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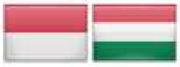

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GEOGEBRA-ASSISTED TEACHING EXPERIMENT ON STUDENTS' CONCEPTUAL CHANGE IN MULTIVARIABLE CALCULUS

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ABSTRACT

This study explores students' conceptual change in understanding gradients and tangent planes through a GeoGebra-assisted teaching experiment in a multivariable calculus course. The study involved eight second year undergraduate mathematics students enrolled at the Mathematics Department of a university in West Java, Indonesia. Data were collected through diagnostic tests, classroom observations, students' written work, and reflective notes, and semi structured interviews, then analyzed qualitatively using coding, triangulation, and retrospective analysis.The findings revealed that, at the outset, students were able to perform symbolic differentiation of partial derivatives but struggled to interpret the gradient vector conceptually or relate it to the tangent plane. Through guided use of GeoGebra, students gradually reconceptualized the gradient as representing both the direction and rate of steepest ascent, demonstrating a shift from procedural to structural understanding. This study contributes to the theory of conceptual change by illustrating how dynamic visualization and guided reinvention can facilitate the transformation of students' procedural knowledge into coherent conceptual models. Practically, the findings provide insights for designing technology-assisted instruction to address persistent misconceptions in multivariable calculus learning.

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INTRODUCTION

Multivariable calculus is a core subject in undergraduate mathematics and science programs, playing a crucial role in developing students' higher-order thinking, spatial reasoning, and modeling skills. Despite its importance, many students face persistent difficulties when transitioning from single-variable to multivariable contexts, particularly in understanding abstract concepts such as partial derivatives, gradients, and tangent planes (Borji et al., 2024; Hahn & Klein, 2025; Kashefi et al., 2011; Rodríguez-Nieto & Moll, 2025). These difficulties often stem from an overreliance on procedural computation, with limited ability to integrate algebraic procedures with geometric interpretation (Borji et al., 2024; Cheong et al., 2023; Hahn & Klein, 2025). As a result, students may compute derivatives correctly but struggle to articulate the conceptual significance of gradients or interpret the geometric meaning of tangent planes.

Recent diagnostic data collected from calculus students at the Mathematics Department of a university in West Java, Indonesia, further confirmed these persistent difficulties. In a preliminary assessment involving gradient and tangent plane tasks, only three out of eight students (37.5%) were able to correctly interpret the geometric meaning of a gradient vector, and none could connect the symbolic expression $\nabla V(x, y)$ with its graphical representation. Most participants performed symbolic differentiation correctly but failed to describe the gradient as representing the direction and rate of maximum change. These findings align with previous research reporting similar misconceptions, where students often treat the gradient as a mere algebraic output rather than a geometric or physical quantity (Borji et al., 2024; Hahn & Klein, 2025; Rodríguez-Nieto & Moll, 2025). Such consistent evidence underscores the need for instructional designs that integrate visualization and conceptual scaffolding to promote deeper understanding.Mathematics education researchers have highlighted the importance of instructional strategies that foster conceptual understanding alongside procedural fluency. Conceptual growth involves transforming operational procedures into structural objects, enabling students to reason about mathematical entities more flexibly (Andiani et al., 2024, 2025; E. D. Dubinsky & Mcdonald, 2001; E. Dubinsky & Mcdonald, 2001; Matindike, 2025; Shvarts et al., 2024; Trigueros et al., 2024; Villabona et al., 2024). In this regard, technology enhanced learning environments, particularly dynamic visualization tools such as GeoGebra, have been shown to support students' conceptual development by linking symbolic computations with graphical and interactive representations (Baye et al., 2021; Gurmu et al., 2024; Suparman et al., 2024). Such tools serve as mediating devices that can bridge students' fragmented understanding of multivariable functions, gradients, and tangent planes (Borji et al., 2022; Cheong et al., 2024; Herrera et al., 2024; Schoenherr et al., 2024). Click or tap here to enter text.

Several studies have demonstrated the benefits of technology in calculus instruction. For example, Schoenherr found that interactive visualizations improved students' reasoning about multivariable functions (Cheong et al., 2023; Herrera et al., 2024; Rao et al., 2023; Schoenherr et al., 2024), while Karabay (2025) highlighted the role of scaffolding in managing cognitive load during problem solving (Faber et al., 2024; Karabay & Meşe, 2025; Li et al., 2023; van Nooijen et al., 2024). However, much of the existing research emphasizes quantitative outcomes, such as test performance, with limited attention to qualitative aspects of conceptual change or the dynamics of classroom interactions. Additionally, few studies have examined how iterative teaching experiment designs can trace students' conceptual difficulties and the pathways of conceptual change over time.

The novelty of this study lies in combining a teaching experiment approach with retrospective analysis to explore students' conceptual difficulties and subsequent changes in understanding gradients and tangent planes through GeoGebra-assisted learning. Teaching experiments enable iterative refinement of instructional interventions while closely observing students' reasoning (Kelly & Lesh, 2000). Retrospective analysis allows systematic identification of shifts in conceptual understanding across multiple data sources.

In designing the instructional intervention, this study was also inspired by the core principle of Realistic Mathematics Education (RME), particularly guided reinvention, which emphasizes that students should be supported in reconstructing mathematical concepts through exploration and teacher-guided reflection rather than direct instruction (Freudenthal, 1991; Gravemeijer & Cobb, 2006). Within the GeoGebra-based environment, guided reinvention is realized through dynamic visualization and scaffolding that allow students to gradually develop conceptual understanding of gradients and tangent planes from their own explorations. This approach aligns with the goal of promoting conceptual change, as students actively reorganize their prior procedural knowledge into deeper structural understanding. Previous research by Andiani et al., (2020) demonstrated that students face challenges in solving geometric problems on competency-based assessments, highlighting difficulties in understanding and reasoning mathematically (Andiani et al., 2020). Furthermore, Andiani et al., (2025) found that realistic mathematics education approaches can improve students' mathematical representation skills, essential in multivariable calculus contexts (Andiani et al., 2025). These findings suggest that explicit scaffolding and visualization tools are crucial for fostering conceptual understanding.

Therefore, this research focuses on exploring students' conceptual difficulties and conceptual change in multivariable calculus through a qualitative teaching experiment approach supported by GeoGebra. The main aim is to describe how students develop an understanding of gradients and tangent planes when exposed to visualization-based instruction and to analyze how this process bridges the gap between procedural fluency and conceptual comprehension.

The findings of this study are expected to contribute both theoretically and practically. Theoretically, it may deepen understanding of how visualization and guided reinvention promote conceptual change in higher mathematics. Practically, it can inform the development of effective, technology-enhanced learning materials and instructional strategies for calculus education.

This study is theoretically grounded in the Conceptual Change framework (Vosniadou et al., 2008) which explains how learners restructure their prior knowledge when confronted with cognitive conflict or anomalous data. Within this perspective, conceptual understanding develops through the gradual reorganization of students' existing mental models toward more coherent and scientifically accurate ones. In the context of multivariable calculus, this framework is particularly relevant because students often rely on procedural schemas, such as differentiating symbolically, without connecting them to geometric or physical meanings. The GeoGebra-assisted learning design in this study was intended to trigger such cognitive conflict and guide students toward reconceptualizing gradients and tangent planes as dynamic representations of rate and direction, thereby fostering meaningful conceptual change.

Recent studies have confirmed that GeoGebra-assisted visualizations can significantly enhance students' conceptual understanding and engagement in multivariable calculus and related mathematical topics (Bedada & Machaba, 2022; Cheong et al., 2024; Sari et al., 2024). Building on these insights, the present study focuses on examining how GeoGebra-assisted instruction can facilitate students' conceptual change in understanding gradients and tangent planes. Specifically, it aims to trace the progression of students' reasoning as they move from procedural manipulation toward conceptual comprehension within a dynamic learning environment.

METHOD

This study employed a qualitative descriptive design through a teaching experiment and retrospective analysis to investigate students' conceptual difficulties and conceptual change in multivariable calculus. The researchers chose this design because it allowed them to explore students' thinking processes and capture how their understanding evolved during the learning activities.

The participants were second year undergraduate students (semester IV, Academic Year 2023/2024) enrolled in a multivariable calculus course at the Mathematics Department of a university in West Java, Indonesia. A total of eight ($n = 8$) students participated in the main teaching experiment phase, selected purposively based on their prior exposure to basic calculus concepts and their availability to engange in all instructional sessions. The small group size was intended to enable intensive observation of students' learning processes.

The main instruments used in this study was a diagnostic test consisting of a series of structured problems on the concept of limits of two variables and GeoGebra-based activities designed to promote visualization and conceptual exploration. This diagnostic test aimed to identify students' conceptual understanding and misconceptions related to gradients and tangent planes while simultaneously encouraging visual reasoning through dynamic representations. The researchers employed observation sheets to document classroom interactions and student strategies, while semi-structured interview guidelines were used to explore students' reasoning, misconceptions, and conceptual changes in greater depth.

Semi-structured interviews were conducted after each teaching experiment session to elicit students' reflections on their learning processes. The questions focused on how students interpreted the meaning of gradient vectors, related symbolic differentiation to graphical visualization, and perceived the role of GeoGebra in helping them reconstruct their understanding. These interviews provided qualitative evidence to support the interpretation of students' conceptual change.

The procedures consisted of two stages of teaching experiments followed by retrospective analysis. Each teaching experiment involved introducing learning tasks, classroom interaction supported by GeoGebra visualization, and reflection activities. The researchers collected data through diagnostic tests, classroom transcripts, students' written work, and reflective notes.

Data analysis was conducted through qualitative coding of students' responses, identifying recurring misconceptions, and comparison across diagnostic test results, classroom observations, and written work. Triangulation between these data sources was applied to ensure the credibility and validity of the findings. The analysis was conducted iteratively, allowing them to refinement of interpretations and alignment with the theoretical framework of conceptual change in multivariable calculus.

RESULT AND DISCUSSION

Overview of Data Coding and Categories

Data from diagnostic tests, written work, classroom observations, and semi-structured interviews were analyzed qualitatively through thematic coding. Three major categories of student reasoning were identified: (1) procedural responses, focused only on symbolic computation, (2) partial conceptual awareness, showing fragmented interpretations, and (3) integrated conceptual understanding, demonstrating coordinated algebraic and geometric reasoning. These categories guided the structure of the findings presented below and formed the analytical lens through which students' conceptual development was interpreted.

Building on these categories, the following section presents students' initial conceptual difficulties, illustrating the dominance of procedural reasoning before the GeoGebra-assisted intervention.

Students' Initial Conceptual Difficulties

At the beginning of the task, most students successfully computed the partial derivatives of the potential function $V(x, y) = x^2 + 2y^2$, yielding the gradient $\nabla V(x, y) = (2x, 4y)$. When evaluated at the point (1,2), this became $\nabla V(1,2) = (2,8)$. However, their written responses tended to stop at the numerical level, showing little evidence of conceptual interpretation. This finding resonates with Borji (2022), who noted that many students demonstrate strong procedural skills while lacking structural understanding in multivariable calculus (Borji et al., 2022).

A recurring pattern emerged where students simply wrote, *"gradient* = (2,8), *finished."* Such utterances illustrate what Sfard (2008) describes as *operational thinking*, where mathematical objects are treated merely as the results of procedures. Click or tap here to enter text. Students in this stage failed to recognize that the gradient is not only a pair of numbers but a vector with both magnitude and direction. Figures 1 and 2 show examples of students' written responses. Both students correctly derived the gradient function $\nabla V(x, y) = (2x, 4y)$ and substituted values to obtain $\nabla V(1,2) = (2,8)$, yet neither provided any explanation of its meaning.

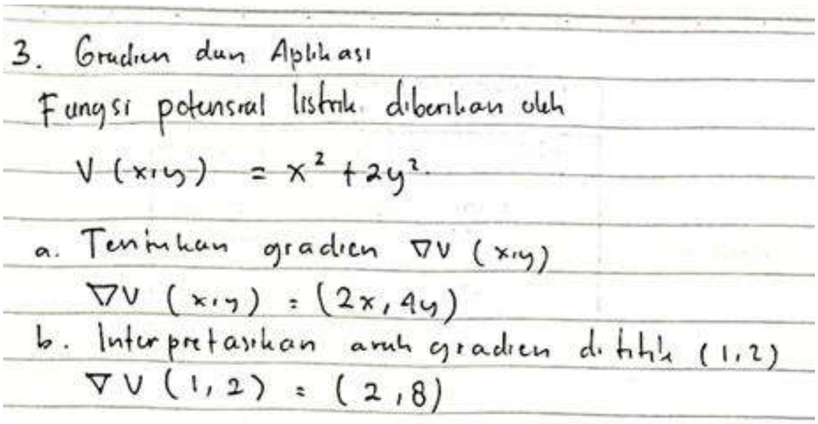

Figure 1. Example Of Student S1's Written Response To The Gradient Task $V(x, y) = x^2 + 2y^2$, Illustrating Purely Procedural Reasoning.

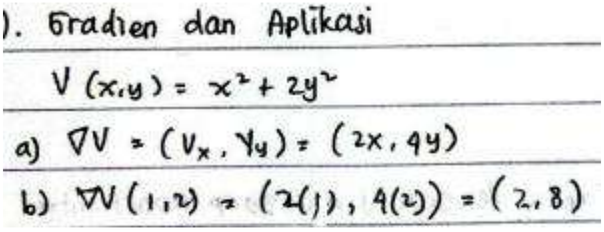

Figure 2. Example Of Student S2's Written Response To The Same Task, Showing Correct Computation But Lacking Conceptual Explanation.

When asked for further interpretation, some students vaguely described the result as "the slope in x is two and in y is 8," without recognizing its vectorial nature. This fragmentation mirrors Cheong et al. (2023), who found that many students treated partial derivatives as isolated algebraic procedures rather than components of a unified vector concept (Cheong et al., 2023). As Herera et al. (2024) srgue, such disconnections limit students' ability to transfer abstract computation to applied contexts, preventing them from developing a coherent understanding of multivariable relationships (Herrera et al., 2024) .Click or tap here to enter text.

This difficulty is particularly important in vector calculus, where gradients play a central role in understanding directional change, optimization, and physical phenomena. Without conceptual grounding, students cannot connect gradients to applied contexts, such as interpreting potential fields in physics or optimization problems in engineering. Medina Herrera et al. (2024) highlight that such gaps hinder the transition from abstract calculation to practical application (Herrera et al., 2024).

A closer analysis of written work confirmed this gap. While algebraic manipulations were accurate, annotations explaining the meaning of the results were rare. Students did not spontaneously refer to the gradient as the direction of steepest ascent or discuss its physical implications in the context of a potential function. Students did not spontaneously refer to the gradient as the direction of steepest ascent or discuss its physical implications in the context of a potential function, as confirmed by Abreu et al. (2021), who emphasize that symbolic fluency alone is insufficient without scaffolding that links symbols to concepts (Abreu-Mendoza et al., 2021).

In retrospective discussions, some students admitted being unsure how to proceed after obtaining the ordered pair. They expressed uncertainty about substituting values for the original function, drawing a graph, or leaving the result. Such hesitation illustrates the lack of an internalized schema for interpreting gradient vectors.

Excerpts from semi-structured interview (Participant S3):

Session 1 (before intervention):

Interviewer (I): *What does the pair (2, 8) mean to you?*

S3: “It’s just the slope — so x is 2 and y is 8, that’s it.”

I: *Can you explain how to draw it on the surface?*

S3: “I don’t know, maybe draw a line? I usually stop after computing numbers.”

Session 2 (after GeoGebra intervention):

I: *Now that you used the GeoGebra applet, what does (2,8) represent?*

S3: “This is an arrow at (1,2), it shows which direction the surface goes up fastest. The 8 means it increases faster in y than x.”

These excerpts clearly illustrate the change in conceptualization before and after the GeoGebra-based intervention. Initially, the student viewed the gradient as a mere numerical pair, but later began to interpret it as a vector that conveys direction and rate of change. These findings underscore the necessity of carefully designed instructional interventions. As Tall (2008) argues, advanced mathematical thinking requires students to move beyond computational comfort zones and integrate multiple representations (Tall, 2008). Here, the initial failure to articulate conceptual meaning highlights the importance of explicitly embedding interpretation tasks in calculus instruction.

Table 1 summarizes students' initial approaches to illustrate the distribution of responses. Categories include purely procedural responses (e.g., numerical computation only), partial interpretations (mentioning “slope” without directionality), and meaningful interpretations (rare at this stage).

Table 1. Categories Of Student Responses To The Gradient Task At (1,2)

Response Type	**Example Student Statement**	**Frequency (N=8)**
Procedural only	“Gradient = (2,8), finished”	3
Partial interpretation	“Slope in x is 2, in y is 8”	0
Full interpretation	“The vector (2,8) shows the direction of steepest increase at (1,2); its rate of increase per unit length is $2\sqrt{17}$.”	5

The predominance of procedural responses highlights the extent to which conceptual understanding was initially lacking. Such a distribution sets the stage for examining how scaffolding and visualization can support a deeper grasp of gradient meaning.

Overall, this first phase demonstrates that students could compute but not interpret. Without explicit prompts or visual tools, they remained anchored in procedural reasoning, which validates earlier findings by Borji (2022) and Cheong et al. (2023) that procedural fluency does not automatically translate into conceptual mastery (Borji et al., 2022; Cheong et al., 2023).

Transition from Symbolic Computation to Geometric Meaning

After the researchers introduced scaffolding strategies, students moved beyond numerical computation toward geometric meaning. The teacher encouraged them to see the gradient as a vector pointing toward the steepest ascent. This process of guided reinvention mirrors Sfard's (2008) description of *reification*, where operational processes gradually become conceived as structural objects (Sfard, 1991).

For example, after further questioning, a student remarked, *"So, the arrow shows the fastest way up."* What the student said illustrates a pivotal shift from perceiving the gradient as an inert ordered pair to recognizing it as a directional indicator. Rochaminah et al. (2025) emphasize that such discursive transitions are critical for advancing from procedural competence to conceptual understanding (Rochaminah et al., 2025).

The case of (2,8) at (1,2) was especially instructive. The researchers prompted students to interpret not only the numerical values but also the relative weights of each component. They concluded that the vector points northeast, with a stronger bias toward the y-axis due to the larger coefficient. This interpretation aligns with Cheong et al. (2023), who found that relative component magnitudes often served as an entry point for students in grasping geometric meaning (Cheong et al., 2023).

Discussions also emphasized physical interpretations. The researchers asked students to imagine the potential function representing an electric field. In this analogy, the gradient indicated the direction in which a test charge would experience the greatest force. This contextualization supported Medina Herrera et al. (2021), who argue that situating symbolic results in applied domains enhances engagement and meaning making (Herrera et al., 2024).

At this stage, evidence of conceptual change was visible in classroom transcripts. One student said, *"The 2 and 8 mean the function increases faster in y than in x."* Such statements go beyond restating numerical results and reveal an emerging awareness of the gradient's functional role.

This development was further supported by interview data. When asked to explain what the gradient vector (2, 8) represents, the following exchange occurred:
Researcher: "Can you explain what the numbers 2 and 8 mean in your result?"
Student A: "It means the surface goes up faster in the y direction than in x. So, the arrow should be longer toward y."
Researcher: "How do you know it's the direction of the fastest increase?"
Student A: "Because when we move in that direction, the value of the function increases most quickly. GeoGebra showed the arrow pointing that way."
This excerpt illustrates how visualization and guided questioning facilitated the student's shift from procedural to geometric reasoning, marking the early stages of conceptual change.

This development illustrates Tall's (2008) assertion that students benefit from explicit opportunities to link symbolic expressions with graphical and contextual interpretations (Tall, 2008). The researchers facilitated the connection through structured questioning and prompts that required students to articulate their thinking.

GeoGebra was later employed to reinforce this connection. Visualizing the vector (2,8) at (1,2) provided immediate geometric confirmation of analytic calculations. Figure 1 shows a screenshot of the vector superimposed on the graph of the potential function. This use of technology resonates with Medina Herrera et al. (2021), who emphasize the value of dynamic software in bridging representational gaps (Herrera et al., 2024).

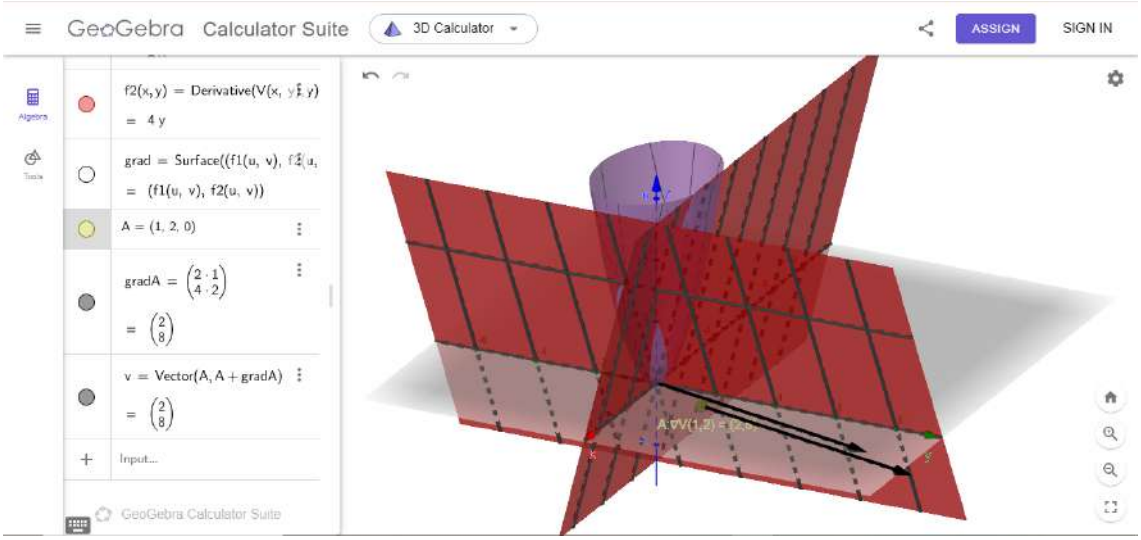

Figure 3. Geogebra Visualization Of The Gradient Vector (2,8) Anchored At Point (1,2) On The Surface $V(x,y) = x^2 + 2y^2$, Illustrating The Direction Of Steepest Ascent

Figure 4. Students Exploring Mathematics Concepts With Geogebra On Mobile Phones

Such visual aids confirmed symbolic computations and expanded students' reasoning. The visual of a steep arrow pointing northeast made the abstract idea of “steepest ascent” concrete. Sung et al. (2025) highlight that such multimodal engagements promote deeper conceptualization (Sung & Nathan, 2025; Yeh et al., 2025).

By the end of this phase, students were increasingly able to articulate the meaning of gradients in symbolic and geometric terms. Increased students demonstrate the potential of scaffolding and visualization in fostering representational integration.

The transition from symbolic to geometric reasoning marked a critical conceptual breakthrough. Students no longer viewed the gradient as a final numerical product but as an active object that connects algebraic, graphical, and physical dimensions. This evolution of students' reasoning exemplifies conceptual change, defined as the restructuring of prior mental models into more coherent scientific understanding (Vosniadou et al., 2008). Students moved from perceiving gradients as isolated numeric results to viewing them as integrated geometric and physical entities.

Difficulties in Visualizing Gradient Vectors

Despite notable progress, visualization remained a challenge for many students. Several could describe the meaning of (2,8) but hesitated to represent it graphically. Some drew arrows incorrectly, while others avoided sketching altogether. This difficulty echoes Moreno (2021), who reported persistent struggles with translating symbolic gradients into graphical form (Moreno-Arotzena et al., 2021).

One common error involved placing the vector at the origin instead of at the point (1,2). The error of the activity suggests a limited understanding of gradient vectors as *anchored* at specific points on the surface. Similar misconceptions were identified by Cheong et al. (2023), who observed that students often treated gradient vectors as free-floating arrows (Cheong et al., 2023).

At this point, transcript evidence further illustrated this difficulty.

Researcher: "Where should the arrow start?"

Student B: "From zero, I think, because it's a vector."

Researcher: "What happens if you start it at (1,2) instead?"

Student B: "Oh, that makes more sense, it shows the slope at that point!"This short exchange demonstrates how guided scaffolding and visualization in GeoGebra helped students reconstruct meaning through interaction, a clear instancce of guided reinvention (Freudenthal, 1991; Gravemeijer & Cobb, 2006), where conceptual understanding emerges from exploration rather than direct instruction.

Another difficulty was scaling. Students who attempted sketches sometimes drew the components with incorrect proportions, e.g., equal lengths for 2 and 8. This indicates partial recognition of directionality but incomplete attention to magnitude. Bollen et al. (2017) caution that without attention to scale, students risk losing the quantitative meaning of vectors (Bollen et al., 2017; Dubey & Barniol, 2023; Sabah, 2023)(Bollen et al., 2017; Dubey & Barniol, 2023; Sabah, 2023). To address these issues, the class used GeoGebra's vector tools to plot (2,8) accurately at (1,2). Figure 3 illustrates how this technology provided precise positioning and scaling. Such scaffolding resonates with Medina Herrera et al. (2021), who show that dynamic software reduces cognitive load and reinforces representational accuracy (Herrera et al., 2024).

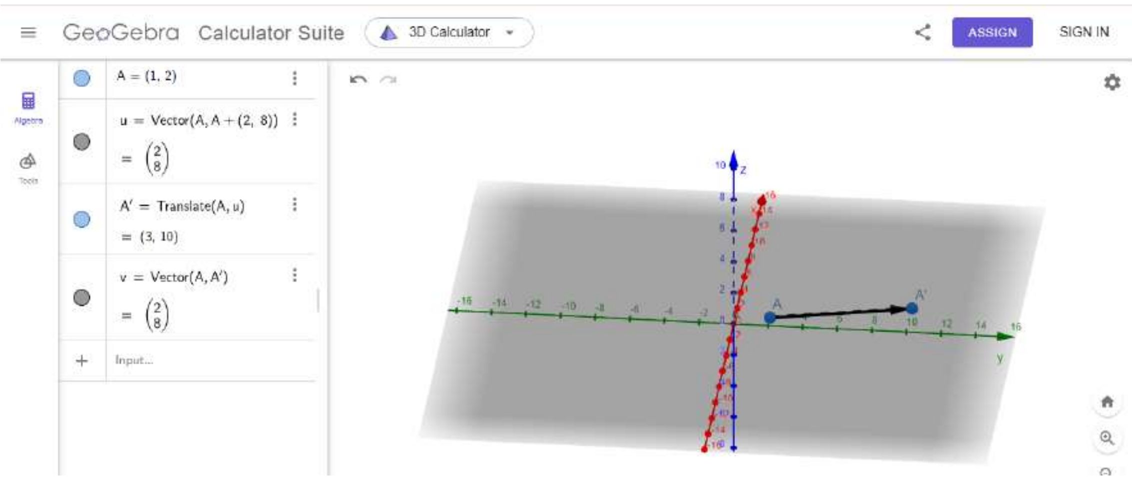

Figure 5. Geogebra Plot Of Gradient Vector (2,8) Anchored At (1,2)

Transcript evidence also showed that visualization helped students reconcile earlier errors. After seeing the GeoGebra output, one student said, *“Oh, the arrow is not from zero, but from (1,2).”* The student said that visualization supported correcting misconceptions and facilitated representational alignment. Nevertheless, reliance on technological scaffolding revealed another challenge: students struggled to visualize vectors without software support. Students' struggles to visualize vectors without software support suggest that internalization of geometric reasoning requires repeated practice, beyond one-time demonstrations. As Sfard (1991) warns that without reification, students may remain dependent on external representations (Sfard, 1991).

Class discussions further emphasized the need to interpret vector length. During this phase, students were guided to compute its magnitude, $\sqrt{2^2 + 8^2} = \sqrt{68}$, and to relate it to the rate of change of the surface. This activity addressed earlier gaps where students equated the gradient merely with “slope,” without recognizing its multidimensional meaning. A classroom transcript captures this moment: Student A: *“Because when we move in that direction, the value of the function increases most quickly.”*

This statement reflects students’ growing awareness that magnitude encodes the rate of change, evidence of conceptual reorganization in progress. Integrating magnitude into interpretation led students to view the gradient as a holistic object with both geometric and physical significance. This shift corresponds to the reification process described by Sfard (1991), in which operational procedures become structural mathematical objects (Sfard, 1991).

Likewise, this progression aligns with Tall’s (2002, 2004) framework of moving from fragmented knowledge toward connected understanding through multiple representationsClick or tap here to enter text. (Tall, 2002, 2004). In the context of this teaching experiment, GeoGebra functioned as a mediating tool supporting this transition. Although visualization difficulties persisted, the combination of symbolic computation, guided scaffolding, and dynamic software effectively mediated reasoning and supported the development of conceptual insight.

Overall, the retrospective analysis provides concrete empirical evidence that systematic scaffolding and dynamic visualization can transform procedural reasoning into a richer conceptual understanding of gradient vectors, illustrating guided reinvention and conceptual change in advanced calculus learning.

While these findings are consistent with previous studies (Cheong et al., 2023; Herrera et al., 2024), this research extends existing knowledge by uncovering the mechanisms underlying students’ conceptual change in multivariable calculus. The data suggest that conceptual restructuring did not occur solely because of GeoGebra’s visual affordances, but through cycles of guided reflection, questioning, and reinterpretation of prior misconceptions. In particular, the shift from viewing the gradient as a mere numerical output to understanding it as a geometric and physical entity illustrates the transformation of operational knowledge into structural understanding, a process central to conceptual change (Sfard, 1991; Vosniadou et al., 2008). This study therefore moves beyond confirming GeoGebra’s effectiveness and contributes theoretically by demonstrating that meaningful conceptual change requires the interplay of visualization, guided reinvention, and metacognitive engagement, rather than visualization alone.

Despite these contributions, several limitations of the study should be acknowledged. The small number of participants ($n = 8$) restricts the generalizability of the findings to broader populations. The short duration of the teaching experiment may also limit insights into the long-term stability of students’ conceptual change. Furthermore, the exclusive use of GeoGebra as the visualization tool might have shaped the learning dynamics in specific ways. Future research should therefore involve larger and more diverse participant groups, extend the intervention period, and compare different visualization or instructional frameworks to provide a more comprehensive understanding of how conceptual change evolves and endures in advanced mathematics learning.

CONCLUSION

This study identified persistent conceptual difficulties among students in interpreting gradients and tangent planes. Initially, students demonstrated strong procedural fluency in computing partial derivatives but failed to connect symbolic results with geometric meaning. These findings confirm the central issue highlighted in the study's title and established a basis for examining conceptual change.

Through iterative GeoGebra-assisted instruction and guided reflection, students gradually reconstructed their understanding, shifting from viewing gradients as mere numerical results to recognizing them as vectors representing both direction and rate of change. This transformation exemplifies conceptual change as the reorganization of prior mental models into coherent conceptual structures. The study extends the Conceptual Change framework by showing that lasting conceptual transformation requires not only visualization but also guided reinvention, metacognitive engagement, and representational integration.

Practically, the findings suggest that instructional design in advanced calculus should integrate dynamic visualization with reflective discussion and scaffolding tasks to promote representational understanding rather than procedural repetition.

However, the study's limitations include its small participant group ($n = 8$), brief intervention period, and focus on a single digital platform (GeoGebra). Future research should adopt longitudinal and multicontext investigations to explore how conceptual change develops and stabilizes across varied technological and instructional environments.

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AUTHOR CONTRIBUTIONS

Author One contributed to the conceptualization, methodology, formal analysis, and writing of the original draft. **Author Two** was responsible for data curation, investigation, and writing, review & editing. **Author Three** contributed to software, supervision, validation, project administration, and funding acquisition.

REFERENCES

Abreu-Mendoza, R. A., Coulanges, L., Ali, K., Powell, A. B., & Rosenberg-Lee, M. (2021). From non-symbolic to symbolic proportions and back: A cuisenaire rod proportional reasoning intervention enhances continuous proportional reasoning skills. *Frontiers in Psychology*, *12*, 1. https://doi.org/10.3389/fpsyg.2021.633077.

Andiani, D., Darhim, & Nurlaelah, E. (2025). *Local Instruction Theory Perbandingan Trigonometri dengan Pendekatan Pendidikan Matematika Realistik untuk Mengembangkan Kemampuan Representasi Matematis* [Thesis, Universitas Pendidikan Indonesia]. http://repository.upi.edu/134627/.

Andiani, D., Hajizah, M. N., & Dahlan, J. A. (2020). Analisis Rancangan Assesmen Kompetensi Minimum (Akm) Numerasi Program Merdeka Belajar. *Majamath: Jurnal Matematika Dan Pendidikan Matematika*, *4*(1), 80–90. https://ejurnal.unim.ac.id/index.php/majamath/article/view/1010/544.

Andiani, D., Nurlaelah, E., & Darhim, D. (2024). Development of a Local Instruction Theory for Trigonometric Ratios. *Jurnal Matematika Kreatif-Inovatif*, *15*(2), 481–500. https://doi.org/https://doi.org/10.15294/dk629522.

Baye, M. G., Ayele, M. A., & Wondimuneh, T. E. (2021). Implementing GeoGebra integrated with multi-teaching approaches guided by the APOS theory to enhance students' conceptual understanding of limit in Ethiopian Universities. *Heliyon*, *7*(5), 1–13. https://doi.org/10.1016/j.heliyon.2021.e07012.

Bedada, T. B., & Machaba, M. F. (2022). The Effect of GeoGebra on Students' Abilities to Study Calculus. *Hindawi Education Research International*, *2022*, 1–14. https://doi.org/10.1155/2022/4400024.

Bollen, L., Van Kampen, P., Baily, C., Kelly, M., & De Cock, M. (2017). Student difficulties regarding symbolic and graphical representations of vector fields. *Physical Review Physics Education Research*, *13*(2), 1–7. https://doi.org/10.1103/PhysRevPhysEducRes.13.020109.

Borji, V., Martínez-Planell, R., & Trigueros, M. (2022). Student Understanding of Functions of Two Variables: A Reproducibility Study. *The Journal of Mathematical Behavior*, *66*. https://doi.org/10.1016/j.jmathb.2022.100950.

Borji, V., Martínez-Planell, R., & Trigueros, M. (2024). Students' geometric understanding of partial derivatives and the locally linear approach. *Educational Studies in Mathematics*, *115*(1), 69–91. https://doi.org/10.1007/s10649-023-10242-z.

Cheong, K. H., Chen, J. S., Kang, K., & Yeo, D. J. (2023). Supporting students' visualization of multivariable calculus partial derivatives via virtual reality. *Mathematics*, *11*(4), 1–11. https://doi.org/10.3390/math11040831.

Cheong, K. H., Chu, C. E., Ng, W. K., & Yeo, D. J. (2024). Implementing GeoGebra 3D Calculator With Augmented Reality in Multivariable Calculus Education. *IEEE Access*, *12*, 85455–85464. https://doi.org/10.1109/ACCESS.2024.3394531.

Dubey, A., & Barniol, P. (2023). Enhancing students' understanding of vectors through personalized adaptive learning. *European Journal of Physics*, *44*(5), 1–17. https://doi.org/10.1088/1361-6404/ace826.

Dubinsky, E. D., & Mcdonald, M. A. (2001). APOS: A constructivist theory of learning in undergraduate mathematics education research. In D. , Holton, M. Artigue, U. Kirchgräber, J. , N. M. Hillel, & A. Schoenfeld (Eds.), *The Teaching and Learning of Mathematics at University Level. New ICMI Study Series* (Vol. 7). Springer. https://doi.org/https://doi.org/10.1007/0-306-47231-7_25.

Dubinsky, E., & Mcdonald, M. A. (2001). APOS: A Constructivist Theory of Learning in Undergraduate Mathematics Education Research. In D. Holton, M. Artigue, U. Kirchgräber, J. Hillel, M. Niss, & A. Schoenfeld (Eds.), *The Teaching and Learning of Mathematics at University Level: An ICMI Study* (pp. 275–282). Springer Netherlands. https://doi.org/10.1007/0-306-47231-7_25.

Faber, T. J. E., Dankbaar, M. E. W., van den Broek, W. W., Bruinink, L. J., Hogeveen, M., & van Merriënboer, J. J. G. (2024). Effects of adaptive scaffolding on performance, cognitive load and engagement in game-based learning: a randomized controlled trial. *BMC Medical Education*, *24*(1). https://doi.org/10.1186/s12909-024-05698-3.

Freudenthal, H. (1991). *Revisiting Mathematics Education: China Lectures* (A. J. Bishop, Ed.; Vol. 9). Kluwer Academic Publishers. https://p4mriunismuh.wordpress.com/wp-content/uploads/2010/08/revisiting-mathematics-education.pdf.

Gravemeijer, K., & Cobb, P. (2006). *Design Research from a Learning Design Perspective*. https://www.researchgate.net/publication/46676722.

Gurmu, F., Tuge, C., & Hunde, A. B. (2024). Effects of GeoGebra-assisted instructional methods on students' conceptual understanding of geometry. *Cogent Education*, *11*(1), 1–18. https://doi.org/10.1080/2331186X.2024.2379745.

Hahn, L., & Klein, P. (2025). Clustering eye-movement data uncovers students' strategies for coordinating equations and diagrams of vector fields. *Educational Studies in Mathematics*, *118*(3), 359–385. https://doi.org/10.1007/s10649-023-10243-y.

Herrera, L. M. M., Ordóñez, S. J., & Ruiz-Loza, S. (2024). Enhancing Mathematical Education with Spatial Visualization Tools. *Frontiers in Education*, *9*, 1–13. https://doi.org/10.3389/feduc.2024.1229126.

Karabay, F. H., & Meşe, C. (2025). The effect of mobile scaffolding on academic achievement and cognitive load of third grade students in mathematical problem solving. *Asia Pacific Education Review*, *26*(1), 227–246. https://doi.org/10.1007/s12564-024-09951-8.

Kashefi, H., Yusof, Y. M., & Ismail, Z. (2011). Students' difficulties in multivariable calculus through mathematical thinking approach. *Journal of Edupres*, *1*, 77–86. https://doi.org/https://www.researchgate.net/publication/277758735.

Kelly, A. E., & Lesh, R. A. (2000). *Handbook of Research Design in Mathematics and Science Education* (1st ed.). https://doi.org/https://doi.org/10.4324/9781410602725.

Li, W., Feng, Q., Zhu, X., Yu, Q., & Wang, Q. (2023). Effect of summarizing scaffolding and textual cues on learning performance, mental model, and cognitive load in a virtual reality environment: An experimental study. *Computers & Education*, *200*, 104793. https://doi.org/https://doi.org/10.1016/j.compedu.2023.104793.

Matindike, F. (2025). Deepening learners' conceptualization of functions using autograph technology: An APOS theory perspective. *Asian Journal for Mathematics Education*, 1–25. https://doi.org/10.1177/27527263251375522.

Moreno-Arotzena, O., Pombar-Hospitaler, I., & Barragués, J. I. (2021). University student understanding of the gradient of a function of two variables: an approach from the perspective of the theory of semiotic representation registers. *Educational Studies in Mathematics*, *106*(1), 65–89. https://doi.org/10.1007/s10649-020-09994-9.

Rao, V. N. V., Legacy, C., Zieffler, A., & delMas, R. (2023). Designing a sequence of activities to build reasoning about data and visualization. *Teaching Statistics*, *45*(S1), S80–S92. https://doi.org/10.1111/test.12341.

Rochaminah, S., Wahyono, U., & Badjeber, R. (2025). Conceptual and procedural knowledge of prospective mathematics teacher in solving derivative problems. *Jurnal Pendidikan MIPA*, *26*(1), 105–117. https://doi.org/10.23960/jpmipa/v26i1.pp105-117.

Rodríguez-Nieto, C. A., & Moll, V. F. (2025). Mathematical connections promoted in multivariable calculus' classes and in problems-solving about vectors, partial and directional derivatives, and applications. *Eurasia Journal of Mathematics, Science and Technology Education*, *21*(4), 1–27. https://doi.org/10.29333/ejmste/16187.

Sabah, S. (2023). Science and engineering students' difficulties in understanding vector concepts. *Eurasia Journal of Mathematics, Science and Technology Education*, *19*(8), 1–7. https://doi.org/10.29333/ejmste/13431.

Sari, N. H. M., Ayusman, M. H., Agus, M., & Nahdi, A. L. (2024). Pemanfaatan GeoGebra dalam pembelajaran kalkulus: Systematic literature review. *Prosiding Santika 4: Seminar Nasional Tadris Matematika Uin K.H. Abdurrahman Wahid*, 429–438. https://proceeding.uingusdur.ac.id/index.php/santika/article/view/santika42434/1505.

Schoenherr, J., Strohmaier, A. R., & Schukajlow, S. (2024). Learning with Visualizations Helps: A Meta-Analysis of Visualization Interventions in Mathematics Education. *Educational Research Review*, *45*. https://doi.org/10.1016/j.edurev.2024.100639.

Sfard, A. (1991). On the Dual Nature of Mathematical Conceptions: Reflections on Processes and Objects as Different Sides of the Same Coin. *Educational Studies in Mathematics*, *22*(1), 1–36. https://doi.org/10.1007/BF00302715.

Shvarts, A., Bos, R., Doorman, M., & Drijvers, P. (2024). Reifying actions into artifacts: process–object duality from an embodied perspective on mathematics learning. *Educational Studies in Mathematics*, *117*(2), 193–214. https://doi.org/10.1007/s10649-024-10310-y.

Sung, H., & Nathan, M. J. (2025). Unraveling temporally entangled multimodal interactions: investigating verbal and nonverbal contributions to collaborative construction of embodied math knowledge. *International Journal of Educational Technology in Higher Education*, *22*(8), 1–28. https://doi.org/10.1186/s41239-025-00504-6.

Suparman, Marasabessy, R., & Helsa, Y. (2024). Fostering spatial visualization in GeoGebra-assisted geometry lesson: A systematic review and meta-analysis. *Eurasia Journal of Mathematics, Science and Technology Education*, *20*(9), 1–18. https://doi.org/10.29333/ejmste/15170.

Tall, D. (2002). *Three Worlds of Mathematics*. http://www.davidtall.com/papers/three-worlds-of%20math-taipei.pdf.

Tall, D. (2004). Thinking through three worlds of mathematics. *Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education*, *4*, 281–288.

Tall, D. (2008). The Transition to Formal Thinking in Mathematics. *Mathematics Education Research Journal*, *20*(2), 5–24. https://files.eric.ed.gov/fulltext/EJ820231.pdf.

Trigueros, M., Badillo, E., Sánchez-Matamoros, G., & Hernández-Rebollar, L. A. (2024). Contributions to the characterization of the Schema using APOS theory: Graphing with derivative. *ZDM - Mathematics Education*, *56*(6), 1093–1108. https://doi.org/10.1007/s11858-024-01615-6.

van Nooijen, C. C. A., de Koning, B. B., Bramer, W. M., Isahakyan, A., Asoodar, M., Kok, E., van Merrienboer, J. J. G., & Paas, F. (2024). A cognitive load theory approach to understanding expert scaffolding of visual problem-solving tasks: A scoping review. *Educational Psychology Review*, *36*(12), 1–42.

https://doi.org/10.1007/s10648-024-09848-3.

Villabona, D., Oktaç, A., & Roa-Fuentes, S. (2024). Acting on totalities of infinite processes: constructing facets of an object conception. *ZDM - Mathematics Education*, *56*(7), 1375–1387. https://doi.org/10.1007/s11858-024-01631-6.

Vosniadou, S., Vamvakoussi, X., & Skopeliti, I. (2008). *The framework theory approach to the problem of conceptual change*. Rotledge, Taylor and Francis Group. https://www.researchgate.net/publication/284495667.

Yeh, C., Reinholz, D. L., Lee, H. H., & Moschetti, M. (2025). Beyond verbal: A methodological approach to highlighting students' embodied participation in mathematics classroom. *Educational Researcher*, *54*(2), 103–110. https://doi.org/10.3102/0013189X241310169.