Education (RME) methodology. A sequence of researcher actions served as a point of reference for the researcher in developing the Hypothetical Learning Trajectory (HLT). The Hypothetical Learning Trajectory (HLT) comprises three elements: learning objectives that will be focused on during the learning process, learning activities, and **hypotheses that will forecast the cultivation of students' cognitive abilities in the learning process** (Simon, 1993). The learning process hypothesis serves as the fundamental basis for constructing learning activities. This document is a compilation of learning activities designed by the researcher.

Learning objectives	Learning Activities	Student Thinking Conjecture
Students can identify the position of the right angle, opposite side, and hypotenuse in a right triangle.	Stretch the raffia from the table's base to the top of the head to form a right triangle.	A group of students can model the activity into a right triangle. One of the students was able to sketch the right triangle that was formed. Students can identify the position of right angles, front sides, hypotenuses, and side sides in sketches formed with various answers without knowing the reasons. Students need to be corrected in identifying the position of right angles.
Students can apply the definition of trigonometric ratios $\sin \theta$, $\cos \theta$, and $\tan \theta$ to	Measuring students' height and shadow	Students can draw sketches formed from the models presented. The definitions of trigonometric ratios $\tan \theta$, $\sin \theta$, and $\cos \theta$ can be used by students to the model of measuring the heights of both students and their shadows.

Commented [LS8]: -Some terms about design research are not commonly used, perhaps this is because you are translating from your language to English, just keep using the terminology according to the reference you are using.
-You need to add the source of the trigonometric ratio theory that you used to design the learning trajectory.

Commented [LS9]: Preliminary stage

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Commented [LS11]: As we know, in RME there is mathematization, whose theory do you use to design the learning trajectory. I didn't find it.

Commented [LS12]: this is what I meant from the previous comment, learning trajectory based on the theory from previous research. unfortunately, I didn't find this in your research.

Commented [LS13]: Where is the theory?

Commented [LS14]: Where is the theory?

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Learning objectives	Learning Activities	Student Thinking Conjecture
contextual problems using the concept of ratios in right triangles.		Students can determine the position θ of different angles to obtain different trigonometric ratio values.
Students can understand the relationship between the concept of similarity and trigonometric ratios.	Experiment to make three right triangles from paper	Students can make three similar right triangles from paper. Students can make sketches from paper formed into three similar right triangles. Students can identify the shortest and longest sides of each triangle formed. Students can determine the ratio between the shortest side and the longest side of each right triangle. Students can relate the concept of similarity with trigonometric ratios.
Students can determine the value of trigonometric ratios at particular angles (0° , 30° , 45° , 60° , and 90°).	Observations on the calendar image in the form of an equilateral triangular prism.	Students can sketch a calendar model as an equilateral triangular prism. Students can construct a right triangle in which one of the angles corresponds to a specifically defined angle. Students can determine the value of trigonometric ratios at particular angles (0°, 30°, 45°, 60°, and 90°).
The application of trigonometric ratios enables students to ascertain the length of sides in a right triangle that are not known.	Bed or bed observation	Students can make sketches or drawings of bed models. Students can identify the position of right angles, oblique angles, side angles, and front side positions in sketches. Students can apply trigonometric ratios to determine the length of unknown sides in problems presented related to beds.
Students can solve contextual problems related to right triangles using trigonometric ratios.	Observation of the image of a skier descending an iceberg	Students can identify the position of the angles, opposite sides and hypotenuses in a right triangle formed as a sketch of a picture of a skier descending an iceberg. Students can apply the definition of trigonometric ratios to skier problems. Students can solve the problems given in the case of skiers.

Table 1: Hypothetical Learning Trajectory in Teaching Experiment 1

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Within cycle 1, the research participants consisted of 25 10th-grade students who attended a high school located in Bandung City. In cycle 2, the research participants consisted of 25 10th-grade students from a different high school within the same municipal area. The HLT Design was first implemented in teaching experiment cycle 1 to assess the efficacy of the prepared learning activity design in investigating students' cognitive patterns. Moreover, the results of teaching experiment 1 indicated an enhancement in HLT. A refined Hypothetical Learning Trajectory (HLT) was implemented in phase 2 of the teaching experiment. The data collecting protocols encompass classroom observations, videos documenting learning activities, student workbooks, and recordings of interviews conducted with students.

Phase 1 of the retrospective analysis involved the collection and analysis of all the data by the researcher. An analysis was conducted to evaluate the outcomes of including the learning trajectory in teaching experiment 1 with the original HLT design. Moreover, the researcher investigated and examined the learning process activities included to determine the level of students' comprehension of the idea of trigonometric ratios. An analysis was conducted on the outcomes of the HLT enhancements implemented in teaching experiment 2, by comparing the initial HLT with the execution of the learning process.

The researcher selected 6 participant subjects to conduct interviews. The six participants had various abilities; two had high, medium, and low abilities. In addition, interviews were also conducted with the mathematics teacher of trigonometric ratios at the school that taught the students.

Results

The learning trajectory design discussed in this paper is a sequence of student activities aimed at comprehending the subject of trigonometric ratios through the Realistic Mathematics Education approach. The trigonometric ratios learning trajectory comprises six learning activities: elongating a raffia from the table's base to the student's head, measuring the student's height and shadow, creating three right triangles from paper, visually examining a calendar image in the shape of an equilateral triangular prism, observing a bed or bed, and observing an image of a skier descending an iceberg.

Activity 1: Observation of "Stretching the Raffia from the Base of the Table to the Top of the Student's Head"

The primary aim of activity 1 is to enable students to accurately determine the location of the right angle, the opposing side, and the hypotenuse within a right triangle. During this exercise, the teacher instructs students to showcase their use of contextual equipment found in the classroom, specifically a raffia, a table base, and one of their peers. The students are segregated into multiple groups. Each group comprises from 4 to 5 students. A single student from each group grasps one end of the raffia, while the other end is fastened to the bottom of the table foundation.

Furthermore, the other students in the group observed the shape between the student's height, the floor, and the raffia. The observation process is shown in Figure 1. Furthermore, by describing the teaching materials, the teacher directs students to sketch the shapes in the demonstration.

Each group sketched their demonstration and represented the information in a picture or symbol. One of the sketch results is shown in Figure 2. Students sketched the table legs, the student's height, and the raffia stretched to form a right triangle. They represented the student's height in an illustration of the student, and the raffia was represented as a line.

After sketching in demonstration activity 1, students began to identify the position of the right angle, the position of the angle θ , the position of the hypotenuse, the front side, and the side. Different groups of students exhibit variations in their results in identifying the position of the angle θ and the front and side.

In this case, students are asked to explain the reasons. Some groups can explain their reasons. For those who cannot explain, other groups are allowed to present their reasons so that the understanding they gain

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Commented [LS15]: In your term, what the difference between cycle and phase?

Commented [LS16]: HLT was first trialled in a pilot experiment. Please chechck the following book:

helps others build the same understanding.

To obtain findings through teacher guidance, the teacher provides guided reinvention for groups that do not yet understand the reasons for the position of the corners θ and the position of the front and side sides as in the following conversation.

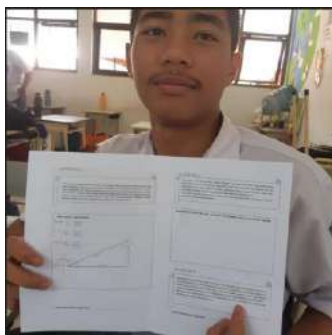


Figure 1: Activity 1 "Stretching a Raffia from the Base of the Table to the Top of the Student's Head"

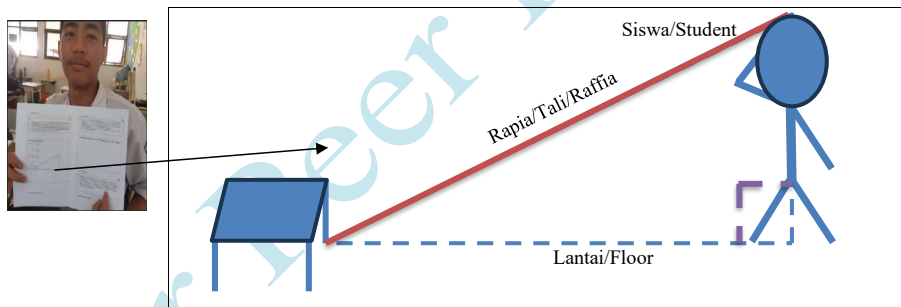


Figure 2: Example of Student Sketches in Activity 1

- Teacher : "Okay, S-1, please explain what you drew."
- S-1 : "Okay, ma'am, if you look at the instructions in the worksheet, we are asked to practice what is written in activity 1, and then we share roles; S-2 ties the raffia raffia to the base of the bottom table, S-3 holds the end of the raffia that is the result of extending S-2's raffia, S-4 reads what is on the worksheet, and I make the sketch."
- Teacher : "Okay, have you considered whether your sketch will be a right triangle?"
- S-1 : "At first, I did not, Ma'am, but when you said, "Look, if you draw it, is a right angle formed?" I remembered."
- Teacher : "Okay, how do you determine the corner points?"
- S-1 : "The benchmarks are at the end of the table legs, Leg S-3, and head S-3, Ma'am."

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- Teacher : "Why is that, S-1?"
- S-1 : "Yes, that is what it looks like when drawn, ma'am."
- Teacher : "Okay, the benchmarks that S-1 mentioned earlier make a move from the rapid raffia, right?"
- S-1 : "What do you mean, ma'am?"
- Teacher : "Mom, look, you represent the rapier raffia, the floor, and the height of S-2 with a line, right?"
- S-1 : "That is right, ma'am."
- Teacher : "Well, when you make that line, there is a phase where you turn; that is called a benchmark."
- S-1 : "Oh yes, that is right, ma'am, that is right."
- Teacher : "S-1, try to determine the right angle, slanted side, front side, and side in the sketch you have made!"
- Student : "This one is a right angle, this one is the hypotenuse, this one is the front side, and this one is the side side." (pointing to the picture).
- Teacher : "How do you determine a right angle, S-1?"
- Student : "There is a line like this and like this, ma'am." (pointing at the picture).
- Teacher : "Now, try to observe; what degree angle do the two lines form, or what angle is it called then?"
- Student : "Oh yes, ma'am, right angles."
- Teacher : "How do you determine the hypotenuse?"
- Student : "The side in front of the right angle, ma'am."
- Teacher : "How do you determine the front side?"
- Student : "This one, ma'am? Yes, the one in front of this corner (pointing to one of the corners of the picture)".
- Teacher : "Okay, how do you determine the side?"
- Student : "The one on the front side, ma'am."

In this activity, students have determined the expected position even though it is not specific because it still depends on the position of the angle θ . This identification procedure enhances pupils' ability to determine the precise location of the angles and sides within a right triangle. Upon completing the task, the teacher and students jointly draw conclusions and engage in reflections regarding the process of determining the position of the right angle, the hypotenuse, and the angles θ that impact the identification of the front side and side of a right triangle.

Activity 2: Observation "Measurement of Students' Height and Shadow"

The aim of activity 2 is to enable students to utilize the definition of trigonometric ratios $\sin \theta$, $\cos \theta$, and $\tan \theta$ in practical situations by employing the notion of comparison in right triangles. As part of this exercise, students are required to transform issues involving comparison in right triangles into tasks that may be directly practiced in groups. The teacher instructs a single student from a group to face away from the lamp until a shadow forms in front of the said student. Moreover, the teacher instructs students to illustrate the encountered shape in this exercise.

A student in his/her group is assigned to calculate the height of the student whose height will be measured and the height of the shadow that occurs. This activity is carried out alternately so that all group members have their height and shadow height measured. The measurement data is shown in a tabular format on the student's designated worksheet. Figure 4 is an example of student work in representing it in table form.

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Furthermore, the teacher determines the angle θ as the angle formed between the shadow and the oblique line. In this exercise, the teacher instructs the group to calculate the ratio of the student's height to the shadow height using the available data. This task aims to help students recall the process of determining the value of trigonometric ratios by drawing the resulting shapes.

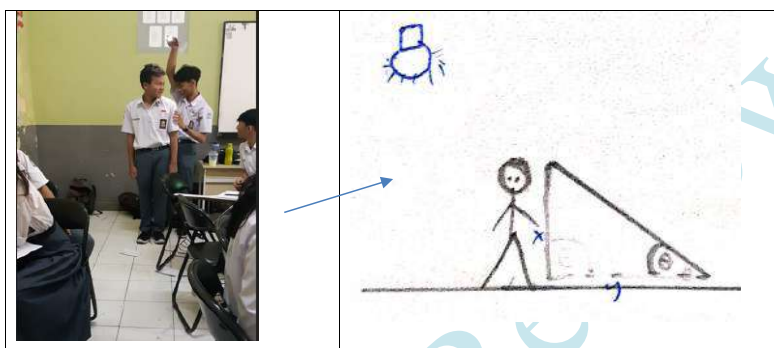


Figure 3: Activity 2 and Example of Student Sketches in Activity 2

	Kamu	Nama temanmu	Nama temanmu	Nama temanmu	Nama temanmu
	<u>Nabila</u>	<u>Bunga</u>	<u>Naufal M</u>	<u>Jahsyi</u>	<u>Nalvin</u>
Tinggi badan	155	153	173	169	161
Tinggi bayangan	165	163	183	179	171

	Your name	Your Classmate's name	Your Classmate's name	Your Classmate's name	Your Classmate's name
	<u>Nabila</u>	<u>Bunga</u>	<u>Naufal M</u>	<u>Jahsyi</u>	<u>Nalvin</u>
Height	155	153	173	169	161
Shadow Height	165	163	183	179	171

Figure 4: Example of a table of results measuring height and shadow in activity 2.

The objective of this project is for students to accurately calculate the trigonometric ratio values of $\sin \theta$, $\cos \theta$, and $\tan \theta$ in every triangle sketch they create (Figure 5). In each activity, the teacher monitors students through visits to each group and guides students in solving problems. Each student can use their way as an alternative, both formally and informally. The teacher facilitates students in revisiting the knowledge they have acquired about determining trigonometric ratio values by providing scaffolding.

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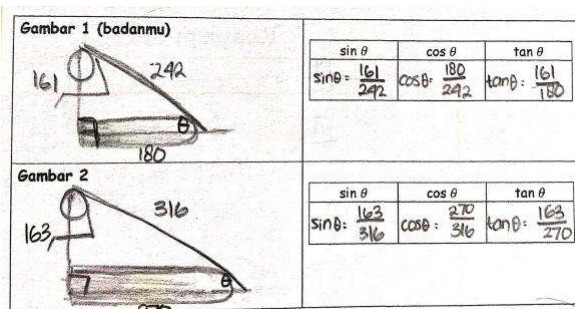


Figure 5: Example of Sketch and Application of Trigonometric Ratios Values in Activity 2

One of the students (S-5), as a representative of one of the group members, participated in presenting activity two and explained how the values of $\sin \theta$, $\cos \theta$, and $\tan \theta$ were obtained. After sketching the experiment, this group identified the right angle to determine the other positions. The following is a dialogue explaining the concept of trigonometric ratios described by the students.

Teacher : "Okay, S-5. Explain how your group determines each trigonometric ratio value requested in the table."

S-5 : "Okay, Ma'am, first we sketch according to our height in the group, then, from the sketch formed, we first determine the right angle, draw the symbol on the sketch, then determine the angle θ"

Teacher : "Okay, do you mean identifying the position of a right angle?"

S-5 : "Yes, ma'am."

Teacher : "What do you mean by determining the angle?"

S-5 : "So that we can determine the front and side, ma'am."

Teacher : "Okay, what is the next process?"

S-5 : "Earlier, after determining the angle position θ , we then wrote down each of our heights according to what was measured earlier, and wrote down the height of the shadow."

Teacher : "Okay, how do you determine the hypotenuse?"

S-5 : "Using the Pythagorean theorem, ma'am, with the help of a calculator."

Teacher : "Okay, how do you calculate $\sin \theta$, $\cos \theta$, and $\tan \theta$?"

S-5 : "Well, because the value of $\sin \theta$ is the front per hypotenuse, look at the picture (while pointing to picture 1 in UBA 2 Activity 2); the front of the angle θ is the front side, which is 161, and the hypotenuse is 242 so the $\sin \theta$ is 161 per 242."

Teacher : "How to calculate $\cos \theta$ and $\tan \theta$?"

S-5 : "Same here, ma'am, just a different position according to what was learned previously: $\cos \theta$ means side per hypotenuse, and $\tan \theta$ means front per side."

The conversation above helped all students understand the concept of trigonometric ratios correctly. At the end of the activity, one student reflected on the learning that day and concluded how to apply trigonometric ratios correctly.

Activity 3: Experiment with "Making Three Right Triangles from Paper"

The teaching aim of exercise 3 is to enable students to comprehend the correlation between the notion of

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resemblance and trigonometric ratios. The students are divided into multiple groups. Each group comprises from 4 to 5 students.

In this activity, the teacher asks each group to prepare the necessary tools, scissors, paper, rulers, and stationery through student worksheets. Furthermore, each group makes a picture through the experiment of cutting rectangular paper so that two equal and similar right triangles are obtained. Students can determine the similarity in three equal and similar right triangles. The teacher guides students in finding the equal and similar sides of three equal and similar right triangles.

Students use several alternative methods to find the concept of similarity, namely diagrams and teaching aids. Examples of activities that occur as in Figure 6 and 7. The results of the presentation in one group are the results of group discussions using diagrams, while the other group uses concrete teaching aid models made by the group. The results of both alternatives in each group find a way to build the concept of similarity and trigonometric ratios in right triangles.

Students are guided to find formal patterns of similarity in right triangles through guided reinvention through directed questions. Students are guided to find a formal way to use Geometry theorems by comparing three similar right triangles. To facilitate a gradual comprehension of the notion of similarity, participants are encouraged to identify the shortest and longest sides of each right triangle under comparison.



Figure 6. Activities and Examples of Student Responses using Teaching Aids in Activity 3.

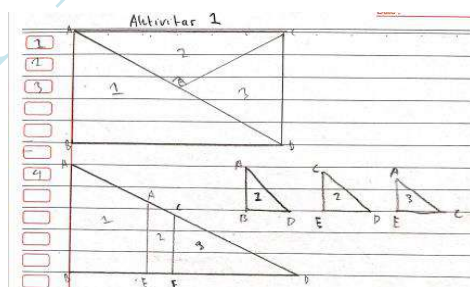


Figure 7. Example of Student Responses using Diagrams in Activity 3.

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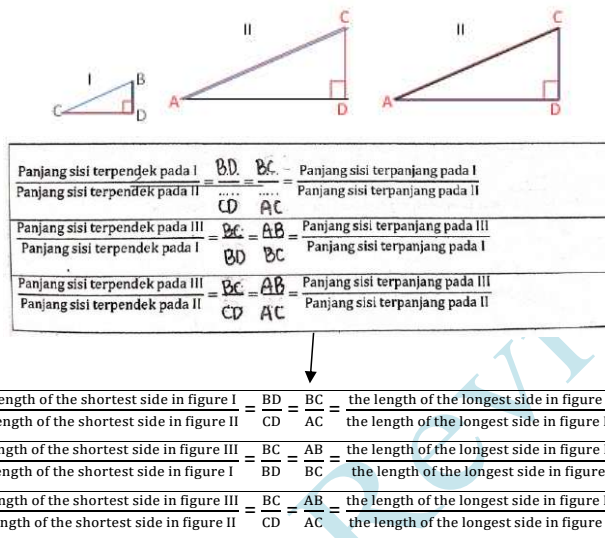


Figure 8. Example of Student Responses in Using the Concept of Comparison in Activity 3.

The teacher directs students in determining the location of the shortest and longest sides of each triangle, enabling them to articulate the resulting comparative thinking patterns. One example of a group response is presented in Figure 8.

In this exercise, the teacher presents questions to facilitate students' comprehension of the analogy between the notion of similarity and trigonometric ratios. Figure 9 displays the provided questions. In this inquiry, the teacher presents contextual inquiries formatted like a garbage can including a logo. The logo is situated on a base of an equilateral triangle of 6 centimetres in side length. The teacher asks students to calculate the logo's height, sketch the triangle formed, and explain the concept used in solving the given problem.



The trash can logo featured below is designed as an equilateral triangle. The dimension of the side measures 6 centimeters. What is the estimated vertical dimension of the logo? Begin by creating a preliminary outline of the triangle. Which concept are you employing? Are there any other concepts available apart from the one you have employed? What is the procedure?

Logo tempat sampah berikut berbentuk segitiga sama sisi dengan panjang sisi 6 cm. Berapa kira-kira tinggi logo tersebut? Buatlah sketsa segitiga terlebih dahulu. Konsep apa kah yang kamu gunakan? Adakah alternatif konsep selain yang sudah kamu gunakan? Bagaimana caranya?

Figure 9. Example of Questions on the Relationship between the Concept of Similarity and Trigonometric Ratios in Activity 3.

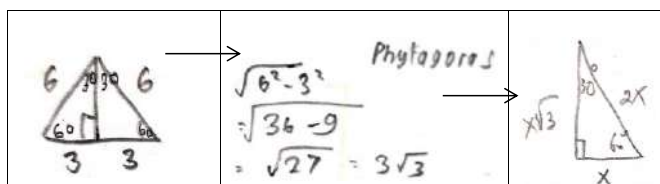


Figure 10. Example of Student Responses to Contextual Questions in Activity 3.

In Figure 10, one group sketches the shape formed and then calculates the height of the logo using the Pythagorean Theorem. They then use the concept of similarity to check the results they obtained.

At the conclusion of the activity, the teacher and students synthesize the idea under study concerning the correlation between similarity and trigonometric ratios.

Activity 4: Observation of "Calendar Image in the form of an Equilateral Triangular Prism"

The learning objective of activity 4 is that students can determine the magnitude of the trigonometric ratio at particular angles (0° , 30° , 45° , 60° , and 90°). For this exercise, the teacher organizes students into groups of 4-5. Using the student worksheet, the teacher instructs each group of students to visually analyze a calendar image as an equilateral triangular prism, as depicted in Figure 11. Subsequently, the teacher directs the students to examine the face of the calendar in order to derive an equilateral triangle. Students are instructed to construct a perfectly symmetrical triangle measuring 2 units in length and annotate each corner. Subsequently, the teacher instructs the students to draw a vertical line from one of the highest points of the triangle to the center of the side directly in front of the triangle, resulting in two equal sections of the triangle.

The students are instructed to redraw a portion of the triangular again, note the length of each side, and record the known angles. Utilising their acquired knowledge, students employ the Pythagorean Theorem to determine the length of the side that is not known. Subsequently, students are required to ascertain the numerical values of the trigonometric ratios $\sin \theta$, $\cos \theta$, and $\tan \theta$ for the particular angles 30° and 60° . Students compile the findings into a tabular format. At the conclusion of the question, the teacher makes an assumption about the length of the shortest side of the triangle, denoted as x . Students are then tasked with determining the lengths of the remaining sides while still utilizing the variable x .

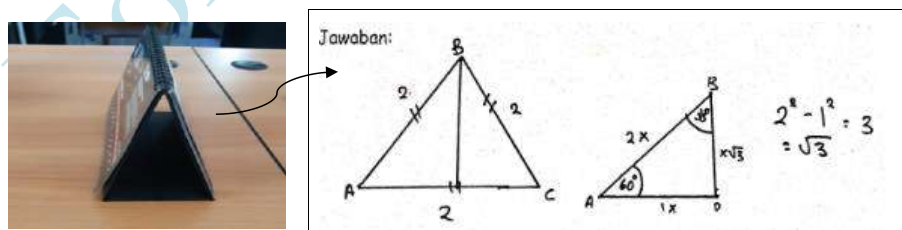


Figure 11. Calendar Image and Example of Student Responses in Making Sketches in Activity 4

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By employing guided innovation with variables x , students are direct to ascertain the comparison of the lengths of the sides of the triangle they have constructed. The student effort in activity 4 of creating a calendar sketch from the front is exemplified in Figure 11. Figure 12 illustrates student replies in activity 4 when answering the problem of calculating the value of trigonometric ratios at particular angles of 30° and 60° in a right triangle.

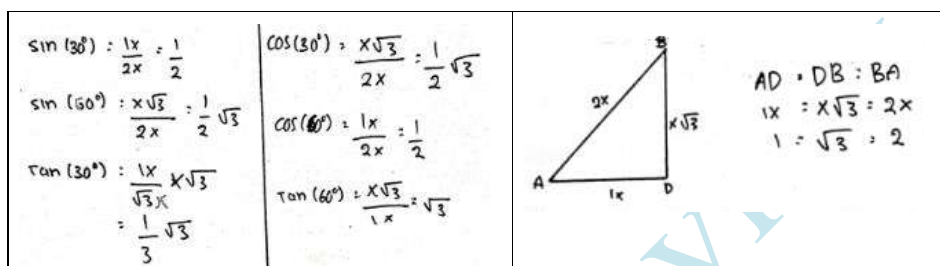


Figure 12. Example of Student Responses regarding memorable and unique 30° angles 60° in Activity 4

In the student worksheet activity 4, the teacher also guides students to determine the magnitude of the trigonometric ratio values at the particular angles of 45° in a right triangle. In this case, students are asked to draw an isosceles right triangle with each side measuring 1 unit. After that, students label each angle. Using the knowledge they have built, students determine the hypotenuse length using the Pythagorean Theorem. After that, students determine the trigonometric ratio values $\sin \theta$, $\cos \theta$, and $\tan \theta$ for the particular angle 45° . Figure 13 shows one of the student responses in their group.

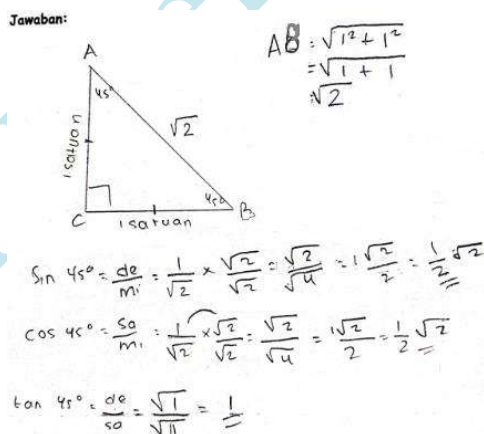


Figure 13. Example of Student Responses Regarding Particular angles 45° in Activity 4

In the initial stage, students sketch an isosceles triangle with a length of 1 unit. Students mark one line on each side that has the same length. In this case, students can already represent according to the question's

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intent. Furthermore, students are accustomed to representing the information obtained in a sketch. Figure 13 displays students' labels indicating the angle size of 45 degrees for all angles except the right angle. Subsequently, the image illustrates the process by which students get the hypotenuse value by the use of the Pythagorean Theorem. By the conclusion of the exercise, students ascertain the numerical value of the trigonometric ratio at the particular angles of 45 degrees and 0 degrees in a right triangle. Lastly, students display the outcomes for numerous specific angles in a tabular format, as depicted in Figure 13.

	Rumus	30°	45°	60°
sin	sin θ = $\frac{\text{sisi depan}}{\text{sisi miring}}$	$\frac{1}{2}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}\sqrt{3}$
cos	cos θ = $\frac{\text{sisi samping}}{\text{sisi miring}}$	$\frac{1}{2}\sqrt{3}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}$
tan	tan θ = $\frac{\text{sisi depan}}{\text{sisi samping}}$	$\frac{1}{3}\sqrt{3}$	1	$\sqrt{3}$

Figure 14. Example of Student Responses Represented in a Table in Activity 4

By the conclusion of the exercise, the teacher and students reached a general conclusion on the determination of the value of trigonometric ratios at particular angles.

Activity 5: Observation of "Bed"

The purpose of exercise 5 is to enable students to use trigonometric ratios in order to ascertain the length of dimensions of unknown sides in a right triangle. For this exercise, the teacher instructs students to be organized into many groups. Each group comprises from 4 to 5 students. The instructor distributes a worksheet to the students that includes instructions for action 5. Each group scrutinizes the image (refer to Figure 15). Figure 16 depicts a triangular figure encompassing the stairs, bed, and floor. Students are instructed to perform the horizontal mathematization method by creating a diagram that represents the triangle produced by the three elements.



Figure 15. Activities and Questions in Activity 5

Furthermore, students are asked to identify whether the shape formed is a right triangle. The teacher

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determines the angle size θ at one corner of the triangle and the distance from the bed to the stairs. Students are asked to represent the known data in a sketch, and the teacher instructs the students to determine the displacement between the floor and the terminus of the stairs that constitute a right triangle by employing the tangent ratio.

From activity 5, the findings of each group are poured into a sketch, as shown in Figure 16.

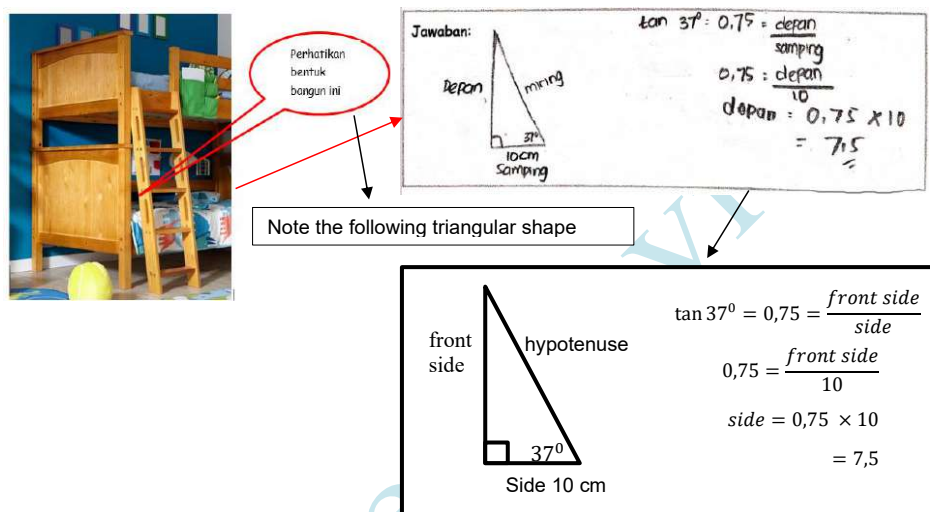


Figure 16. Questions and Student Responses Representing Data into Sketches in Activity 5

Through teacher guidance, students pour out their understanding by sketching a right triangle between the bed, floor, and stairs. At the beginning of the worksheet, students are given questions to identify whether the sketch formed is a right triangle. Students are asked to give a right angle sign if the triangle formed is a right triangle. Furthermore, the teacher provides information through the worksheet that the angle formed between the stairs and the floor is 37° and the distance from the bed to the stairs is 10 cm. Students are asked to calculate the distance from the floor to the end of the stairs that form a right triangle using the tangent ratio.

Following the completion of the given problems by all groups, one group will deliver the outcomes of their group discussion to the entire class. When employing trigonometric ratios to calculate the length of the unknown side of a right triangle, students articulate their understanding of the concept. At the conclusion of the activity, the teacher poses questions to assess the extent to which student's knowledge has been developed, as shown in the dialogue below.

- Teacher : "Okay, how do you find the length of the unknown side in a right triangle?"
- S-6 : "First, draw a sketch formed from the existing problem, then determine what information is contained in the problem, whether the value of the front side, hypotenuse, or side, even whether the angle value is also known. Next, from the information we get, we see

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what principles fulfill the information collected so that we know the principle of trigonometric ratios that we use."

Teacher : "Can you give an example?"

S-6 : "For example, based on the bed problem earlier, after we sketch a right triangle, we obtain information that the angle formed between the stairs and the floor is 37° and the distance from the bed to the stairs is 10 cm. That means we label the angle as large 37° on the sketch and label 10 cm on the sketch for the side between the bed and the stairs. Then, we refer to the sketch to determine the information we obtain, namely the side, the size of the angle, and the tangent value 37° . This information means we use the trigonometric ratios of tangent, which is the comparison between the value of the front side and the value of the side.

Activity 6: Observation of "Picture of a Skier Descending an Iceberg"

The aim of task 6 is to enable students to accurately solve contextual situations involving right triangles by applying trigonometric ratios. This activity divides students into several groups consisting of 4-5 students. The teacher provides student worksheets as learning materials for activity 6. Each group observes the images presented (see Figure 17).



Figure 17. Questions in Activity 6

The student worksheet in this activity presents open-ended questions. Each group or student has the opportunity to have a different value. This activity encourages students to freely determine the angle's value, the opposite side's value, and the hypotenuse. This question is taken from a Geometry book by Larson (Larson et al., 2007). In this question, students are asked to observe a picture of a skier going down an iceberg. Through the worksheet, the teacher asks students to determine the angle size according to their wishes. In addition, students determine the height of the iceberg themselves. Furthermore, from the data and values they obtain, students determine the track length traveled by the iceberg skier. The teacher allows students to ask the teacher if they need a trigonometric ratio value from a particular angle value.

In activity 6, students express their ability to think logically through questions. The teacher guides students in determining values so that they can think logically. Referring to the values they present, students must determine for themselves the principle of trigonometric ratio values that they must use to resolve the issue. Figure 18 shows an example of student responses in activity 6. The results of the students' responses show that the student assumes the size of the angle requested in the question with the value 75° . The height of the iceberg is 90 units. From the value given by the student, he calculates the length of the track traveled by the skier using the comparison value $\sin \theta$ so that the desired track length value is obtained.

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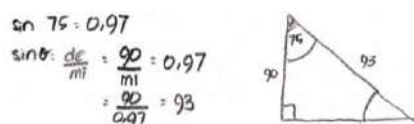


Figure 18. Example of Student Responses to Activity 6

From several findings from each group, one group presents the results of their discussion and simultaneously concludes the learning outcomes.

DISCUSSION

Learning activities using the Realistic Mathematics Education approach regarding trigonometric ratios followed by students consist of six activities, namely raffia stretching activities, student height measurement activities, activities to make three right triangles from paper, activities to observe calendar images, activities to observe bed images, and activities to observe skier images. Each activity has its purpose and is interrelated and continuous. The goals of each action are as outlined below: Firstly, stretching the raffia allows students to determine the location of the angles and sides of a right triangle. Secondly, measuring the height of students and their shadows allows students to apply the concept of trigonometric ratios. 3) Creating a teaching tool including three right triangles is intended to facilitate students' comprehension of the correlation between the notion of resemblance and trigonometric ratios. 4) The purpose of observing the calendar image is to permit students to ascertain the numerical value of trigonometric ratios at specific angles. The objective of viewing the bed image is to facilitate students in applying trigonometric ratios to ascertain the length of the desired side in a right triangle. Similarly, observing the skier image seeks to empower students in solving contextual difficulties by employing the concept of trigonometric ratios. Every worksheet provided incorporates the specific features of the Realistic Mathematics Education pedagogy in relation to trigonometric ratios. These findings corroborate Subekti's assertion that student worksheets will enhance the learning process (Subekti & Prahmana, 2021).

The implementation of the Realistic Mathematics Education approach in different learning process activities can facilitate students' comprehension of mathematical skills and enable teachers to effectively assist students towards attaining learning objectives (Ekowati & Nenohai, 2017). The reason for this is that the mathematics learning process, which follows the Realistic Mathematics Education approach, leverages the surrounding world to encourage students to generate ideas and allow them to develop their knowledge (Nirawati et al., 2021).

According to research conducted by Sukendra (2020), a sequence of learning process activities utilizing the RME approach significantly facilitates students' comprehension of the idea of trigonometric ratios (Sukendra, 2020). Thus, the deliberate learning trajectory promotes students to construct their knowledge, enhance their current skills, and effectively address challenges across many subject areas (Prahmana et al., 2017; Nursyahidah et al., 2020; Wijaya et al., 2021; Kuncoro et al., 2023).

CONCLUSIONS

The objective of this work is to develop a learning trajectory for trigonometric ratios by utilizing the RME approach. The findings of this study suggest that implementing the RME approach in designing a learning trajectory for trigonometric ratios can enhance students' mathematical proficiency in comprehending the concepts of trigonometric ratios. The learning trajectory designed using the RME approach consists of six

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activities. Through a series of continuous and ongoing activities in the six activities, students find it easier to build their understanding in understanding the concept of trigonometric ratios. Students build their understanding starting from their ability to identify angle and side positions, apply the definition of trigonometric ratios to contextual problems, understand the relationship between similarity and trigonometric ratios, determine the magnitude of trigonometric ratios values at particular angles, apply trigonometric ratios, and solve contextual problems using trigonometric ratios.

This research is expected to encourage other researchers to design learning trajectories using the same topic but different approaches. The researcher hopes that the results of this design can contribute to the researchers' contribution to building a generation according to the nation's expectations.

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**3. Bukti Konfirmasi Submit Revisi
Pertama, Respon Kepada
Reviewer, dan Artikel Yang
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DINI ANDIANI <diniandiani@upi.edu>

to Bronisław ▾

Mar 28, 2025, 6:35 AM



Subject: Resubmission of Revised Manuscript for MTRJ

Dear Chief Editor, Bronisław Czarnocha,

I hope this email finds you well.

Following the reviewers' comments and the MTRJ submission guidelines, we have carefully revised our manuscript titled "**Learning Trajectory of Trigonometric Ratios Applying Realistic Mathematics Education**". We have addressed all the reviewers' comments to improve the clarity and quality of our work.

To the resubmission requirements, we are submitting the following documents:

1. A revised manuscript without tracked changes.
2. A revised manuscript with tracked changes.
3. A correction matrix detailing the modifications, including the page and line numbers, original text, reviewers' comments, and the revised text or rebuttal, where necessary.

We appreciate the valuable feedback provided by the reviewers and the editorial team, which has greatly enhanced our manuscript. Please let us know if any further revisions or clarifications are needed.

Thank you for your time and consideration. We look forward to your response.

Best regards,

Dini Andiani

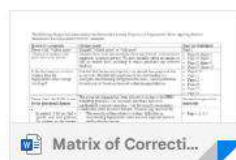
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diniandiani@upi.edu

diniandiani367@gmail.com

diniandiani367@gmail.com

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Learning Trajectory of Trigonometric Ratios Applying Realistic Mathematics Education

Dini Andiani^{1,2}, Darhim¹, Elah Nurlaelah¹

¹Department of Mathematics Education, Universitas Pendidikan Indonesia, Indonesia,

² Department of Mathematics, Universitas Bale Bandung, Indonesia

diniandiani@upi.edu, diniandiani367gmail.com, darhim0355@gmail.com

Abstract: Trigonometry materials are still considered complex materials at the secondary level. It is necessary to design an appropriate trigonometric ratios learning trajectory so that the students can quickly build the concept of trigonometric ratios. This study aims to design a learning trajectory for trigonometric ratio material through Realistic Mathematics Education (RME). This study uses a research design involving 10th-grade high school students in Bandung City in the 2023-2024 academic year. The research data was collected through data collection using written test instruments, student worksheets, observations, interviews, and documentation. The results of the study showed that the trigonometric ratios learning trajectory consisted of raffia stretching activities from the table base to the student's head, measuring the student's height and shadow, experimenting with making three right triangles from paper, observing a picture of a calendar in the form of an equilateral triangular prism, observing a bed or bunk bed, and observing a picture of a skier going down an iceberg. Students can quickly build the concept of trigonometric ratios from several activities provided. In addition, various learning flows from various activities make it easier for students to build knowledge about trigonometric ratios and solve problems presented contextually.

Keywords: Learning Trajectory, Trigonometric Ratios, Realistic Mathematics Education

INTRODUCTION

Trigonometry materials are vital in everyday applications such as statistics, surveying, architecture, and other engineering fields (Weber, 2005a). However, building knowledge about trigonometry is challenging (Blackett & Tall, 1991; Demir et al., 2012; Kendal & Stacey, 1997; Ngu & Phan, 2020; Sugianto et al., 2023). To understand the concept of trigonometry, students need to learn several prerequisite mathematical concepts that serve as a foundation. These

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prerequisite concepts include the Pythagorean theorem and the properties of right triangles, which are essential in understanding trigonometric ratios. Kamber's Research (2017) stated that students struggle to grasp trigonometry through the unit circle (Kamber & Takaci, 2017). Additionally, they face difficulties distinguishing between trigonometry in right triangles, the unit circle, and the relationship between the two representations (Maknun et al., 2019). As a result, many students fail to understand trigonometric ratios properly, even though these ratios are fundamental in learning trigonometry (Weber, 2005; Nordlander, 2022). Several studies (Hershkowitz et al., 2016; Lessani et al., 2017; Lu'lu'il Maknun et al., 2022; Maphutha et al., 2023; Weber, 2005b) also indicate that traditional approaches lead students to often rely on memorizing formulas rather than understanding the reasoning behind trigonometric ratio concepts.

One example of students' ability to master trigonometric ratios is Andiani's Research results (Andiani & Nurlaelah, 2024). The test questions given to students are presented in Figure 1.

Seseorang berdiri di geladak kapal yang berada 10 m di atas permukaan air, mengamati bahwa sudut elevasi puncak bukit adalah 60° dan sudut depresi dasar bukit adalah 30° . Hitunglah jarak bukit dari kapal dan tinggi bukit tersebut.

An individual stands on the deck of a ship 10 meters above the water's surface and observes a particular distance of a hill. The angle of elevation from the observer's position to the top of the hill is 60 degrees, while the angle of depression to the base of the hill is 30 degrees. Based on this information, determine the horizontal distance between the ship and the hill, and also the total height of the hill.

Figure 1. Example of Test Questions in Andiani et al.'s Research (2024)

The results of one student's answer in solving the problem are shown in Figure 2.

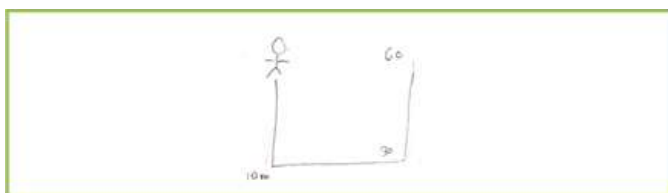


Figure 1.4. Example of Test Answers in Andiani et al.'s Research (2024)

The results of students' answers show that students are starting to be able to bring up their ideas or concepts in the form of mathematical models. In examining these errors, questions arise (Do students not understand the correct symbols to use? Do students not understand trigonometry? Do

students understand the meaning of elevation angles? Do students make the errors due to their prior knowledge, or are errors occurring during the learning process?). Student errors occur when students understand a concept in a particular context and adapt it to be applied in another context (Brousseau, 2002). The procedures that students should have written cannot be skipped because students have difficulty in the initial steps of representing the problem in a model. In this case, students have gone through the horizontal mathematization process but have not gone through the vertical mathematization process in its entirety; this error dominates students (Meika, 2018).

Student errors indicate that the students have difficulty solving contextual problems. This finding is in line with Haryani's research that the students still find it difficult to construct an understanding of trigonometry material, especially in the context of story problems or real life (Saputri et al., 2020; Widyanti et al., 2022). In addition, a similar opinion was expressed by Insani and Kadarisma in their research that in studying trigonometry, the students face epistemological obstacles caused by the limitations of their knowledge tied to a particular context. As a result, when faced with different situations, the students struggle to apply or adapt this knowledge to the new context (Insani & Kadarisma, 2020; Nasrulloh & Zahiro, 2023). Therefore, teachers need to know the causes of the difficulties experienced by the students and what the students think in their learning process so that these difficulties occur (Pramesti et al., 2024; Rianasari & Guzon, 2024; Wilson et al., 2013, 2015).

Learning trajectory represents students' cognitive processes in understanding concepts (Clements, 2011), with each student following a unique path (Chahine & Grinshpon, 2020; Ellis et al., 2014; Tahiri et al., 2017; Zhao et al., 2024). While previous studies have explored learning trajectories for trigonometric ratios (Hamdani et al., 2023; Hidayati & Armiati, 2024; Saputra et al., 2021), none have specifically applied the Realistic Mathematics Education (RME) approach. This study aims to develop a learning trajectory for trigonometric ratios using RME and answer the following main questions: How is the learning trajectory for trigonometric ratios of high school students with the application of Realistic Mathematics Education?

This article is structured into several sections. The next section explores the theoretical framework and the methods employed in the study. Subsequently, the research findings are presented and analyzed in depth. The article concludes by summarising the key findings and the study's implications.

THEORETICAL FRAMEWORK

Structured learning trajectories provide a clear pathway for students to develop mathematical understanding. Simon (1993) introduced the concept of Hypothetical Learning Trajectories (HLT), later expanded by Clements (2011) to describe students' cognitive processes in mathematics. This

framework consists of guided learning activities that foster conceptual development, helping the students progress through different reasoning levels and achieve specific learning goals.

This study adopted the Realistic Mathematics Education (RME) approach to help the students better understand trigonometric ratios. The Realistic Mathematics Education (RME) approach aligns with the student's understanding of trigonometric ratios because it emphasizes contextual learning, allowing the students to build concepts based on real-world experiences (Gravemeijer, 1994). Through authentic situations, such as calculating the height of a building with a shadow or determining the elevation angle in navigation, the students can understand the concept of trigonometric ratios more meaningfully than simply memorizing formulas (M. van D. H. Panhuizen, 2003). In addition, RME encourages visual exploration using tools such as triangle diagrams, interactive math software, or physical props so that the students can directly observe the relationship between angles and side lengths in triangles (Freudenthal, 1991). This approach also emphasizes mathematical modelling, where the students learn and apply the theory to solve real problems, improving their conceptual understanding and critical thinking skills (Gravemeijer & Doorman, 1999). Thus, RME enables students to understand how to calculate trigonometric ratios, why these concepts are important, and how to use them in real-world situations.

Realistic Mathematics Education (RME) emphasizes mathematization as the main principle to help students understand mathematical concepts gradually, starting from real situations to achieving formal forms. Hans Freudenthal (1991) developed the theory of mathematization in RME by distinguishing two types of mathematization, namely horizontal mathematization, which transforms real-world problems into mathematical models, and vertical mathematization, which processes mathematical models into more formal forms (Freudenthal, 1991).

In addition, the concept of mathematization in RME is also strengthened by Treffers (1987), who emphasized that mathematization is the core of the RME approach, allowing the students to explore and construct mathematical concepts through meaningful activities (Treffers, 1987). Meanwhile, Gravemeijer (1994) emphasized the importance of designing learning trajectories in RME that consider how students develop understanding progressively through the mathematization process (Gravemeijer, 1994).

In the Realistic Mathematics Education (RME) approach, horizontal and vertical mathematization support modelling activities into right triangles (Freudenthal, 1991; Treffers, 1987). Horizontal mathematization occurs when the students translate real-world situations, such as measuring the length of a shadow and the height of a student and then recognizing the proportional relationship in the situation. Furthermore, through vertical mathematization, the students construct more formal mathematical models, for example, by comparing the height and shadow length ratio using the concept of triangle similarity or trigonometric ratios such as the tangent of an elevation angle.

In addition, the concept of a model as a bridge in RME helps students move from informal understanding to more abstract mathematical models (Gravemeijer, 1994). The students begin

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learning from real-world situation models, such as shadows formed by light in the classroom, then gradually move to formal mathematical models, such as right triangles and trigonometric ratios. These models serve as a tool to help the students understand how the relationship between elements in a physical system can be represented in a more abstract mathematical form. In this learning, a real-world context is an important starting point to facilitate the students' understanding of mathematical concepts (De Lange, 1996). Measuring the students' heights and shadows allows the students to understand the concepts of triangle similarity and trigonometric ratios more intuitively.

RME is based on the philosophy of Hans Freudenthal, who views mathematics as a human activity involving creativity and problem-solving (Freudenthal, 1973; Gravemeijer & Terwel, 2000; Treffers, 1987; Van den Heuvel-Panhuizen & Drijvers, 2020). This approach allows students to develop mathematical understanding through real-life contexts and experiences (Yuanita et al., 2018). Research by Aiyub et al. (2024) also highlights that learning designs incorporating real-world problems can improve students' algebraic thinking and overall mathematical understanding (Aiyub et al., 2024).

The students build understanding through informal processes. They observe, see, and understand the environment wherever they are (Meika, 2018). Realistic Mathematics Education encompasses three key concepts: directed reinvention and progressive mathematical processes, didactic phenomenology, and model construct development (Gravemeijer, 1994). Within the teaching and learning process, these three concepts are further developed into six distinct features of RME: 1) The principle of activity, 2) The principle of reality, 3) The principle of level, 4) The principle of intertwining, 5) The principle of interactivity, and 6) The principle of guidance (M. V. den H. Panhuizen & Drijvers, 2014).

Based on Freudenthal's view of mathematics as a human activity, the activity principle emphasizes the student's active role in learning. The reality principle encourages learning through real-world problems that can be mathematized. The level principle describes the students' progression through different levels of understanding. The interconnection principle highlights integrating various concepts using multiple tools and methods. The interaction principle sees learning as a social and reflective process. Lastly, the guidance principle, rooted in Freudenthal's "guided reinvention," ensures learning aligns with a structured long-term trajectory. (M. V. den H. Panhuizen & Drijvers, 2014).

The students often have difficulty understanding the concept of trigonometric ratios, especially in relating the lengths of the sides of a triangle to the angles (Maknun et al., 2019). In addition, several studies have shown that they fail to understand the concept of trigonometric ratios even though they have been taught formally (Weber, 2005; Nordlander, 2022). Another difficulty often faced is distinguishing between the right triangle approach and the unit circle (Kamber & Takaci, 2017). Given the importance of understanding this concept, this study aims to develop a learning

trajectory that helps students overcome these difficulties, with the Realistic Mathematics Education (RME) approach as the main theoretical basis.

METHOD

The methodology employed is design research as described by Gravemeijer and Van Eerde in 2009 (Gravemeijer & Van Eerde, 2009). This research methodology is focused on learning trajectory to enhance the quality of learning by fostering effective collaboration between researchers and teachers (Gravemeijer & Van Eerde, 2009). Design research will enable the attainment of the goals of the research paradigm in the development of sequential activities and the enhancement of learning quality. The primary objective of this work is to develop a comparative trigonometry learning trajectory by utilizing the RME approach. The study was carried out in two cyclical phases, each following the design research stages outlined by Gravemeijer (2004): 1) preliminary design, 2) teaching experiment, and 3) retrospective analysis. The outcomes of the initial phase activities that have been executed and the subsequent analytical review have contributed valuable insights (Gravemeijer, 2004).

In the initial design stage, the researcher observed the school and interviewed teachers and students. Additionally, the researcher collected and analyzed various sources related to the challenges the students and teachers faced in understanding trigonometric ratios and reviewed materials on learning activities based on the Realistic Mathematics Education (RME) approach. These analyses served as a reference for developing the Hypothetical Learning Trajectory (HLT), which includes learning objectives, learning activities, and hypotheses on students' cognitive development (Simon, 1993). These learning hypotheses form the foundation for designing the learning activities developed by the researcher.

Learning objectives	Learning Activities	Student Thinking Conjecture
The students can identify the position of the right angle, opposite side, and hypotenuse in a right triangle.	Stretch the raffia from the table's base to the top of the head to form a right triangle.	A group of students can model the activity into a right triangle. One of the students was able to sketch the formed right triangle. The students can identify the position of right angles, front sides, hypotenuses, and side sides in sketches formed with various answers without knowing the reasons. The students need to be corrected in identifying the position of right angles.
The students can apply the definition of trigonometric	Measuring students' height and shadow	The students can draw sketches formed from the models presented.

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Learning objectives	Learning Activities	Student Thinking Conjecture
ratios $\sin \theta$, $\cos \theta$, and $\tan \theta$ to contextual problems using the concept of ratios in right triangles.		The students can use the definitions of the trigonometric ratios $\tan \theta$, $\sin \theta$, and $\cos \theta$ to measure the heights of both students and their shadows. The students can determine the position θ of different angles to obtain different trigonometric ratio values.
The students can understand the relationship between similarity and trigonometric ratios.	Experiment with making three right triangles from paper	The students can make three similar right triangles from paper. The students can make sketches from paper formed into three similar right triangles. The students can identify the shortest and longest sides of each triangle formed. The students can determine the ratio between the shortest side and the longest side of each right triangle. The students can relate the concept of similarity to trigonometric ratios.
The students can determine the value of trigonometric ratios at particular angles (0° , 30° , 45° , 60° , and 90°).	Observations on the calendar image in the form of an equilateral triangular prism.	The students can sketch a calendar model as an equilateral triangular prism. The students can construct a right triangle in which one of the angles corresponds to a specifically defined angle. The students can determine the value of trigonometric ratios at particular angles (0° , 30° , 45° , 60° , and 90°).
Applying trigonometric ratios enables students to ascertain a right triangle's unknown length of sides	Bed or bunk bed observation	The students can observe bed or bunk beds by sketching or drawing bed models. The students can identify the position of right angles, oblique angles, side angles, and front side positions in sketches. The students can apply trigonometric ratios to determine the length of unknown sides in problems presented related to beds.
The students can solve contextual problems related to right	Observing an image of a skier descending an iceberg	The students can identify the position of the angles, opposite sides, and hypotenuses in a right triangle, which is formed as a sketch of a picture of a skier descending an iceberg.

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Learning objectives	Learning Activities	Student Thinking Conjecture
triangles using trigonometric ratios.		The students can apply the definition of trigonometric ratios to skier problems. The students can solve the problems given in the case of skiers.

Table 1: Hypothetical Learning Trajectory in Teaching Experiment 1

Within **phase 1**, the research participants comprised 25 10th-grade students attending a Bandung City high school. In **phase 2**, the research participants comprised 25 10th-grade students from a different high school within the same municipal area. **The initial implementation of the HLT took place in the first phase of the teaching experiment to assess the efficacy of the prepared learning activity design in investigating students' cognitive patterns.**

Moreover, the teaching experiment 1 results indicated an enhancement in **HLT**. The Hypothetical Learning Trajectory (HLT) enhancement was conducted based on the students' difficulties while implementing the Local Instructional Theory (LIT) on trigonometric ratios. In particular, students struggled with constructing right-angled triangles, applying mathematical models in contextual problems, and solving mathematical representation tasks. The HLT was refined using the principles of Realistic Mathematics Education (RME) to address these challenges.

First, the instructional design was improved by embedding more meaningful real-world contexts to help the students relate trigonometric concepts to their daily experiences, reinforcing the idea of mathematics as a human activity. Second, scaffolding strategies were strengthened by structuring learning activities progressively, guiding students from informal visual representations (e.g., real-life diagrams and sketches) to more formal mathematical models. Third, interactive learning moments were emphasized by incorporating collaborative discussions and teacher questioning techniques to encourage student reflection and active participation. These refinements maintain the core activities of the previous HLT while improving students' engagement with mathematical representations through a more realistic and interactive learning approach.

The revised HLT will be tested in a different school setting to assess its effectiveness in diverse learning environments. A refined Hypothetical Learning Trajectory (HLT) was implemented in phase 2 of the teaching experiment. The data-collecting protocols encompass classroom observations, videos documenting learning activities, student workbooks, and recordings of interviews conducted with students. A refined Hypothetical Learning Trajectory (HLT) was implemented in phase 2 of the teaching experiment. The data-collecting protocols encompass classroom observations, videos documenting learning activities, student workbooks, and recordings of interviews conducted with students.

Commented [bc1]: What did the enhancement of HLT consist in, precisely, please

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Before implementing the developed Hypothetical Learning Trajectory (HLT), a diagnostic test was conducted, which showed that 47% of students had difficulty constructing right triangles, while 86% had difficulty applying trigonometric ratios in contextual problems; after the intervention, the results of the final test showed a significant improvement, with 83% of the students successfully constructing right triangles correctly and 50% of students being able to apply trigonometric ratios correctly. Classroom observations also showed increased student engagement and accuracy in representing mathematical concepts. These results indicate that the updated HLT, by integrating real-world contexts, scaffolding, and interactive discussions, effectively helps the students bridge understanding from informal thinking to formal mathematical representations, aligning with Realistic Mathematics Education (RME) principles.

Phase 1 of the retrospective analysis involved the researcher collecting and analyzing all the data. An analysis was conducted to evaluate the outcomes of including the learning trajectory in teaching experiment 1 with the original HLT design. Moreover, the researcher investigated and examined the learning process activities included to determine the level of the student's comprehension of the idea of trigonometric ratios. An analysis was conducted on the outcomes of the HLT enhancements implemented in teaching experiment 2, by comparing the initial HLT with the execution of the learning process.

The researchers selected six participant subjects to interview. The six participants had various abilities; two had high, medium, and low abilities. In addition, interviews were also conducted with the mathematics teacher of trigonometric ratios at the school that taught the students.

RESULTS

The learning trajectory design discussed in this paper involves a series of student activities to help students understand trigonometric ratios through a Realistic Mathematics Education approach. The trajectory developed in this research comes from Empirical data obtained through direct observation of students, interviews, and worksheet analysis during the learning process. Findings showed that the students experienced gradual progress in understanding trigonometric ratios through a series of exploration-based activities.

Based on observation data, the students initially experienced difficulty connecting the concept of trigonometric ratios with the right triangles they encountered in everyday life. However, after gradual activities, their understanding develops from concrete exploration to a formal introduction to trigonometric ratios.

Activity 1: Observation of “Stretching the Raffia from the Base of the Table to the Top of the Student's Head”

Activity 1 aims to help the students accurately determine the right angle, opposite side, and hypotenuse in a right triangle. The students, divided into groups of 4–5, use contextual equipment

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like raffia, a table, and a peer. One of the students holds one end of the raffia while the other is attached to the table base, forming a right triangle with the student's height and the floor. Observations are illustrated in Figure 1. Each group sketches their demonstration (Figure 2), identifying key triangle components. Students explain their reasoning for positioning angle θ and triangle sides. Those struggling receive guided reinvention through teacher-led discussions.



Figure 1: Activity 1 "Stretching a Raffia from the Base of the Table to the Top of the Student's Head"

Commented [bc2]: Faces of students have to be blurred

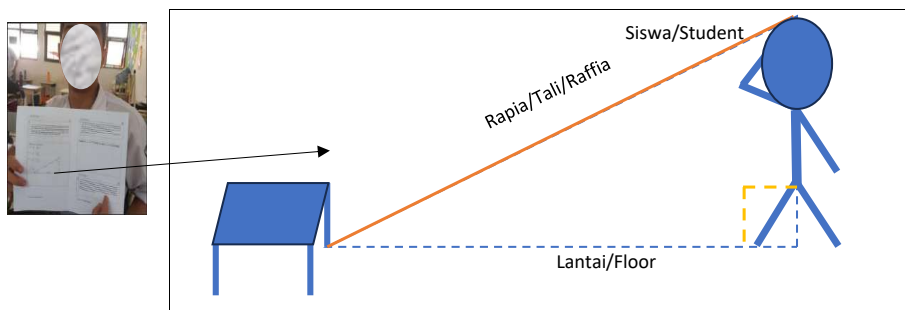


Figure 2: Example of the Student Sketches in Activity 1

- Teacher : "Okay, S-1, please explain what you drew."
- S-1 : "Okay, ma'am, if you look at the instructions in the worksheet, we are asked to practice what is written in activity 1, and then we share roles; S-2 ties the raffia to the base of the bottom table, S-3 holds the end of the raffia that results from extending S-2's raffia, S-4 reads what is on the worksheet, and I make the sketch."
- Teacher : "Okay, have you considered whether your sketch will be a right triangle?"
- S-1 : "At first, I did not, Ma'am, but when you said, 'Look if you draw it, is a right angle formed?' I remembered."
- Teacher : "Okay, how do you determine the corner points?"
- S-1 : "The benchmarks are at the end of the table legs, Leg S-3, and head S-3, Ma'am."

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- Teacher : "Why is that, S-1?"
- S-1 : "Yes, that is what it looks like when drawn, ma'am."
- Teacher : "Okay, the benchmarks that S-1 mentioned earlier make a move from the rapid raffia, right?"
- S-1 : "What do you mean, ma'am?"
- Teacher : "Mom, look, you represent the rapier raffia, the floor, and the height of S-2 with a line, right?"
- S-1 : "That is right, ma'am."
- Teacher : "Well, when you make that line, there is a phase where you turn; that is called a benchmark."
- S-1 : "Oh yes, that is right, ma'am, that is right."
- Teacher : "S-1, try to determine the right angle, slanted side, front side, and side in the sketch you have made!"
- S-1 : "This one is a right angle, this one is the hypotenuse, this one is the front side, and this one is the side side." (pointing to the picture).
- Teacher : "How do you determine a right angle, S-1?"
- S-1 : "There is a line like this and like this, ma'am." (pointing at the picture).
- Teacher : "Now, try to observe what degree angle the two lines form, or what angle is it called then?"
- S-1 : "Oh yes, ma'am, right angles."
- Teacher : "How do you determine the hypotenuse?"
- S-1 : "The side in front of the right angle, ma'am."
- Teacher : "How do you determine the front side?"
- S-1 : "This one, ma'am? Yes, the one in front of this corner (pointing to one of the corners of the picture)".
- Teacher : "Okay, how do you determine the side?"
- S-1 : "The one on the front side, ma'am."

In this activity, the students identify the expected position, which depends on the angle θ . This process helps them determine angles and sides in a right triangle. After completing the task, the teacher and the students reflect on positioning the right angle, hypotenuse, and θ , which influence side identification. The raffia stretching activity from the table to a student's head helps visualize the relationship between height and shadow length. Observations indicate that the students begin recognizing this pattern, even without fully grasping the ratio concept, providing a concrete experience before introducing the formal definition.

Activity 2: Observation "Measurement of The Students' Height and Shadow"

Activity 2 aims to help the students apply trigonometric ratios ($\sin \theta$, $\cos \theta$, and $\tan \theta$) in real-life contexts using right triangle comparisons. The students work in groups to transform comparison problems into hands-on tasks. One student stands with their back to a lamp to create a shadow, which the group then sketches. Each member takes turns measuring their height and shadow length,

recording the data in a table (Figure 4). The teacher guides the students in determining angle θ and calculating the height-to-shadow length ratio. This activity reinforces the concept of trigonometric ratios through visual and practical learning.

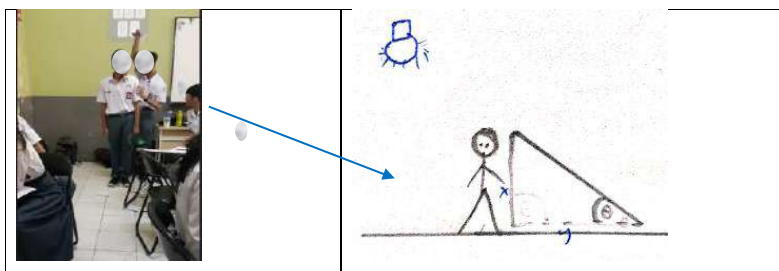


Figure 3: Activity 2 and Example of The Student Sketches in Activity 2

This project aims to help the students accurately calculate $\sin \theta$, $\cos \theta$, and $\tan \theta$ in their triangle sketches (Figure 5). The teacher monitors each group, guiding the students in problem-solving while allowing them to use formal or informal methods. Scaffolding is provided to help students recall and apply their knowledge of trigonometric ratios.

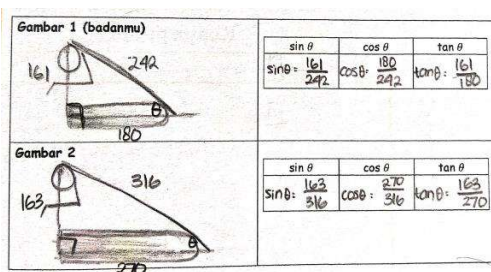


Figure 5: Example of Sketch and Application of Trigonometric Ratios Values in Activity 2

One of the students (S-5), as a representative of one of the group members, participated in presenting activity two and explained how the values of $\sin \theta$, $\cos \theta$, and $\tan \theta$ were obtained. After sketching the experiment, this group identified the right angle to determine the other positions. The following is a dialogue explaining the concept of trigonometric ratios described by the students.

Teacher : "Okay, S-5. Explain how your group determines each trigonometric ratio value requested in the table."

- S-5 : "Okay, Ma'am, first we sketch according to our height in the group, then, from the sketch formed, we determine the right angle, draw the symbol on the sketch, then determine the angle θ "
- Teacher : "Okay, do you mean identifying the position of a right angle?"
- S-5 : "Yes, ma'am."
- Teacher : "What do you mean by determining the angle?"
- S-5 : "So we can determine the front and side, ma'am."
- Teacher : "Okay, what is the next process?"
- S-5 : "Earlier, after determining the angle position θ , we then wrote down each of our heights according to what was measured earlier and wrote down the height of the shadow."
- Teacher : "Okay, how do you determine the hypotenuse?"
- S-5 : "Using the Pythagorean theorem, ma'am, with the help of a calculator."
- Teacher : "Okay, how do you calculate $\sin \theta$, $\cos \theta$, and $\tan \theta$?"
- S-5 : "Well, because the value of $\sin \theta$ is the front per hypotenuse, look at the picture (while pointing to picture 1 in UBA 2 Activity 2); the front of the angle θ is the front side, which is 161, and the hypotenuse is 242 so the $\sin \theta$ is 161 per 242."
- Teacher : "How to calculate $\cos \theta$ and $\tan \theta$?"
- S-5 : "Same here, ma'am, just a different position according to what was learned previously: $\cos \theta$ means side per hypotenuse, and $\tan \theta$ means front per side."

The conversation above helped all the students understand the concept of trigonometric ratios correctly. At the end of the activity, one of the students reflected on the lesson and concluded how to apply trigonometric ratios correctly. **Measuring the height and shadow length helped the students understand that the ratio between an object's height and its image can be quantified. After multiple measurements, most students recognized a consistent pattern in this relationship, reinforcing the concept of constant ratios as the foundation for sine, cosine, and tangent.**

Activity 3: Experiment with "Making Three Right Triangles from Paper"

Exercise 3 aims to help the students understand the relationship between similarity and trigonometric ratios. In groups of 4–5, the students use scissors, paper, rulers, and worksheets to cut rectangular paper into two equal right triangles and identify similarities in three right triangles. They explore similarity using diagrams and concrete teaching aids, as illustrated in Figures 6 and 7, where some groups present findings through diagrams while others use models. The teacher guides the students in recognizing corresponding sides and discovering formal patterns of similarity through directed questions, leading them to apply Geometry theorems by comparing three similar right triangles.

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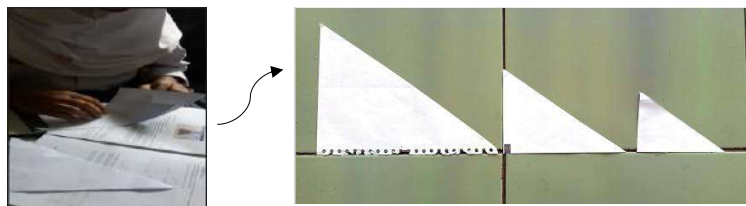


Figure 6. Activities and Examples of the Student Responses using Teaching Aids in Activity 3.

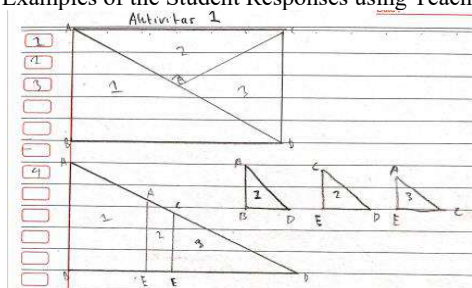


Figure 7. Example of the Student Responses using Diagrams in Activity 3

The teacher guides the students in identifying the shortest and longest sides of each triangle, helping them develop comparative thinking patterns (Figure 8). To deepen understanding, the teacher poses questions that draw analogies between similarity and trigonometric ratios (Figure 9). One example involves a logo on a garbage can placed on an equilateral triangle with a 6 cm side length. The students are asked to calculate the logo's height, sketch the triangle, and explain the concepts used in their solution.



The trash can logo featured below is designed as an equilateral triangle. The dimension of the side measures 6 centimeters. What is the estimated vertical dimension of the logo? Begin by creating a preliminary outline of the triangle. Which concept are you employing? Are there any other concepts available apart from the one you have employed? What is the procedure?

Logo tempat sampah berikut berbentuk segitiga sama sisi dengan panjang sisi 6 cm. Berapa kira-kira tinggi logo tersebut? Buatlah sketsa segitiga terlebih dahulu. Konsep apa kah yang kamu gunakan? Adakah alternatif konsep selain yang sudah kamu gunakan? Bagaimana caranya?

Figure 9. Example of Questions on the Relationship between the Concept of Similarity and Trigonometric Ratios in Activity 3.

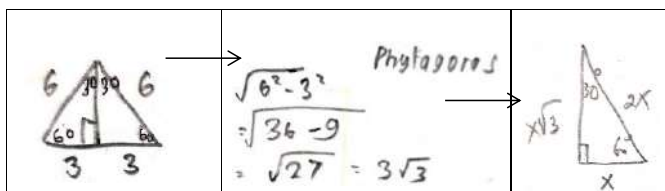


Figure 10. Example of the Student Responses to Contextual Questions in Activity 3.

In Figure 10, a group sketches the shape and calculates the logo's height using the Pythagorean Theorem, then verifies their results through similarity. The teacher and the students discuss the relationship between similarity and trigonometric ratios. By creating three right triangles of different sizes, the students observe that side length ratios remain constant for equal angles. Some believed the ratios would change initially, but experimentation confirmed their invariance, reinforcing a key principle of trigonometric functions.

Activity 4: Observation of "Calendar Image in the form of an Equilateral Triangular Prism"

Activity 4 helps the students determine trigonometric ratios for specific angles (0° , 30° , 45° , 60° , and 90°). Working in groups of 4–5, they analyze a calendar image as an equilateral triangular prism (Figure 11), extract an equilateral triangle, construct a symmetrical one with a 2-unit side, and divide it into two right triangles. Using the Pythagorean Theorem, they calculate unknown side lengths and determine trigonometric ratios for 30° and 60° , compiling their results in a table (Figure 12). For a 45° angle, the students draw an isosceles right triangle with 1-unit sides, calculate the hypotenuse, and determine $\sin \theta$, $\cos \theta$, and $\tan \theta$ (Figure 13). By the end of the activity, the students summarize their findings in a table, reinforcing their understanding of trigonometric ratios.

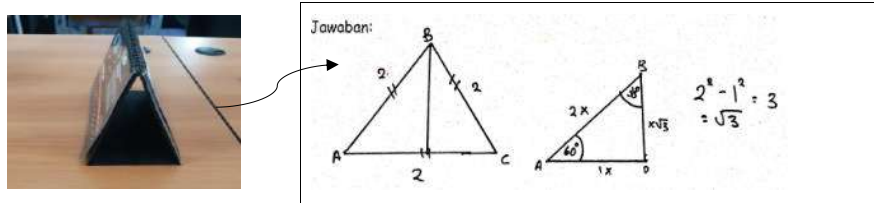


Figure 11. Calendar Image and Example of the Student Responses in Making Sketches in Activity 4

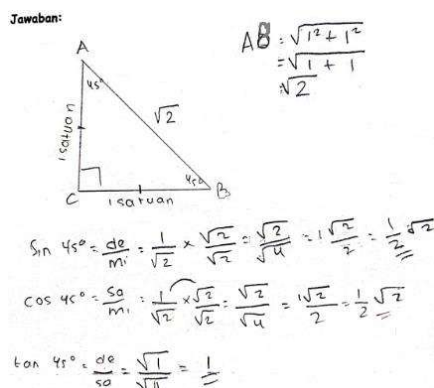


Figure 13. Example of the Student Responses Regarding Particular Angles 45° In Activity 4

	Rumus	30°	45°	60°
$\sin$	$\sin \theta = \frac{\text{sisl. depan}}{\text{sisl. miring}}$	$\frac{1}{2}$	$\frac{1}{2} \sqrt{2}$	$\frac{1}{2} \sqrt{3}$
$\cos$	$\cos \theta = \frac{\text{sisl. samping}}{\text{sisl. miring}}$	$\frac{1}{2} \sqrt{3}$	$\frac{1}{2} \sqrt{2}$	$\frac{1}{2}$
$\tan$	$\tan \theta = \frac{\text{sisl. depan}}{\text{sisl. samping}}$	$\frac{1}{3} \sqrt{3}$	1	$\sqrt{3}$

Figure 14. Example of Student Responses Represented in a Table in Activity 4

By the end of the exercise, the teacher and the students had reached a general conclusion on determining the value of trigonometric ratios at particular angles.

Observing the calendar image as an equilateral triangle helps the students recognize right triangles in various visual contexts. This activity increases their ability to identify right triangles in everyday objects, making it easier for them to connect trigonometric ratios to real-world applications.

Activity 5: Observation of “Bunk Bed”

Exercise 5 helps the students use trigonometric ratios to determine unknown side lengths in a right triangle. The students work in groups of 4-5, analyzing an image (Figure 15) and a triangular figure formed by stairs, a bunk bed, and the floor (Figure 16). They then apply horizontal mathematization by creating a diagram representing the triangle. The students identify whether the shape is a right triangle and describe the given data in a sketch. Using the tangent ratio, they calculate the displacement between the floor and the end of the stairs.

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Figure 15. Activities and Questions in Activity 5

Each group's findings from activity five are poured into a sketch, as shown in Figure 16.

With teacher guidance, the students sketch a right triangle from the bunk bed, floor, and stairs, identifying the right angle and using the given values (37° and 10 cm) to calculate the height with the tangent ratio. After solving the problems, one group presents their results, explaining their understanding of trigonometric ratios in determining unknown side lengths. The teacher then asks questions to assess the student's comprehension, as shown in the dialogue below.

- Teacher : "Okay, how do you find the length of the unknown side in a right triangle?"
- S-6 : "Begin by sketching the problem, identifying given information such as side lengths (opposite, hypotenuse, or adjacent) and known angles. Then, analyze the available data to determine the appropriate trigonometric ratio to apply."
- Teacher : "Can you give an example?"
- S-6 : "In the bunk bed problem, after sketching a right triangle, we label the given values: a 37° angle between the stairs and floor and a 10 cm distance from the bunk bed to the stairs. Using the tangent ratio, we calculate the unknown side length, which relates the opposite and adjacent sides."

Observing the bed frame helps the students see how trigonometric ratios apply in construction. Those who initially struggled with trigonometry begin to connect triangle-side ratios to real-world structures. This activity reinforces the practical significance of trigonometric ratios in design and construction.

Activity 6: Observation of "Picture of a Skier Descending an Iceberg"

Task 6 helps the students apply trigonometric ratios to real-world problems by analyzing worksheet images (Figure 17) and solving open-ended questions in groups. Referring to Larson et al. (2007),

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they calculate a skier's track length on an iceberg (Larson et al., 2007), and one group presents their findings to conclude the learning outcomes (Figure 18).



Figure 17. Questions in Activity 6

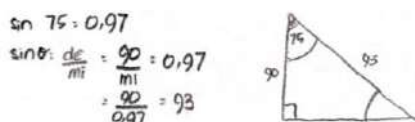


Figure 18. Example of the Student Responses to Activity 6

Observing an image of a skier descending an iceberg introduces the concept of slope about terrain and movement. The students connect slope angles to right triangle ratios, enhancing their understanding of the tangent function. This activity reinforces that trigonometry applies not only to static shapes but also to dynamic phenomena.

DISCUSSION

Learning activities using the Realistic Mathematics Education approach regarding trigonometric ratios followed by the students consist of six activities, namely raffia stretching activities, the student height measurement activities, activities to make three right triangles from paper, activities to observe calendar images, activities to observe bunk bed images, and activities to observe skier images. Each activity has its purpose and is interrelated and continuous. The goals of each action are outlined below: Firstly, stretching the raffia allows the students to determine the location of the angles and sides of a right triangle. Secondly, measuring the height of the students and their shadows allows the students to apply the concept of trigonometric ratios. 3) Creating a teaching tool including three right triangles is intended to facilitate the student's comprehension of the correlation between resemblance and trigonometric ratios. 4) Observing the calendar image permits the students to ascertain the numerical value of trigonometric ratios at specific angles. The objective of viewing the bunk bed image is to help the students apply trigonometric ratios to ascertain the length of the desired side in a right triangle. Similarly, observing the skier image seeks

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to empower the students in solving contextual difficulties by employing the concept of trigonometric ratios. Every worksheet provided incorporates the specific features of the Realistic Mathematics Education pedagogy about trigonometric ratios. These findings corroborate Subekti's assertion that student worksheets will enhance learning (Subekti & Prahmana, 2021).

Applying a realistic approach to teaching mathematics in various activities in the learning process should facilitate the student's understanding of mathematical skills and allow teachers to help the students effectively achieve learning goals (Ekowati & Nenohai, 2017). Students' understanding improves because the process of learning mathematics that follows Realistic Mathematics Education utilizes the environment around the world to encourage students to produce ideas and develop their knowledge (Nirawati et al., 2021).

According to research done by Sukendra (2020), a series of learning process activities with the RME approach can significantly facilitate students' understanding of the trigonometric comparison concept (Sukendra, 2020). Therefore, deliberate learning paths encourage students to build their knowledge, improve their current skills, and effectively overcome challenges in many subjects (Prahmana et al., 2017; Nursyahidah et al., 2020; Wijaya et al., 2021; Kuncoro et al., 2023).

CONCLUSIONS

This work aims to develop a learning trajectory for trigonometric ratios using the RME approach. The findings of this study suggest that implementing the RME approach in designing a learning trajectory for trigonometric ratios can enhance the students' mathematical proficiency in comprehending the concepts of trigonometric ratios. The learning trajectory designed using the RME approach consists of six activities. Through a series of continuous and ongoing activities in the six activities, the students find it easier to build their concepts and understand the concept of trigonometric ratios. The students develop their understanding starting from their ability to identify angle and side positions, apply the definition of trigonometric ratios to contextual problems, understand the relationship between similarity and trigonometric ratios, determine the magnitude of trigonometric ratio values at particular angles, apply trigonometric ratios, and solve contextual problems using trigonometric ratios.

This research is expected to encourage other researchers to design learning trajectories using the same topic but different approaches. The researcher hopes that the results of this design can contribute to the researchers' contribution to building a generation according to the nation's expectations.

Commented [bc3]: In order to assess the improvement you need to show us how students understood the trig c oncepts before the intervention. Otherwise how do you know they improved?

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APPENDIX

	Nama <u>Nabila</u>	Nama temanmu <u>Bunga</u>	Nama temanmu <u>Naufal M</u>	Nama temanmu <u>Jahsyi</u>	Nama temanmu <u>Nalvin</u>
Tinggi badan	155	153	173	169	161
Tinggi bayangan	165	163	183	179	171



	Your name <u>Nabila</u>	Your Classmate's name <u>Bunga</u>	Your Classmate's name <u>Naufal M</u>	Your Classmate's name <u>Jahsyi</u>	Your Classmate's name <u>Nalvin</u>
Height	155	153	173	169	161
Shadow Height	165	163	183	179	171

Figure 4: Example of a table of results measuring height and shadow in activity 2.

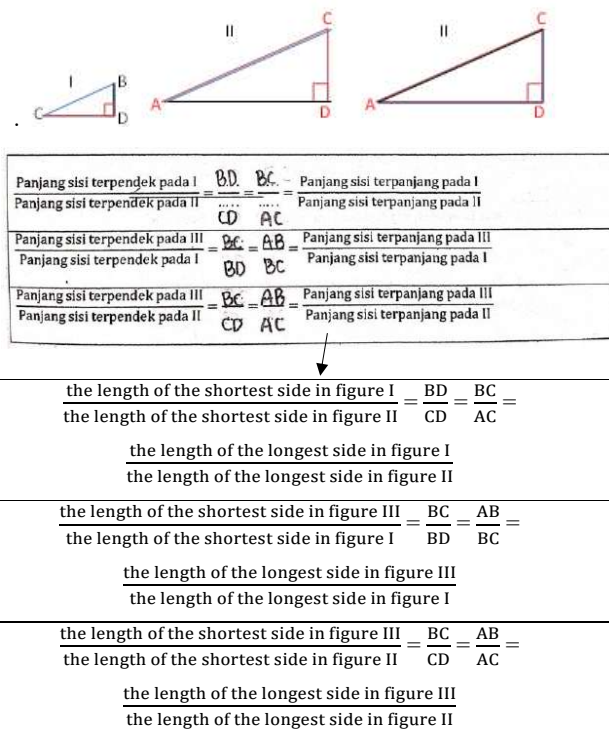


Figure 8. Example of the Student Responses in Using the Concept of Comparison in Activity 3.

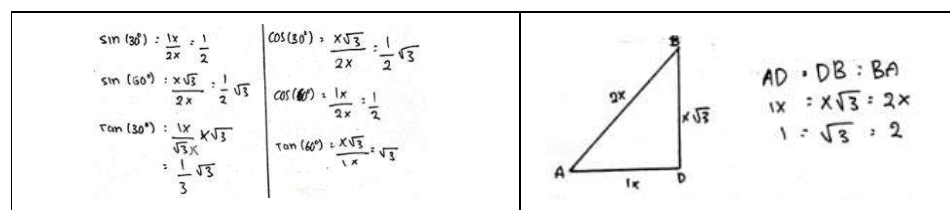


Figure 12. Example of the Student Responses regarding memorable and unique 30° angles 60° in Activity 4



Perhatikan
bentuk
bangun ini

Note the following triangular shape

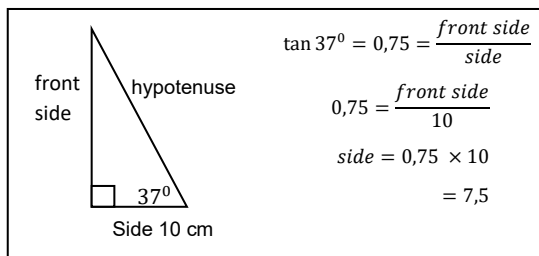
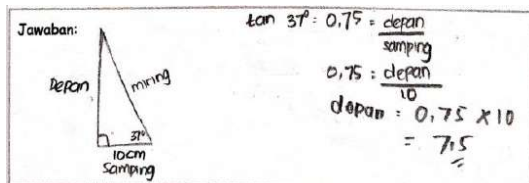


Figure 16. Questions and the Student Responses Representing Data into Sketches in Activity 5

The following changes have been made to the Manuscript *Learning Trajectory of Trigonometric Ratios Applying Realistic Mathematics Education* under reviewers' comments

Reviewer's comments	Changes made	Page (see highlights)
Please write "without space"	Changed "without space" to "with space"	Page 1
"The faces of students in the photos have to be blurred."	Students' faces in all relevant photos have been blurred, as the reviewer suggested, to protect privacy. We have carefully edited the images to blur all student faces according to ethical guidelines and reviewer feedback.	• Page 10, figure 1 • Page 10, figure 2 • Page 12, figure 3 • Page 17, figure 15
Is this the trajectory in which students learn the trigonometric ratios concept you found?	Clarified that the learning trajectory was derived from empirical data in our study. Detailed the progression of activities leading to a conceptual understanding of trigonometric ratios. Added justification for each activity based on observed student learning behavior.	• Page 9, 11 • Page 13, • Page 15, • Page 16, • Page 17, • Page 18
Please check the MTRJ format for the paragraph alignment	The paragraph alignment has been adjusted according to the MTRJ formatting guidelines. The necessary corrections have been implemented to ensure consistency with the journal's requirements.	Revised throughout the manuscript
• In general, I did not find a specific and clear problem for students on the concept of trigonometric ratios in your article. • There are no research questions, research gap, and also the novelty. I assume, it is because you didn't explore a specific and clear problem for students	Clarification of the Research Problem, Research Gap, and Novelty: • The research problem related to students' difficulties in understanding trigonometric ratios has been explicitly stated in the Introduction section. • A discussion on the research gap has been added, emphasizing the lack of studies that specifically address students' misconceptions and challenges in learning trigonometric ratios. • The novelty of this study has been highlighted by explaining how the approach differs from previous research and contributes to the improvement of students' conceptual understanding. • The research questions have been explicitly formulated and incorporated into the Introduction section to guide the study's objectives.	• Page 1, 2, 3, 5 • Page 2, 3 • Page 3 • Page 3

Proofreading from professional is needed	Proofreading: - The manuscript has been professionally proofread to improve clarity, coherence, and adherence to academic writing standards. - Grammatical errors and unclear sentences have been revised to enhance readability and accuracy.	All pages, revised for grammar and clarity
Do you mean “One concept and another” in the domain of trigonometry concept or in the broader mathematics concept?	The paragraph has been revised to clarify that understanding trigonometry requires prerequisite mathematical concepts, such as the Pythagorean theorem and right triangles. This revision enhances coherence and ensures a more systematic explanation of the challenges students face in learning trigonometric ratios	Page 1
You can explore this one as a problem	The suggested issue has been explored and elaborated further in the manuscript. Additional explanations have been included to enhance clarity and depth. These revisions can be found in Section Introduction (page 2), where we have emphasized their relevance within the study context.	Page 2
- From my point of view, you don't need to talk about this, because it's too specific. just talk about the content at the junior high school level. - Only one sentence in this paragraph.	The mentioned paragraph has been removed as per the reviewer's suggestion to maintain the focus on the senior high school level. This adjustment ensures that the content remains relevant and appropriately	The paragraph has been removed, See previous version for reference
Why RME in accordance with the student's understanding trigonometry ratio?	A detailed explanation has been added to justify why Realistic Mathematics Education (RME) aligns with students' understanding of trigonometric ratios. Specifically, RME emphasizes contextual learning, allowing students to construct mathematical concepts based on real-world experiences. Since trigonometric ratios are closely related to spatial reasoning and proportional thinking, RME provides an intuitive approach that enhances conceptual understanding.	Section Theoretical Framework Page 4 & Page 5
Some terms about design research are not commonly used, perhaps this is because	We have revised the terminology related to design research by aligning it with the standard terms found in relevant references. The	Section Method Page 6

you are translating from your language to English, just keep using the terminology according to the reference you are using. You need to add the source of the trigonometric ratio theory that you used to design the learning trajectory.	revised terms are now consistent with the established literature to ensure clarity and accuracy in the explanation. We have added the appropriate references for the trigonometric ratio theory used in designing the learning trajectory. The sources have been cited in the relevant section to provide theoretical support for the study.	Section Theoretical Framework Page 4
- Preliminary stage - Without bracket	These revisions have been applied in Section Method to align with the reviewer's suggestions	Section Method Page 6
As we know, in RME there is mathematisation, whose theory do you use to design the learning trajectory. I didn't find it.	The theory of mathematization used in this study is based on [Freudenthal, 1991], which has now been cited in the relevant section to support the learning trajectory design	Section Theoretical Framework Page 4, page 5
this is what I meant from the previous comment, learning trajectory based on the theory from previous research. unfortunately, I didn't find this in your research.	The learning trajectory in this study is now explicitly grounded in the theories proposed by Freudenthal (1991), Treffers (1987), and Gravemeijer (1994). These references have been cited in Section Theoretical Framework to clarify the theoretical underpinning of the trajectory design.	Section Theoretical Framework Page 4
Where is the theory? [.... into a right triangle.]	We have added the relevant theoretical foundation in Section Introduction to support the study explicitly. The revised manuscript now includes references to key theories, such as Freudenthal (1991), Treffers (1987), Gravemeijer (1994), and De Lange (1996), to clarify the conceptual basis underpinning the research.	Section Theoretical Framework Page 4

Commented [LS1]: Where is the theory?

Where is the theory? [.... measuring the heights of both students and their shadows.]	We have incorporated an explanation of the theory supporting the relationship between measuring heights and shadows as a contextual problem in Realistic Mathematics Education (RME). This discussion explicitly refers to horizontal mathematization, where students interpret real-world relationships as mathematical objects, such as comparing their height with the length of their shadow. Furthermore, it involves vertical mathematization, in which students refine their understanding by developing a formal mathematical model, including similar right triangles and trigonometric ratios. To strengthen the theoretical foundation, we have included relevant references, such as Freudenthal (1991), Treffers (1987), Gravemeijer (1994), and De Lange (1996), which highlight the role of contextual problems in fostering meaningful mathematical learning.	Section Theoretical Framework Page 4
In your term, what the difference between cycle and phase?	All occurrences of "cycle" have been replaced with "phase" to ensure consistency and avoid misinterpretation.	Section Method Page 8
HLT was first trialled in a pilot experiment. Please check the following book:	The sentence "The HLT Design was first implemented in teaching experiment cycle 1..." has been revised to: "The initial implementation of the HLT took place in the first phase of the teaching experiment to assess the efficacy of the prepared learning activity design in investigating students' cognitive patterns." This revision ensures that our intended meaning is clear and distinguishes our research process from a pilot experiment.	Section Method Page 8
Remove [;]	The semicolon ";" has been removed from the relevant sentence to ensure proper punctuation.	Section Discussion Page 19
What did the enhancement of HLT consited in, precisely, please	Enhancing the Hypothetical Learning Trajectory (HLT) was conducted based on students' difficulties observed during the implementation of the Local Instructional Theory (LIT) on trigonometric ratios. The	Section Method Page 8

Commented [LS2]: Where is the theory?

	improvements were made using the principles of Realistic Mathematics Education (RME). 1. Embedding real-world contexts: More meaningful real-world situations were introduced to help students relate trigonometric concepts to daily experiences, reinforcing the idea of mathematics as a human activity. 2. Structuring a progressive learning path: Learning activities were redesigned to guide students from informal visual representations (e.g., sketches and real-life diagrams) to more formal mathematical models, improving conceptual understanding. 3. Enhancing interactivity: The role of collaborative discussions and teacher questioning strategies was expanded to encourage active participation and student reflection. These refinements ensure a more effective and engaging learning experience. The revised HLT maintains the core activities of the previous version but strengthens the scaffolding and contextualization to support students' mathematical representation skills. The updated HLT will be tested in different schools to assess its effectiveness in diverse learning environments.	
Faces of students have to be blurred	Students' faces in all relevant photos have been blurred, as the reviewer suggested, to protect privacy. We have carefully edited the images to blur all student faces according to ethical guidelines and reviewer feedback.	• Page 10, figure 1 • Page 10, figure 2 • Page 12, figure 3 • Page 18, figure 15
In order to assess the improvement you need to show us how students understood the trig concepts before the intervention. Otherwise how do you know they improved?	To address this issue, we have included an analysis of students' baseline understanding of trigonometric concepts before the intervention. The assessment was conducted through a diagnostic test administered before the implementation of HLT, which measured students' ability to represent and apply trigonometric ratios in contextual problems. The results showed that many students struggled with constructing right triangles, transitioning from real-world situations to mathematical models, and applying trigonometric ratios correctly.	Section Method Page 9

	This baseline data has now been included in Section Method of the manuscript, providing a clear reference point for assessing improvement. By comparing students' pre-intervention performance with their post-intervention results, we show how enhanced HLT helped students develop better mathematical representation skills.	
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Learning Trajectory of Trigonometric Ratios Applying Realistic Mathematics Education

Dini Andiani^{1,2}, Darhim¹, Elah Nurlaelah¹

¹Department of Mathematics Education, Universitas Pendidikan Indonesia, Indonesia,

²Department of Mathematics, Universitas Bale Bandung, Indonesia

diniandiani@upi.edu, diniandiani367gmail.com, darhim0355@gmail.com

Abstract: Trigonometry materials are still considered complex materials at the secondary level. It is necessary to design an appropriate trigonometric ratios learning trajectory so that the students can quickly build the concept of trigonometric ratios. This study aims to design a learning trajectory for trigonometric ratio material through Realistic Mathematics Education (RME). This study uses a research design involving 10th-grade high school students in Bandung City in the 2023-2024 academic year. The research data was collected through data collection using written test instruments, student worksheets, observations, interviews, and documentation. The results of the study showed that the trigonometric ratios learning trajectory consisted of raffia stretching activities from the table base to the student's head, measuring the student's height and shadow, experimenting with making three right triangles from paper, observing a picture of a calendar in the form of an equilateral triangular prism, observing a bed or bunk bed, and observing a picture of a skier going down an iceberg. Students can quickly build the concept of trigonometric ratios from several activities provided. In addition, various learning flows from various activities make it easier for students to build knowledge about trigonometric ratios and solve problems presented contextually.

Keywords: Learning Trajectory, Trigonometric Ratios, Realistic Mathematics Education

INTRODUCTION

Trigonometry materials are vital in everyday applications such as statistics, surveying, architecture, and other engineering fields (Weber, 2005a). However, building knowledge about trigonometry is challenging (Blackett & Tall, 1991; Demir et al., 2012; Kendal & Stacey, 1997; Ngu & Phan, 2020; Sugianto et al., 2023). To understand the concept of trigonometry, students need to learn several prerequisite mathematical concepts that serve as a foundation. These

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prerequisite concepts include the Pythagorean theorem and the properties of right triangles, which are essential in understanding trigonometric ratios. Kamber's Research (2017) stated that students struggle to grasp trigonometry through the unit circle (Kamber & Takaci, 2017). Additionally, they face difficulties distinguishing between trigonometry in right triangles, the unit circle, and the relationship between the two representations (Maknun et al., 2019). As a result, many students fail to understand trigonometric ratios properly, even though these ratios are fundamental in learning trigonometry (Weber, 2005; Nordlander, 2022). Several studies (Hershkowitz et al., 2016; Lessani et al., 2017; Lu'lu'il Maknun et al., 2022; Maphutha et al., 2023; Weber, 2005b) also indicate that traditional approaches lead students to often rely on memorizing formulas rather than understanding the reasoning behind trigonometric ratio concepts.

One example of students' ability to master trigonometric ratios is Andiani's Research results (Andiani & Nurlaelah, 2024). The test questions given to students are presented in Figure 1.

Seseorang berdiri di geladak kapal yang berada 10 m di atas permukaan air, mengamati bahwa sudut elevasi puncak bukit adalah 60° dan sudut depresi dasar bukit adalah 30° . Hitunglah jarak bukit dari kapal dan tinggi bukit tersebut.

An individual stands on the deck of a ship 10 meters above the water's surface and observes a particular distance of a hill. The angle of elevation from the observer's position to the top of the hill is 60 degrees, while the angle of depression to the base of the hill is 30 degrees. Based on this information, determine the horizontal distance between the ship and the hill, and also the total height of the hill.

Figure 1. Example of Test Questions in Andiani et al.'s Research (2024)

The results of one student's answer in solving the problem are shown in Figure 2.

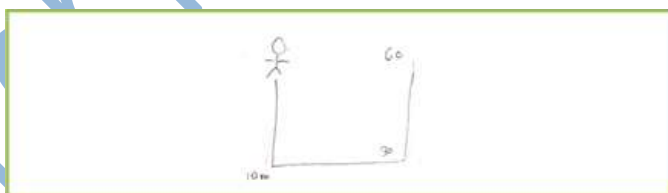


Figure 1.4. Example of Test Answers in Andiani et al.'s Research (2024)

The results of students' answers show that students are starting to be able to bring up their ideas or concepts in the form of mathematical models. In examining these errors, questions arise (Do students not understand the correct symbols to use? Do students not understand trigonometry? Do

students understand the meaning of elevation angles? Do students make the errors due to their prior knowledge, or are errors occurring during the learning process?). Student errors occur when students understand a concept in a particular context and adapt it to be applied in another context (Brousseau, 2002). The procedures that students should have written cannot be skipped because students have difficulty in the initial steps of representing the problem in a model. In this case, students have gone through the horizontal mathematization process but have not gone through the vertical mathematization process in its entirety; this error dominates students (Meika, 2018).

Student errors indicate that the students have difficulty solving contextual problems. This finding is in line with Haryani's research that the students still find it difficult to construct an understanding of trigonometry material, especially in the context of story problems or real life (Saputri et al., 2020; Widyanti et al., 2022). In addition, a similar opinion was expressed by Insani and Kadarisma in their research that in studying trigonometry, the students face epistemological obstacles caused by the limitations of their knowledge tied to a particular context. As a result, when faced with different situations, the students struggle to apply or adapt this knowledge to the new context (Insani & Kadarisma, 2020; Nasrulloh & Zahiro, 2023). Therefore, teachers need to know the causes of the difficulties experienced by the students and what the students think in their learning process so that these difficulties occur (Pramesti et al., 2024; Rianasari & Guzon, 2024; Wilson et al., 2013, 2015).

Learning trajectory represents students' cognitive processes in understanding concepts (Clements, 2011), with each student following a unique path (Chahine & Grinshpon, 2020; Ellis et al., 2014; Tahiri et al., 2017; Zhao et al., 2024). While previous studies have explored learning trajectories for trigonometric ratios (Hamdani et al., 2023; Hidayati & Armiaati, 2024; Saputra et al., 2021), none have specifically applied the Realistic Mathematics Education (RME) approach. This study aims to develop a learning trajectory for trigonometric ratios using RME and answer the following main questions: How is the learning trajectory for trigonometric ratios of high school students with the application of Realistic Mathematics Education?

This article is structured into several sections. The next section explores the theoretical framework and the methods employed in the study. Subsequently, the research findings are presented and analyzed in depth. The article concludes by summarising the key findings and the study's implications.

THEORETICAL FRAMEWORK

Structured learning trajectories provide a clear pathway for students to develop mathematical understanding. Simon (1993) introduced the concept of Hypothetical Learning Trajectories (HLT), later expanded by Clements (2011) to describe students' cognitive processes in mathematics. This

framework consists of guided learning activities that foster conceptual development, helping the students progress through different reasoning levels and achieve specific learning goals.

This study adopted the Realistic Mathematics Education (RME) approach to help the students better understand trigonometric ratios. The Realistic Mathematics Education (RME) approach aligns with the student's understanding of trigonometric ratios because it emphasizes contextual learning, allowing the students to build concepts based on real-world experiences (Gravemeijer, 1994). Through authentic situations, such as calculating the height of a building with a shadow or determining the elevation angle in navigation, the students can understand the concept of trigonometric ratios more meaningfully than simply memorizing formulas (M. van D. H. Panhuizen, 2003). In addition, RME encourages visual exploration using tools such as triangle diagrams, interactive math software, or physical props so that the students can directly observe the relationship between angles and side lengths in triangles (Freudenthal, 1991). This approach also emphasizes mathematical modelling, where the students learn and apply the theory to solve real problems, improving their conceptual understanding and critical thinking skills (Gravemeijer & Doorman, 1999). Thus, RME enables students to understand how to calculate trigonometric ratios, why these concepts are important, and how to use them in real-world situations.

Realistic Mathematics Education (RME) emphasizes mathematization as the main principle to help students understand mathematical concepts gradually, starting from real situations to achieving formal forms. Hans Freudenthal (1991) developed the theory of mathematization in RME by distinguishing two types of mathematization, namely horizontal mathematization, which transforms real-world problems into mathematical models, and vertical mathematization, which processes mathematical models into more formal forms (Freudenthal, 1991).

In addition, the concept of mathematization in RME is also strengthened by Treffers (1987), who emphasized that mathematization is the core of the RME approach, allowing the students to explore and construct mathematical concepts through meaningful activities (Treffers, 1987). Meanwhile, Gravemeijer (1994) emphasized the importance of designing learning trajectories in RME that consider how students develop understanding progressively through the mathematization process (Gravemeijer, 1994).

In the Realistic Mathematics Education (RME) approach, horizontal and vertical mathematization support modelling activities into right triangles (Freudenthal, 1991; Treffers, 1987). Horizontal mathematization occurs when the students translate real-world situations, such as measuring the length of a shadow and the height of a student and then recognizing the proportional relationship in the situation. Furthermore, through vertical mathematization, the students construct more formal mathematical models, for example, by comparing the height and shadow length ratio using the concept of triangle similarity or trigonometric ratios such as the tangent of an elevation angle.

In addition, the concept of a model as a bridge in RME helps students move from informal understanding to more abstract mathematical models (Gravemeijer, 1994). The students begin

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learning from real-world situation models, such as shadows formed by light in the classroom, then gradually move to formal mathematical models, such as right triangles and trigonometric ratios. These models serve as a tool to help the students understand how the relationship between elements in a physical system can be represented in a more abstract mathematical form. In this learning, a real-world context is an important starting point to facilitate the students' understanding of mathematical concepts (De Lange, 1996). Measuring the students' heights and shadows allows the students to understand the concepts of triangle similarity and trigonometric ratios more intuitively.

RME is based on the philosophy of Hans Freudenthal, who views mathematics as a human activity involving creativity and problem-solving (Freudenthal, 1973; Gravemeijer & Terwel, 2000; Treffers, 1987; Van den Heuvel-Panhuizen & Drijvers, 2020). This approach allows students to develop mathematical understanding through real-life contexts and experiences (Yuanita et al., 2018). Research by Aiyub et al. (2024) also highlights that learning designs incorporating real-world problems can improve students' algebraic thinking and overall mathematical understanding (Aiyub et al., 2024).

The students build understanding through informal processes. They observe, see, and understand the environment wherever they are (Meika, 2018). Realistic Mathematics Education encompasses three key concepts: directed reinvention and progressive mathematical processes, didactic phenomenology, and model construct development (Gravemeijer, 1994). Within the teaching and learning process, these three concepts are further developed into six distinct features of RME: 1) The principle of activity, 2) The principle of reality, 3) The principle of level, 4) The principle of intertwining, 5) The principle of interactivity, and 6) The principle of guidance (M. V. den H. Panhuizen & Drijvers, 2014).

Based on Freudenthal's view of mathematics as a human activity, the activity principle emphasizes the student's active role in learning. The reality principle encourages learning through real-world problems that can be mathematized. The level principle describes the students' progression through different levels of understanding. The interconnection principle highlights integrating various concepts using multiple tools and methods. The interaction principle sees learning as a social and reflective process. Lastly, the guidance principle, rooted in Freudenthal's "guided reinvention," ensures learning aligns with a structured long-term trajectory. (M. V. den H. Panhuizen & Drijvers, 2014).

The students often have difficulty understanding the concept of trigonometric ratios, especially in relating the lengths of the sides of a triangle to the angles (Maknun et al., 2019). In addition, several studies have shown that they fail to understand the concept of trigonometric ratios even though they have been taught formally (Weber, 2005; Nordlander, 2022). Another difficulty often faced is distinguishing between the right triangle approach and the unit circle (Kamber & Takaci, 2017). Given the importance of understanding this concept, this study aims to develop a learning

trajectory that helps students overcome these difficulties, with the Realistic Mathematics Education (RME) approach as the main theoretical basis.

METHOD

The methodology employed is design research as described by Gravemeijer and Van Eerde in 2009 (Gravemeijer & Van Eerde, 2009). This research methodology is focused on learning trajectory to enhance the quality of learning by fostering effective collaboration between researchers and teachers (Gravemeijer & Van Eerde, 2009). Design research will enable the attainment of the goals of the research paradigm in the development of sequential activities and the enhancement of learning quality. The primary objective of this work is to develop a comparative trigonometry learning trajectory by utilizing the RME approach. The study was carried out in two cyclical phases, each following the design research stages outlined by Gravemeijer (2004): 1) preliminary design, 2) teaching experiment, and 3) retrospective analysis. The outcomes of the initial phase activities that have been executed and the subsequent analytical review have contributed valuable insights (Gravemeijer, 2004).

In the initial design stage, the researcher observed the school and interviewed teachers and students. Additionally, the researcher collected and analyzed various sources related to the challenges the students and teachers faced in understanding trigonometric ratios and reviewed materials on learning activities based on the Realistic Mathematics Education (RME) approach. These analyses served as a reference for developing the Hypothetical Learning Trajectory (HLT), which includes learning objectives, learning activities, and hypotheses on students' cognitive development (Simon, 1993). These learning hypotheses form the foundation for designing the learning activities developed by the researcher.

Learning objectives	Learning Activities	Student Thinking Conjecture
The students can identify the position of the right angle, opposite side, and hypotenuse in a right triangle.	Stretch the raffia from the table's base to the top of the head to form a right triangle.	A group of students can model the activity into a right triangle. One of the students was able to sketch the formed right triangle. The students can identify the position of right angles, front sides, hypotenuses, and side sides in sketches formed with various answers without knowing the reasons. The students need to be corrected in identifying the position of right angles.
The students can apply the definition of trigonometric	Measuring students' height and shadow	The students can draw sketches formed from the models presented.

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Learning objectives	Learning Activities	Student Thinking Conjecture
ratios $\sin \theta$, $\cos \theta$, and $\tan \theta$ to contextual problems using the concept of ratios in right triangles.		The students can use the definitions of the trigonometric ratios $\tan \theta$, $\sin \theta$, and $\cos \theta$ to measure the heights of both students and their shadows. The students can determine the position θ of different angles to obtain different trigonometric ratio values.
The students can understand the relationship between similarity and trigonometric ratios.	Experiment with making three right triangles from paper	The students can make three similar right triangles from paper. The students can make sketches from paper formed into three similar right triangles. The students can identify the shortest and longest sides of each triangle formed. The students can determine the ratio between the shortest side and the longest side of each right triangle. The students can relate the concept of similarity to trigonometric ratios.
The students can determine the value of trigonometric ratios at particular angles (0° , 30° , 45° , 60° , and 90°).	Observations on the calendar image in the form of an equilateral triangular prism.	The students can sketch a calendar model as an equilateral triangular prism. The students can construct a right triangle in which one of the angles corresponds to a specifically defined angle. The students can determine the value of trigonometric ratios at particular angles (0° , 30° , 45° , 60° , and 90°).
Applying trigonometric ratios enables students to ascertain a right triangle's unknown length of sides	Bed or bunk bed observation	The students can observe bed or bunk beds by sketching or drawing bed models. The students can identify the position of right angles, oblique angles, side angles, and front side positions in sketches. The students can apply trigonometric ratios to determine the length of unknown sides in problems presented related to beds.
The students can solve contextual problems related to right	Observing an image of a skier descending an iceberg	The students can identify the position of the angles, opposite sides, and hypotenuses in a right triangle, which is formed as a sketch of a picture of a skier descending an iceberg.

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Learning objectives	Learning Activities	Student Thinking Conjecture
triangles using trigonometric ratios.		The students can apply the definition of trigonometric ratios to skier problems. The students can solve the problems given in the case of skiers.

Table 1: Hypothetical Learning Trajectory in Teaching Experiment 1

Within phase 1, the research participants comprised 25 10th-grade students attending a Bandung City high school. In phase 2, the research participants comprised 25 10th-grade students from a different high school within the same municipal area. The initial implementation of the HLT took place in the first phase of the teaching experiment to assess the efficacy of the prepared learning activity design in investigating students' cognitive patterns.

Moreover, the teaching experiment 1 results indicated an enhancement in HLT. The Hypothetical Learning Trajectory (HLT) enhancement was conducted based on the students' difficulties while implementing the Local Instructional Theory (LIT) on trigonometric ratios. In particular, students struggled with constructing right-angled triangles, applying mathematical models in contextual problems, and solving mathematical representation tasks. The HLT was refined using the principles of Realistic Mathematics Education (RME) to address these challenges.

First, the instructional design was improved by embedding more meaningful real-world contexts to help the students relate trigonometric concepts to their daily experiences, reinforcing the idea of mathematics as a human activity. Second, scaffolding strategies were strengthened by structuring learning activities progressively, guiding students from informal visual representations (e.g., real-life diagrams and sketches) to more formal mathematical models. Third, interactive learning moments were emphasized by incorporating collaborative discussions and teacher questioning techniques to encourage student reflection and active participation. These refinements maintain the core activities of the previous HLT while improving students' engagement with mathematical representations through a more realistic and interactive learning approach.

The revised HLT will be tested in a different school setting to assess its effectiveness in diverse learning environments. A refined Hypothetical Learning Trajectory (HLT) was implemented in phase 2 of the teaching experiment. The data-collecting protocols encompass classroom observations, videos documenting learning activities, student workbooks, and recordings of interviews conducted with students. A refined Hypothetical Learning Trajectory (HLT) was implemented in phase 2 of the teaching experiment. The data-collecting protocols encompass classroom observations, videos documenting learning activities, student workbooks, and recordings of interviews conducted with students.

Commented [bc1]: What did the enhancement of HLT consist in, precisely, please

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Before implementing the developed Hypothetical Learning Trajectory (HLT), a diagnostic test was conducted, which showed that 47% of students had difficulty constructing right triangles, while 86% had difficulty applying trigonometric ratios in contextual problems; after the intervention, the results of the final test showed a significant improvement, with 83% of the students successfully constructing right triangles correctly and 50% of students being able to apply trigonometric ratios correctly. Classroom observations also showed increased student engagement and accuracy in representing mathematical concepts. These results indicate that the updated HLT, by integrating real-world contexts, scaffolding, and interactive discussions, effectively helps the students bridge understanding from informal thinking to formal mathematical representations, aligning with Realistic Mathematics Education (RME) principles.

Phase 1 of the retrospective analysis involved the researcher collecting and analyzing all the data. An analysis was conducted to evaluate the outcomes of including the learning trajectory in teaching experiment 1 with the original HLT design. Moreover, the researcher investigated and examined the learning process activities included to determine the level of the student's comprehension of the idea of trigonometric ratios. An analysis was conducted on the outcomes of the HLT enhancements implemented in teaching experiment 2, by comparing the initial HLT with the execution of the learning process.

The researchers selected six participant subjects to interview. The six participants had various abilities; two had high, medium, and low abilities. In addition, interviews were also conducted with the mathematics teacher of trigonometric ratios at the school that taught the students.

RESULTS

The learning trajectory design discussed in this paper involves a series of student activities to help students understand trigonometric ratios through a Realistic Mathematics Education approach. The trajectory developed in this research comes from Empirical data obtained through direct observation of students, interviews, and worksheet analysis during the learning process. Findings showed that the students experienced gradual progress in understanding trigonometric ratios through a series of exploration-based activities.

Based on observation data, the students initially experienced difficulty connecting the concept of trigonometric ratios with the right triangles they encountered in everyday life. However, after gradual activities, their understanding develops from concrete exploration to a formal introduction to trigonometric ratios.

Activity 1: Observation of “Stretching the Raffia from the Base of the Table to the Top of the Student’s Head”

Activity 1 aims to help the students accurately determine the right angle, opposite side, and hypotenuse in a right triangle. The students, divided into groups of 4–5, use contextual equipment

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like raffia, a table, and a peer. One of the students holds one end of the raffia while the other is attached to the table base, forming a right triangle with the student's height and the floor. Observations are illustrated in Figure 1. Each group sketches their demonstration (Figure 2), identifying key triangle components. Students explain their reasoning for positioning angle θ and triangle sides. Those struggling receive guided reinvention through teacher-led discussions.

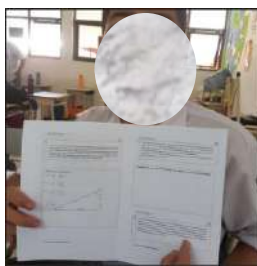


Figure 1: Activity 1 "Stretching a Raffia from the Base of the Table to the Top of the Student's Head"

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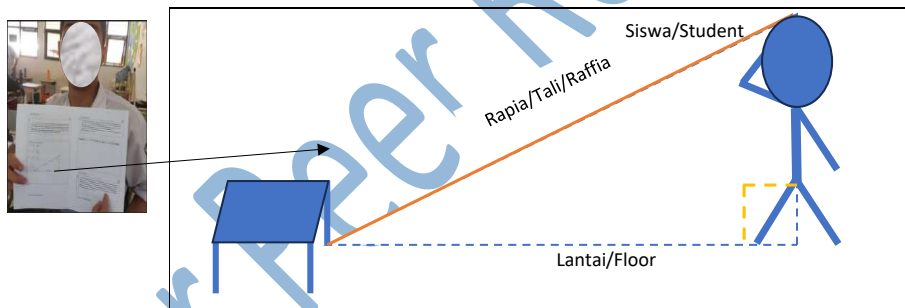


Figure 2: Example of the Student Sketches in Activity 1

- Teacher : "Okay, S-1, please explain what you drew."
- S-1 : "Okay, ma'am, if you look at the instructions in the worksheet, we are asked to practice what is written in activity 1, and then we share roles; S-2 ties the raffia to the base of the bottom table, S-3 holds the end of the raffia that results from extending S-2's raffia, S-4 reads what is on the worksheet, and I make the sketch."
- Teacher : "Okay, have you considered whether your sketch will be a right triangle?"
- S-1 : "At first, I did not, Ma'am, but when you said, 'Look if you draw it, is a right angle formed?' I remembered."
- Teacher : "Okay, how do you determine the corner points?"
- S-1 : "The benchmarks are at the end of the table legs, Leg S-3, and head S-3, Ma'am."

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- Teacher : "Why is that, S-1?"
- S-1 : "Yes, that is what it looks like when drawn, ma'am."
- Teacher : "Okay, the benchmarks that S-1 mentioned earlier make a move from the rapid raffia, right?"
- S-1 : "What do you mean, ma'am?"
- Teacher : "Mom, look, you represent the rapier raffia, the floor, and the height of S-2 with a line, right?"
- S-1 : "That is right, ma'am."
- Teacher : "Well, when you make that line, there is a phase where you turn; that is called a benchmark."
- S-1 : "Oh yes, that is right, ma'am, that is right."
- Teacher : "S-1, try to determine the right angle, slanted side, front side, and side in the sketch you have made!"
- S-1 : "This one is a right angle, this one is the hypotenuse, this one is the front side, and this one is the side side." (pointing to the picture).
- Teacher : "How do you determine a right angle, S-1?"
- S-1 : "There is a line like this and like this, ma'am." (pointing at the picture).
- Teacher : "Now, try to observe what degree angle the two lines form, or what angle is it called then?"
- S-1 : "Oh yes, ma'am, right angles."
- Teacher : "How do you determine the hypotenuse?"
- S-1 : "The side in front of the right angle, ma'am."
- Teacher : "How do you determine the front side?"
- S-1 : "This one, ma'am? Yes, the one in front of this corner (pointing to one of the corners of the picture)".
- Teacher : "Okay, how do you determine the side?"
- S-1 : "The one on the front side, ma'am."

In this activity, the students identify the expected position, which depends on the angle θ . This process helps them determine angles and sides in a right triangle. After completing the task, the teacher and the students reflect on positioning the right angle, hypotenuse, and θ , which influence side identification. The raffia stretching activity from the table to a student's head helps visualize the relationship between height and shadow length. Observations indicate that the students begin recognizing this pattern, even without fully grasping the ratio concept, providing a concrete experience before introducing the formal definition.

Activity 2: Observation "Measurement of The Students' Height and Shadow"

Activity 2 aims to help the students apply trigonometric ratios ($\sin \theta$, $\cos \theta$, and $\tan \theta$) in real-life contexts using right triangle comparisons. The students work in groups to transform comparison problems into hands-on tasks. One student stands with their back to a lamp to create a shadow, which the group then sketches. Each member takes turns measuring their height and shadow length,

recording the data in a table (Figure 4). The teacher guides the students in determining angle θ and calculating the height-to-shadow length ratio. This activity reinforces the concept of trigonometric ratios through visual and practical learning.

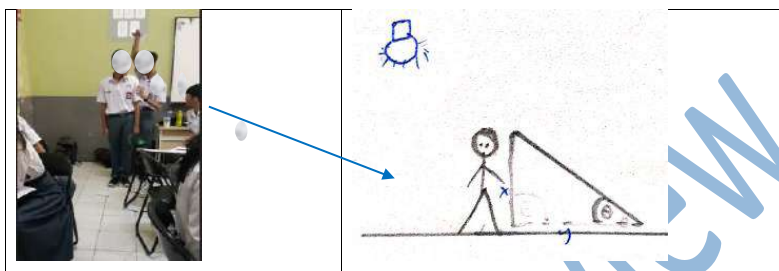


Figure 3: Activity 2 and Example of The Student Sketches in Activity 2

This project aims to help the students accurately calculate $\sin \theta$, $\cos \theta$, and $\tan \theta$ in their triangle sketches (Figure 5). The teacher monitors each group, guiding the students in problem-solving while allowing them to use formal or informal methods. Scaffolding is provided to help students recall and apply their knowledge of trigonometric ratios.

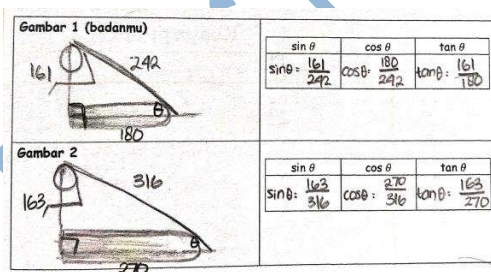


Figure 5: Example of Sketch and Application of Trigonometric Ratios Values in Activity 2

One of the students (S-5), as a representative of one of the group members, participated in presenting activity two and explained how the values of $\sin \theta$, $\cos \theta$, and $\tan \theta$ were obtained. After sketching the experiment, this group identified the right angle to determine the other positions. The following is a dialogue explaining the concept of trigonometric ratios described by the students.

Teacher : "Okay, S-5. Explain how your group determines each trigonometric ratio value requested in the table."

- S-5 : "Okay, Ma'am, first we sketch according to our height in the group, then, from the sketch formed, we determine the right angle, draw the symbol on the sketch, then determine the angle θ "
- Teacher : "Okay, do you mean identifying the position of a right angle?"
- S-5 : "Yes, ma'am."
- Teacher : "What do you mean by determining the angle?"
- S-5 : "So we can determine the front and side, ma'am."
- Teacher : "Okay, what is the next process?"
- S-5 : "Earlier, after determining the angle position θ , we then wrote down each of our heights according to what was measured earlier and wrote down the height of the shadow."
- Teacher : "Okay, how do you determine the hypotenuse?"
- S-5 : "Using the Pythagorean theorem, ma'am, with the help of a calculator."
- Teacher : "Okay, how do you calculate $\sin \theta$, $\cos \theta$, and $\tan \theta$?"
- S-5 : "Well, because the value of $\sin \theta$ is the front per hypotenuse, look at the picture (while pointing to picture 1 in UBA 2 Activity 2); the front of the angle θ is the front side, which is 161, and the hypotenuse is 242 so the $\sin \theta$ is 161 per 242."
- Teacher : "How to calculate $\cos \theta$ and $\tan \theta$?"
- S-5 : "Same here, ma'am, just a different position according to what was learned previously: $\cos \theta$ means side per hypotenuse, and $\tan \theta$ means front per side."

The conversation above helped all the students understand the concept of trigonometric ratios correctly. At the end of the activity, one of the students reflected on the lesson and concluded how to apply trigonometric ratios correctly. Measuring the height and shadow length helped the students understand that the ratio between an object's height and its image can be quantified. After multiple measurements, most students recognized a consistent pattern in this relationship, reinforcing the concept of constant ratios as the foundation for sine, cosine, and tangent.

Activity 3: Experiment with "Making Three Right Triangles from Paper"

Exercise 3 aims to help the students understand the relationship between similarity and trigonometric ratios. In groups of 4–5, the students use scissors, paper, rulers, and worksheets to cut rectangular paper into two equal right triangles and identify similarities in three right triangles. They explore similarity using diagrams and concrete teaching aids, as illustrated in Figures 6 and 7, where some groups present findings through diagrams while others use models. The teacher guides the students in recognizing corresponding sides and discovering formal patterns of similarity through directed questions, leading them to apply Geometry theorems by comparing three similar right triangles.

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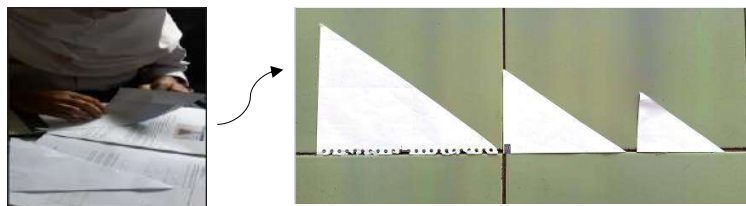


Figure 6. Activities and Examples of the Student Responses using Teaching Aids in Activity 3.

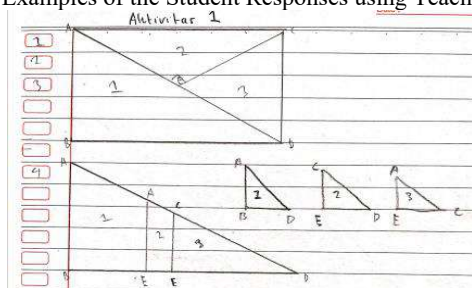


Figure 7. Example of the Student Responses using Diagrams in Activity 3

The teacher guides the students in identifying the shortest and longest sides of each triangle, helping them develop comparative thinking patterns (Figure 8). To deepen understanding, the teacher poses questions that draw analogies between similarity and trigonometric ratios (Figure 9). One example involves a logo on a garbage can placed on an equilateral triangle with a 6 cm side length. The students are asked to calculate the logo's height, sketch the triangle, and explain the concepts used in their solution.



The trash can logo featured below is designed as an equilateral triangle. The dimension of the side measures 6 centimeters. What is the estimated vertical dimension of the logo? Begin by creating a preliminary outline of the triangle. Which concept are you employing? Are there any other concepts available apart from the one you have employed? What is the procedure?

Logo tempat sampah berikut berbentuk segitiga sama sisi dengan panjang sisi 6 cm. Berapa kira-kira tinggi logo tersebut? Buatlah sketsa segitiga terlebih dahulu. Konsep apa kah yang kamu gunakan? Adakah alternatif konsep selain yang sudah kamu gunakan? Bagaimana caranya?

Figure 9. Example of Questions on the Relationship between the Concept of Similarity and Trigonometric Ratios in Activity 3.

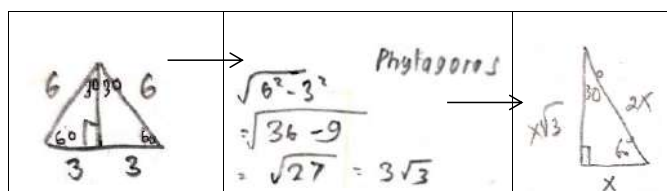


Figure 10. Example of the Student Responses to Contextual Questions in Activity 3.

In Figure 10, a group sketches the shape and calculates the logo's height using the Pythagorean Theorem, then verifies their results through similarity. The teacher and the students discuss the relationship between similarity and trigonometric ratios. By creating three right triangles of different sizes, the students observe that side length ratios remain constant for equal angles. Some believed the ratios would change initially, but experimentation confirmed their invariance, reinforcing a key principle of trigonometric functions.

Activity 4: Observation of "Calendar Image in the form of an Equilateral Triangular Prism"

Activity 4 helps the students determine trigonometric ratios for specific angles (0° , 30° , 45° , 60° , and 90°). Working in groups of 4–5, they analyze a calendar image as an equilateral triangular prism (Figure 11), extract an equilateral triangle, construct a symmetrical one with a 2-unit side, and divide it into two right triangles. Using the Pythagorean Theorem, they calculate unknown side lengths and determine trigonometric ratios for 30° and 60° , compiling their results in a table (Figure 12). For a 45° angle, the students draw an isosceles right triangle with 1-unit sides, calculate the hypotenuse, and determine $\sin \theta$, $\cos \theta$, and $\tan \theta$ (Figure 13). By the end of the activity, the students summarize their findings in a table, reinforcing their understanding of trigonometric ratios.

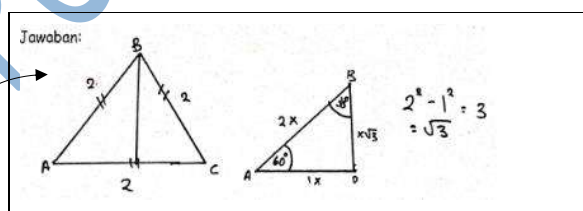


Figure 11. Calendar Image and Example of the Student Responses in Making Sketches in Activity 4

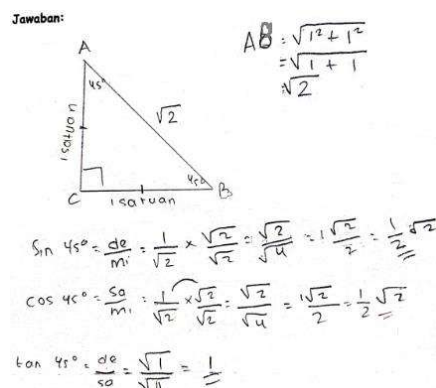


Figure 13. Example of the Student Responses Regarding Particular Angles 45° In Activity 4

	Rumus	30°	45°	60°
$\sin$	$\sin \theta = \frac{\text{sisl. depan}}{\text{sisl. miring}}$	$\frac{1}{2}$	$\frac{1}{2} \sqrt{2}$	$\frac{1}{2} \sqrt{3}$
$\cos$	$\cos \theta = \frac{\text{sisl. samping}}{\text{sisl. miring}}$	$\frac{1}{2} \sqrt{3}$	$\frac{1}{2} \sqrt{2}$	$\frac{1}{2}$
$\tan$	$\tan \theta = \frac{\text{sisl. depan}}{\text{sisl. samping}}$	$\frac{1}{3} \sqrt{3}$	1	$\sqrt{3}$

Figure 14. Example of Student Responses Represented in a Table in Activity 4

By the end of the exercise, the teacher and the students had reached a general conclusion on determining the value of trigonometric ratios at particular angles.

Observing the calendar image as an equilateral triangle helps the students recognize right triangles in various visual contexts. This activity increases their ability to identify right triangles in everyday objects, making it easier for them to connect trigonometric ratios to real-world applications.

Activity 5: Observation of “Bunk Bed”

Exercise 5 helps the students use trigonometric ratios to determine unknown side lengths in a right triangle. The students work in groups of 4-5, analyzing an image (Figure 15) and a triangular figure formed by stairs, a bunk bed, and the floor (Figure 16). They then apply horizontal mathematization by creating a diagram representing the triangle. The students identify whether the shape is a right triangle and describe the given data in a sketch. Using the tangent ratio, they calculate the displacement between the floor and the end of the stairs.



Figure 15. Activities and Questions in Activity 5

Each group's findings from activity five are poured into a sketch, as shown in Figure 16.

With teacher guidance, the students sketch a right triangle from the bunk bed, floor, and stairs, identifying the right angle and using the given values (37° and 10 cm) to calculate the height with the tangent ratio. After solving the problems, one group presents their results, explaining their understanding of trigonometric ratios in determining unknown side lengths. The teacher then asks questions to assess the student's comprehension, as shown in the dialogue below.

- Teacher : "Okay, how do you find the length of the unknown side in a right triangle?"
- S-6 : "Begin by sketching the problem, identifying given information such as side lengths (opposite, hypotenuse, or adjacent) and known angles. Then, analyze the available data to determine the appropriate trigonometric ratio to apply."
- Teacher : "Can you give an example?"
- S-6 : "In the bunk bed problem, after sketching a right triangle, we label the given values: a 37° angle between the stairs and floor and a 10 cm distance from the bunk bed to the stairs. Using the tangent ratio, we calculate the unknown side length, which relates the opposite and adjacent sides."

Observing the bed frame helps the students see how trigonometric ratios apply in construction. Those who initially struggled with trigonometry begin to connect triangle-side ratios to real-world structures. This activity reinforces the practical significance of trigonometric ratios in design and construction.

Activity 6: Observation of "Picture of a Skier Descending an Iceberg"

Task 6 helps the students apply trigonometric ratios to real-world problems by analyzing worksheet images (Figure 17) and solving open-ended questions in groups. Referring to Larson et al. (2007),

they calculate a skier's track length on an iceberg (Larson et al., 2007), and one group presents their findings to conclude the learning outcomes (Figure 18).



Figure 17. Questions in Activity 6

$$\begin{aligned} \sin 75 &= 0.97 \\ \sin \theta &= \frac{dc}{mi} = \frac{90}{mi} = 0.97 \\ &= \frac{90}{0.97} = 93 \end{aligned}$$


Figure 18. Example of the Student Responses to Activity 6

Observing an image of a skier descending an iceberg introduces the concept of slope about terrain and movement. The students connect slope angles to right triangle ratios, enhancing their understanding of the tangent function. This activity reinforces that trigonometry applies not only to static shapes but also to dynamic phenomena.

DISCUSSION

Learning activities using the Realistic Mathematics Education approach regarding trigonometric ratios followed by the students consist of six activities, namely raffia stretching activities, the student height measurement activities, activities to make three right triangles from paper, activities to observe calendar images, activities to observe bunk bed images, and activities to observe skier images. Each activity has its purpose and is interrelated and continuous. The goals of each action are outlined below: Firstly, stretching the raffia allows the students to determine the location of the angles and sides of a right triangle. Secondly, measuring the height of the students and their shadows allows the students to apply the concept of trigonometric ratios. 3) Creating a teaching tool including three right triangles is intended to facilitate the student's comprehension of the correlation between resemblance and trigonometric ratios. 4) Observing the calendar image permits the students to ascertain the numerical value of trigonometric ratios at specific angles. The objective of viewing the bunk bed image is to help the students apply trigonometric ratios to ascertain the length of the desired side in a right triangle. Similarly, observing the skier image seeks

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to empower the students in solving contextual difficulties by employing the concept of trigonometric ratios. Every worksheet provided incorporates the specific features of the Realistic Mathematics Education pedagogy about trigonometric ratios. These findings corroborate Subekti's assertion that student worksheets will enhance learning (Subekti & Prahmana, 2021).

Applying a realistic approach to teaching mathematics in various activities in the learning process should facilitate the student's understanding of mathematical skills and allow teachers to help the students effectively achieve learning goals (Ekowati & Nenohai, 2017). Students' understanding improves because the process of learning mathematics that follows Realistic Mathematics Education utilizes the environment around the world to encourage students to produce ideas and develop their knowledge (Nirawati et al., 2021).

According to research done by Sukendra (2020), a series of learning process activities with the RME approach can significantly facilitate students' understanding of the trigonometric comparison concept (Sukendra, 2020). Therefore, deliberate learning paths encourage students to build their knowledge, improve their current skills, and effectively overcome challenges in many subjects (Prahmana et al., 2017; Nursyahidah et al., 2020; Wijaya et al., 2021; Kuncoro et al., 2023).

CONCLUSIONS

This work aims to develop a learning trajectory for trigonometric ratios using the RME approach. The findings of this study suggest that implementing the RME approach in designing a learning trajectory for trigonometric ratios can enhance the students' mathematical proficiency in comprehending the concepts of trigonometric ratios. The learning trajectory designed using the RME approach consists of six activities. Through a series of continuous and ongoing activities in the six activities, the students find it easier to build their concepts and understand the concept of trigonometric ratios. The students develop their understanding starting from their ability to identify angle and side positions, apply the definition of trigonometric ratios to contextual problems, understand the relationship between similarity and trigonometric ratios, determine the magnitude of trigonometric ratio values at particular angles, apply trigonometric ratios, and solve contextual problems using trigonometric ratios.

This research is expected to encourage other researchers to design learning trajectories using the same topic but different approaches. The researcher hopes that the results of this design can contribute to the researchers' contribution to building a generation according to the nation's expectations.

Commented [bc3]: In order to assess the improvement you need to show us how students understood the trig c oncepts before the intervention. Otherwise how do you know they improved?

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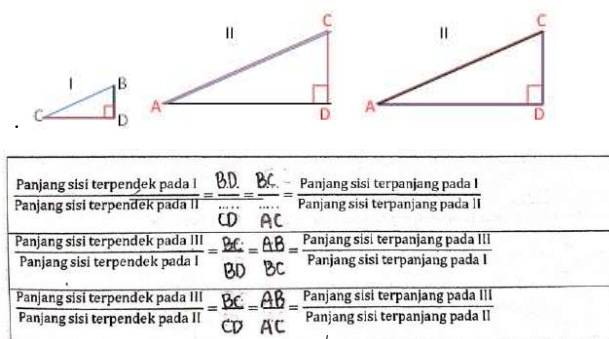
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APPENDIX

Kamu	Nama temanmu	Nama temanmu	Nama temanmu	Nama temanmu
<u>Nabila</u>	<u>Bunga</u>	<u>Naufal M</u>	<u>Jahsyi</u>	<u>Nalvin</u>
Tinggi badan	155	153	173	169
Tinggi bayangan	165	163	183	179

	Your name <u>Nabila</u>	Your Classmate's name <u>Bunga</u>	Your Classmate's name <u>Naufal M</u>	Your Classmate's name <u>Jahsyi</u>	Your Classmate's name <u>Nalvin</u>
Height	155	153	173	169	161
Shadow Height	165	163	183	179	171

Figure 4: Example of a table of results measuring height and shadow in activity 2.



the length of the shortest side in figure I	$\frac{BD}{CD} = \frac{BC}{AC}$
the length of the shortest side in figure II	$\frac{BD}{CD} = \frac{BC}{AC}$
the length of the longest side in figure I	$\frac{BD}{CD} = \frac{BC}{AC}$
the length of the longest side in figure II	$\frac{BD}{CD} = \frac{BC}{AC}$
the length of the shortest side in figure III	$\frac{BD}{CD} = \frac{BC}{AC}$
the length of the shortest side in figure I	$\frac{BD}{CD} = \frac{BC}{AC}$
the length of the longest side in figure III	$\frac{BD}{CD} = \frac{BC}{AC}$
the length of the longest side in figure I	$\frac{BD}{CD} = \frac{BC}{AC}$
the length of the shortest side in figure III	$\frac{BD}{CD} = \frac{BC}{AC}$
the length of the shortest side in figure II	$\frac{BD}{CD} = \frac{BC}{AC}$
the length of the longest side in figure III	$\frac{BD}{CD} = \frac{BC}{AC}$
the length of the longest side in figure II	$\frac{BD}{CD} = \frac{BC}{AC}$

Figure 8. Example of the Student Responses in Using the Concept of Comparison in Activity 3.

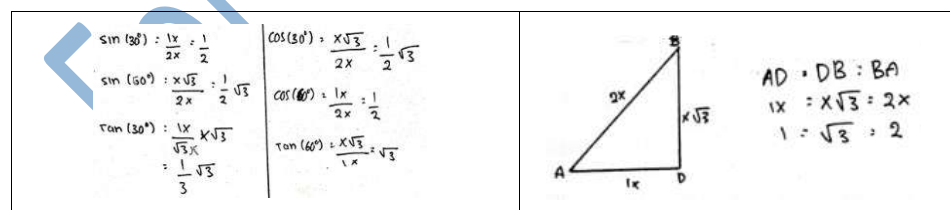


Figure 12. Example of the Student Responses regarding memorable and unique 30° angles 60° in Activity 4

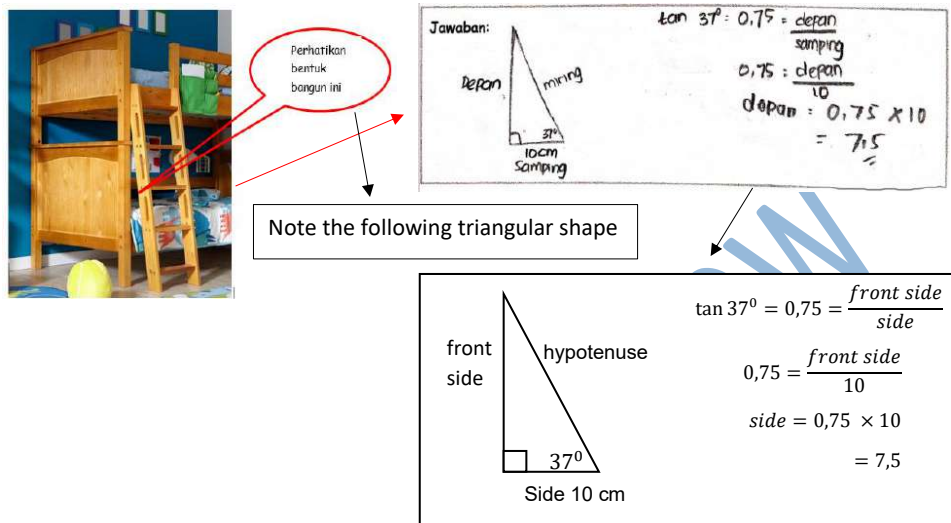


Figure 16. Questions and the Student Responses Representing Data into Sketches in Activity 5

**4. Bukti Konfirmasi Artikel
Accepted
(2 April 2025)**



Bronislaw Czarnocha <bronislawcz2@gmail.com>
to me, darhim0355, diniandiani367, Malgorzata ▾

Wed, Apr 2, 2025, 3:24 AM ☆ 😊 ↶ ⋮

Hello colleagues, it's my pleasure to inform you that your paper

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**5. Bukti Konfirmasi Artikel
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