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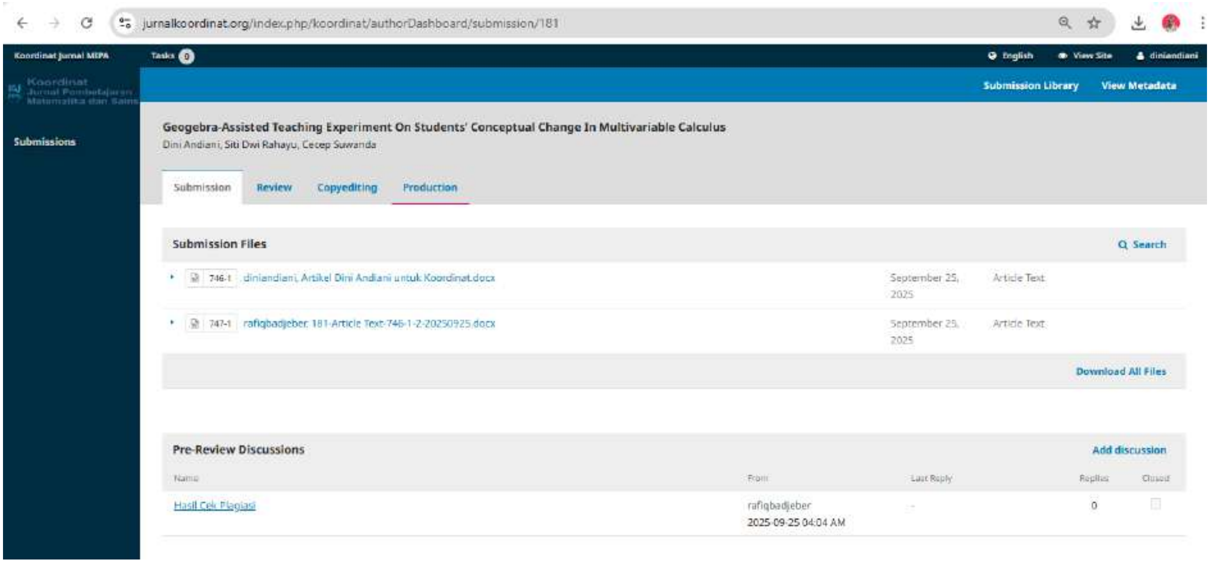

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EXPLORING STUDENTS' CONCEPTUAL DIFFICULTIES AND CONCEPTUAL CHANGE IN MULTIVARIABLE CALCULUS: A TEACHING-EXPERIMENT AND RETROSPECTIVE ANALYSIS

Dini Andiani[1*], Siti Dwi Rahayu [2], Cecep Suwanda [3]

[1, 2, 3] Department of Mathematics, Faculty of Mathematics and Natural Sciences, Universitas Bale Bandung, West Java, Indonesia

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ABSTRACT

This study investigates the effectiveness of GeoGebra-assisted learning in enhancing students' conceptual understanding of multivariable calculus through a teaching experiment design. The research involved undergraduate students enrolled in a multivariable calculus course and was carried out in two iterative stages. Data were collected using diagnostic tests, observation sheets, and reflective notes, while analysis included coding of students' responses, identification of misconceptions, and triangulation of test results, classroom transcripts, and task products.
The findings revealed that, at the outset, students could perform symbolic computations of partial derivatives but often stopped at numerical results without interpreting them as components of a gradient vector. They also experienced difficulties visualizing the geometric meaning of the gradient and its relation to the tangent plane. After introducting of GeoGebra-based visualizations and guided scaffolding, students began to describe gradient components as rates of change and connected them to the direction of steepest ascent. Observations showed that students engaged more and participated actively in collaborative discussions using dynamic visualization tools.
The study demonstrates that GeoGebra-assisted learning, implemented within a teaching experiment framework, can effectively bridge the gap between procedural fluency and conceptual understanding in multivariable calculus. The study contributes to mathematics education by highlighting the role of technological integration and reflective instructional design in addressing students' persistent misconceptions about gradients and tangent planes.

Corresponding Author:
Dini Andiani, Department of Mathematics, Faculty of Mathematics and Natural Sciences, Universitas Bale Bandung, West Java, Indonesia
Email: diniandiani@unibba.ac.id

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INTRODUCTION

Multivariable calculus is a core subject in undergraduate mathematics and science programs, playing a crucial role in developing students' higher-order thinking, spatial reasoning, and modeling skills. Despite its importance, many students face persistent difficulties when transitioning from single-variable to multivariable contexts, particularly in understanding abstract concepts such as partial derivatives, gradients, and tangent planes (Borji et al., 2024; Hahn & Klein, 2025; Kashefi et al., 2011; Rodríguez-Nieto & Moll, 2025). These difficulties often stem from an overreliance on procedural computation, with limited ability to integrate algebraic procedures with geometric interpretation (Borji et al., 2024; Cheong et al., 2023; Hahn & Klein, 2025). As a result, students may compute derivatives correctly but struggle to articulate the conceptual significance of gradients or interpret the geometric meaning of tangent planes.

Mathematics education researchers have highlighted the importance of instructional strategies that foster conceptual understanding alongside procedural fluency. Conceptual growth involves transforming operational procedures into structural objects, enabling students to reason about mathematical entities more flexibly (Andiani et al., 2024, 2025; E. D. Dubinsky & Mcdonald, 2001; E. Dubinsky & Mcdonald, 2001; Matindike, 2025; Shvarts et al., 2024; Trigueros et al., 2024; Villabona et al., 2024). In this regard, technology enhanced learning environments, particularly dynamic visualization tools such as GeoGebra, have been shown to support students' conceptual development by linking symbolic computations with graphical and interactive representations (Baye et al., 2021; Gurmu et al., 2024; Suparman et al., 2024). Such tools serve as mediating devices that can bridge students' fragmented understanding of multivariable functions, gradients, and tangent planes (Borji et al., 2022; Cheong et al., 2024; Herrera et al., 2024; Schoenherr et al., 2024). In particular, the doctoral thesis *Local Instruction Theory Perbandingan Trigonometri dengan Pendekatan Matematika Realistik untuk Mengembangkan Kemampuan Representasi Matematis* (Andiani et al., 2025) demonstrates that an LIT-based RME approach with realistic contexts (*bayangan*, *kemiringan tangga*, *posisi mata terhadap objek*) significantly enhances students' mathematical representation abilities in the trigonometric ratios compared to purely procedural learning.

Several studies have demonstrated the benefits of technology in calculus instruction. For example, Schoenherr found that interactive visualizations improved students' reasoning about multivariable functions (Cheong et al., 2023; Herrera et al., 2024; Rao et al., 2023; Schoenherr et al., 2024), while Karabay (2025) highlighted the role of scaffolding in managing cognitive load during problem solving (Faber et al., 2024; Karabay & Meşe, 2025; Li et al., 2023; van Nooijen et al., 2024). However, much of the existing research emphasizes quantitative outcomes, such as test performance, with limited attention to qualitative aspects of conceptual change or the dynamics of classroom interactions. Additionally, few studies have examined how iterative teaching experiment designs can trace students' conceptual difficulties and the pathways of conceptual change over time.

The novelty of this study lies in combining a teaching experiment approach with retrospective analysis to explore students' conceptual difficulties and subsequent changes in understanding gradients and tangent planes through GeoGebra-assisted learning. Teaching experiments enable iterative refinement of instructional interventions while closely observing students' reasoning (Kelly & Lesh, 2000). Retrospective analysis allows systematic identification of shifts in conceptual understanding across multiple data sources.

Previous research by Andiani et al.,(2020) demonstrated that students face challenges in solving geometric problems on competency-based assessments, highlighting difficulties in understanding and reasoning mathematically (Andiani et al., 2020). Furthermore, Andiani et al., (2025) found that realistic mathematics education approaches can improve students' mathematical representation skills, essential in multivariable calculus contexts (Andiani et al., 2025). These findings suggest that explicit scaffolding and visualization tools are crucial for fostering conceptual understanding.

Therefore, this research focuses on exploring students' conceptual difficulties and conceptual change in multivariable calculus through a qualitative teaching experiment approach supported by GeoGebra. The main aim is to describe how students develop an understanding of gradients and tangent planes when exposed to visualization-based instruction and to analyze how this process bridges the gap between procedural fluency and conceptual comprehension. Recent studies have confirmed that GeoGebra-assisted visualizations can significantly enhance students' conceptual understanding and engagement in multivariable calculus and related mathematical topics (Abdurrahman Wahid Pekalongan et al., n.d.; Bedada & Machaba, 2022; Cheong et al., 2024).

METHOD

This study employed a qualitative descriptive design through a teaching experiment and retrospective analysis to investigate students' conceptual difficulties and conceptual change in multivariable calculus. The researchers chose this design because it allowed them to explore students' thinking processes and capture how their understanding evolved during the learning activities.

The participants were eight undergraduate students enrolled in a multivariable calculus course at a mathematics education program in West Java, Indonesia. Participants were selected purposively based on their prior exposure to basic calculus concepts and their availability to participate in the entire teaching experiment cycle. The small group size was intended to enable intensive observation of students' learning processes.

The instruments used in this study included learning tasks in the form of problem sets related to the concept of limits in two variables and GeoGebra-based activities designed to promote visualization and conceptual exploration. The researchers employed observation sheets to document classroom interactions and student strategies, while semi-structured interview guidelines were used to explore students' reasoning, misconceptions, and conceptual changes in greater depth.

The procedures consisted of two stages of teaching experiments followed by retrospective analysis. Each teaching experiment involved introducing learning tasks, classroom interaction supported by GeoGebra visualization, and reflection activities. The researchers collected data through diagnostic tests, classroom transcripts, students' written work, and reflective notes.

Data analysis was conducted through coding students' responses, identifying recurring misconceptions, and comparing test results, classroom observations, and students' task products. Triangulation between these data sources was applied to ensure the credibility and validity of the findings. The researchers conducted the analysis iteratively, allowing them to refine their interpretations and connect the results with the theoretical framework on conceptual understanding in multivariable calculus.

RESULT AND DISCUSSION

Students' Initial Conceptual Difficulties

At the beginning of the task, most students successfully computed the partial derivatives of the potential function $V(x, y) = x^2 + 2y^2$, yielding the gradient $\nabla V(x, y) = (2x, 4y)$. When evaluated at the point $(1,2)$, this became $\nabla V(1,2) = (2,8)$. However, their written responses tended to stop at the numerical level, showing little evidence of conceptual interpretation. This finding resonates with Borji (2022), who noted that many students demonstrate strong procedural skills while lacking structural understanding in multivariable calculus (Borji et al., 2022).

A recurring pattern emerged where students simply reported, *"gradient* = (2,8), *finished."* Such utterances illustrate what Sfard (2008) describes as *operational thinking* treating mathematical objects as results of procedures rather than entities with independent meaning (Sfard, 1991). Students in this stage failed to recognize that the gradient is more than just a pair of numbers; it is a vector with both magnitude and direction.

When pressed for further interpretation, some students attempted to describe the result in vague terms, such as "the slope in x is two and in y is 8," but without elaborating on its vectorial nature. This suggests that symbolic results were fragmented rather than integrated into a coherent conceptual framework. Cheong et al. (2023) also reported similar tendencies in their study, where students often compartmentalized partial derivatives without connecting them to vectorial interpretation (Cheong et al., 2023).

This difficulty is particularly important in vector calculus, where gradients play a central role in understanding directional change, optimization, and physical phenomena. Without conceptual grounding, students cannot connect gradients to applied contexts, such as interpreting potential fields in physics or optimization problems in engineering. Medina Herrera et al. (2021) highlight that such gaps hinder the transition from abstract calculation to practical application (Herrera et al., 2024).

A closer analysis of written work confirmed this gap. While algebraic manipulations were accurate, annotations explaining the meaning of the results were rare. Students did not spontaneously refer to the gradient as the direction of steepest ascent or discuss its physical implications in the context of a potential function. Students did not spontaneously refer to the gradient as the direction of steepest ascent or discuss its physical implications in the context of a potential function, as confirmed by Abreu et al. (2021), who emphasize that symbolic fluency alone is insufficient without scaffolding that links symbols to concepts (Abreu-Mendoza et al., 2021).

In retrospective discussions, some students admitted being unsure how to proceed after obtaining the ordered pair. They expressed uncertainty about substituting values for the original function, drawing a graph, or leaving the result. Such hesitation illustrates the lack of an internalized schema for interpreting gradient vectors.

These findings underscore the necessity of carefully designed instructional interventions. As Tall (2008) argues, advanced mathematical thinking requires students to move beyond computational comfort zones and integrate multiple representations (Tall, 2008). Here, the initial failure to articulate conceptual meaning highlights the importance of explicitly embedding interpretation tasks in calculus instruction.

Table 1 summarizes students' initial approaches to illustrate the distribution of responses. Categories include purely procedural responses (e.g., numerical computation only), partial interpretations (mentioning "slope" without directionality), and meaningful interpretations (rare at this stage).

Table 1. Categories of student responses to the gradient task at (1,2)

Response Type	Example Student Statement	Frequency (N=20)
Procedural only	"Gradient = (2,8), finished"	3
Partial interpretation	"Slope in x is 2, in y is 8"	0
Full interpretation	"The vector (2,8) shows the direction of steepest increase at (1,2); its rate of increase per unit length is $2\sqrt{17}$."	5

The predominance of procedural responses highlights the extent to which conceptual understanding was initially lacking. Such a distribution sets the stage for examining how scaffolding and visualization can support a deeper grasp of gradient meaning.

Overall, this first phase demonstrates that students could compute but not interpret. Without explicit prompts or visual tools, they remained anchored in procedural reasoning, which validates earlier findings by Borji (2022) and Cheong et al. (2023) that procedural fluency does not automatically translate into conceptual mastery (Borji et al., 2022; Cheong et al., 2023).

Transition from Symbolic Computation to Geometric Meaning

After the researchers introduced scaffolding strategies, students moved beyond numerical computation toward geometric meaning. The teacher encouraged them to see the gradient as a vector pointing toward the steepest ascent. This process of guided reinvention mirrors Sfard's (2008) description of *reification*, where operational processes gradually become conceived as structural objects (Sfard, 1991).

For example, after further questioning, a student remarked, *"So, the arrow shows the fastest way up."* What the student said illustrates a pivotal shift from perceiving the gradient as an inert ordered pair to recognizing it as a directional indicator. Rochaminah et al. (2025) emphasize that such discursive transitions are critical for advancing from procedural competence to conceptual understanding (Rochaminah et al., 2025).

The case of (2,8) at (1,2) was especially instructive. The researchers prompted students to interpret not only the numerical values but also the relative weights of each component. They concluded that the vector points northeast, with a stronger bias toward the y-axis due to the larger coefficient. This interpretation aligns with Cheong et al. (2023), who found that relative component magnitudes often served as an entry point for students in grasping geometric meaning (Cheong et al., 2023).

Discussions also emphasized physical interpretations. The researchers asked students to imagine the potential function representing an electric field. In this analogy, the gradient indicated the direction in which a test charge would experience the greatest force. This contextualization supported Medina Herrera et al. (2021), who argue that situating symbolic results in applied domains enhances engagement and meaning making (Herrera et al., 2024).

At this stage, evidence of conceptual change was visible in classroom transcripts. One student said, *"The 2 and 8 mean the function increases faster in y than in x."* Such statements go beyond restating numerical results and reveal an emerging awareness of the gradient's functional role.

This development illustrates Tall's (2008) assertion that students benefit from explicit opportunities to link symbolic expressions with graphical and contextual interpretations (Tall, 2008). The researchers facilitated the connection through structured questioning and prompts that required students to articulate their thinking.

GeoGebra was later employed to reinforce this connection. Visualizing the vector (2,8) at (1,2) provided immediate geometric confirmation of analytic calculations. Figure 1 shows a screenshot of the vector superimposed on the graph of the potential function. This use of technology resonates with Medina Herrera et al. (2021), who emphasize the value of dynamic software in bridging representational gaps (Herrera et al., 2024).

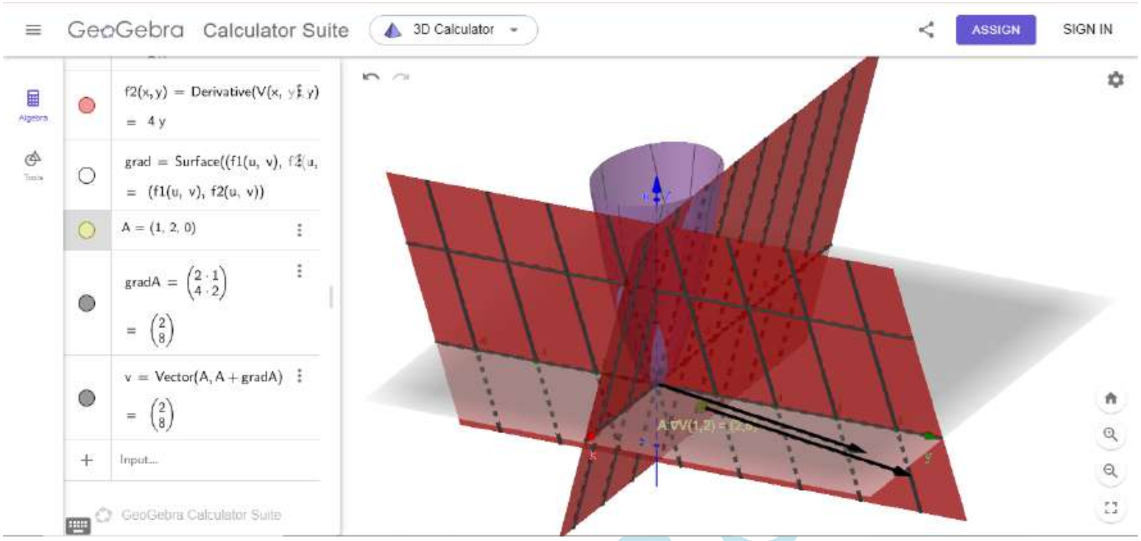

Figure 1. GeoGebra visualization of the gradient vector (2,8) at point (1,2)

Figure 2. Students exploring mathematics concepts with GeoGebra on mobile phones

Such visual aids confirmed symbolic computations and expanded students' reasoning. The visual of a steep arrow pointing northeast made the abstract idea of “steepest ascent” concrete. Sung et al. (2025) highlight that such multimodal engagements promote deeper conceptualization (Sung & Nathan, 2025; Yeh et al., 2025).

By the end of this phase, students were increasingly able to articulate the meaning of gradients in symbolic and geometric terms. Increased students demonstrate the potential of scaffolding and visualization in fostering representational integration.

The transition from symbolic to geometric reasoning marked a critical conceptual breakthrough. Students no longer viewed the gradient as a final numerical product but as an active object that connects algebraic, graphical, and physical dimensions.

Difficulties in Visualizing Gradient Vectors

Despite notable progress, visualization remained a challenge for many students. Several could describe the meaning of (2,8) but hesitated to represent it graphically. Some drew arrows incorrectly, while others avoided sketching altogether. This difficulty echoes Moreno (2021), who reported persistent struggles with translating symbolic gradients into graphical form (Moreno-Arotzena et al., 2021).

One common error involved placing the vector at the origin instead of at the point (1,2). The error of the activity suggests a limited understanding of gradient vectors as *anchored* at specific points on the surface. Similar misconceptions were identified by Cheong et al. (2023), who observed that students often treated gradient vectors as free-floating arrows (Cheong et al., 2023).

Another difficulty was scaling. Students who attempted sketches sometimes drew the components with incorrect proportions, e.g., equal lengths for 2 and 8. This indicates partial recognition of directionality but incomplete attention to magnitude. Bollen et al. (2017) caution that without attention to scale, students risk losing the quantitative meaning of vectors (Bollen et al., 2017; Dubey & Barniol, 2023; Sabah, 2023).

To address these issues, the class used GeoGebra's vector tools to plot (2,8) accurately at (1,2). Figure 3 illustrates how this technology provided precise positioning and scaling. Such scaffolding resonates with Medina Herrera et al. (2021), who show that dynamic software reduces cognitive load and reinforces representational accuracy (Herrera et al., 2024).

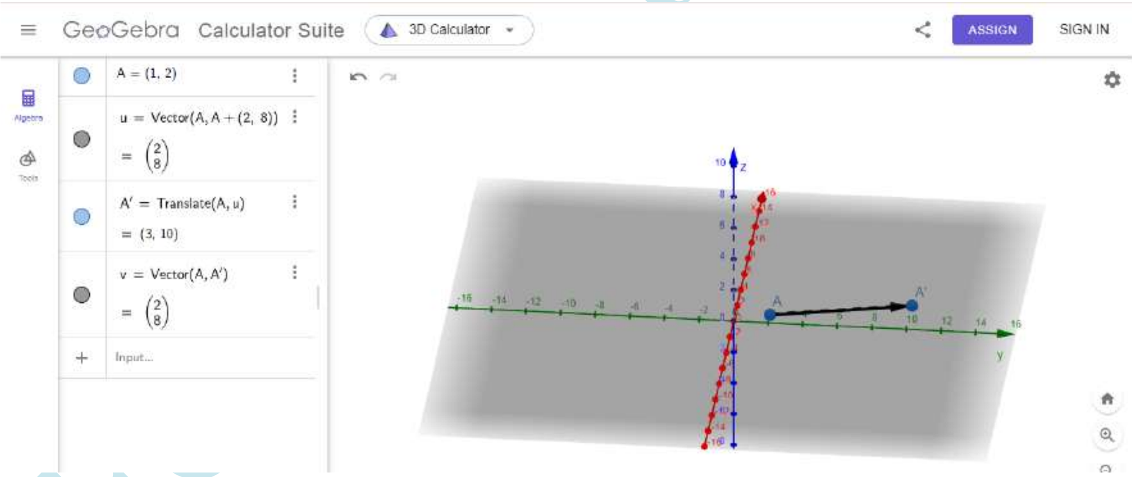

Figure 3. GeoGebra plot of gradient vector (2,8) anchored at (1,2)

Transcript evidence also showed that visualization helped students reconcile earlier errors. After seeing the GeoGebra output, one student said, *"Oh, the arrow is not from zero, but from (1,2)."* The student said that visualization supported correcting misconceptions and facilitated representational alignment. Nevertheless, reliance on technological scaffolding revealed another challenge: students struggled to visualize vectors without software support. Students' struggles to visualize vectors without software support suggest that internalization of geometric reasoning requires repeated practice, beyond one-time demonstrations. Sfard (1991) warns that without reification, students may remain dependent on external representations (Sfard, 1991).

Class discussions further emphasized the need to interpret vector length. The researchers guided students to compute the magnitude $\sqrt{2^2+8^2}=\sqrt{68}$, reinforcing that the gradient encodes direction and rate of change. This mistakes addressed earlier gaps where students only equated the gradient with slope, without recognizing its multidimensional meaning.

By integrating magnitude into interpretation, students began to view the gradient as a holistic object with geometric and physical significance. Tall (2002) notes that such integration is essential for moving from fragmented knowledge to connected understanding (Tall, 2002, 2004). Ultimately, while difficulties in visualization persisted, the combination of symbolic computation, guided scaffolding, and dynamic software fostered meaningful progress. The retrospective analysis suggests that systematic support can gradually shift students from procedural only reasoning to rich conceptualization of gradients as vectors.

CONCLUSION

This study investigated students' understanding of gradient vectors in the context of the potential function $V(x,y) = x^2 + 2y^2$. The results indicate that, although most students could correctly compute partial derivatives and obtain $\nabla V(x,y) = (2x, 4y)$, many initially stopped at procedural manipulation without conceptual articulation. For instance, when students responded with 'gradient = (4,13), done,' it revealed that they had not yet interpreted the numerical result as a directional vector with meaning in the context of surface growth. Through guided scaffolding and the use of GeoGebra visualizations, students gradually transitioned from symbolic computation to a structural understanding of the gradient vector as representing the direction of steepest ascent and its magnitude as the rate of change per unit distance (Sfard, 1991; Tall, 2002, 2004).

The implications of this study suggest that integrating dynamic visualization tools such as GeoGebra can play a crucial role in bridging the gap between procedural fluency and conceptual understanding in multivariable calculus. By anchoring gradient vectors at specific points, students observed the direction and connected the symbolic form $\nabla V(1,2) = (2,8)$ with its geometric representation. This approach facilitated the reification process described by Sfard (2008), helping students to perceive the gradient vector as both a computational output and a conceptual object (Sfard, 1991). The findings are consistent with previous studies highlighting the need for explicit tasks that link algebraic representations with visual and contextual meaning (Borji et al., 2022; Cheong et al., 2023).

Nevertheless, the study has some limitations. The number of participants was relatively small, and the analysis focused primarily on a single problem involving gradient interpretation. Broader exploration across different surfaces and longitudinal data, would strengthen the conclusions and provide deeper insights into how students develop an integrated understanding of gradients in multivariable contexts. Future research could expand on these findings by designing tasks that explicitly connect gradient concepts with applications in physics and engineering, thereby enhancing the relevance and transferability of students' learning (Herrera et al., 2024).

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AUTHOR CONTRIBUTIONS

Author One contributed to the conceptualization, methodology, formal analysis, and writing of the original draft. **Author Two** was responsible for data curation, investigation, and writing, review & editing. **Author Three** contributed to software, supervision, validation, project administration, and funding acquisition.

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2. Bukti Konfirmasi Review dan Hasil Review Pertama (6 Oktober 2025)

[Koordinat] Editor Decision

2025-10-06 10:12 PM

Dini Andiani, Siti Dwi Rahayu, Cecep Suwanda:

We have reached a decision regarding your submission to Koordinat Jurnal MIPA, "EXPLORING STUDENTS' CONCEPTUAL DIFFICULTIES AND CONCEPTUAL CHANGE IN MULTIVARIABLE CALCULUS: A TEACHING-EXPERIMENT AND RETROSPECTIVE ANALYSIS ".

Our decision is: **Revisions Required**

Rafiq Badjeber
State Islamic University Datokarama Palu
rafiqbadjeber@uindatokarama.ac.id

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EXPLORING STUDENTS' CONCEPTUAL DIFFICULTIES AND CONCEPTUAL CHANGE IN MULTIVARIABLE CALCULUS: A TEACHING-EXPERIMENT AND RETROSPECTIVE ANALYSIS

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ABSTRACT

This study investigates the effectiveness of GeoGebra-assisted learning in enhancing students' conceptual understanding of multivariable calculus through a teaching experiment design. The research involved undergraduate students enrolled in a multivariable calculus course and was carried out in two iterative stages. Data were collected using diagnostic tests, observation sheets, and reflective notes, while analysis included coding of students' responses, identification of misconceptions, and triangulation of test results, classroom transcripts, and task products.
The findings revealed that, at the outset, students could perform symbolic computations of partial derivatives but often stopped at numerical results without interpreting them as components of a gradient vector. They also experienced difficulties visualizing the geometric meaning of the gradient and its relation to the tangent plane. After introducting of GeoGebra-based visualizations and guided scaffolding, students began to describe gradient components as rates of change and connected them to the direction of steepest ascent. Observations showed that students engaged more and participated actively in collaborative discussions using dynamic visualization tools.
The study demonstrates that GeoGebra-assisted learning, implemented within a teaching experiment framework, can effectively bridge the gap between procedural fluency and conceptual understanding in multivariable calculus. The study contributes to mathematics education by highlighting the role of technological integration and reflective instructional design in addressing students' persistent misconceptions about gradients and tangent planes.

Corresponding Author:

How to cite:

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Option 2. Revise the abstract to match the title, for example, "State the theory of conceptual change; explain how GeoGebra not only improves understanding but also facilitates conceptual change from misconceptions to scientific concepts; emphasize that this study is not simply a test of GeoGebra's effectiveness, but an analysis of the process of change in students' understanding."

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INTRODUCTION

Multivariable calculus is a core subject in undergraduate mathematics and science programs, playing a crucial role in developing students' higher-order thinking, spatial reasoning, and modeling skills. Despite its importance, many students face persistent difficulties when transitioning from single-variable to multivariable contexts, particularly in understanding abstract concepts such as partial derivatives, gradients, and tangent planes (Borji et al., 2024; Hahn & Klein, 2025; Kashefi et al., 2011; Rodríguez-Nieto & Moll, 2025). These difficulties often stem from an overreliance on procedural computation, with limited ability to integrate algebraic procedures with geometric interpretation (Borji et al., 2024; Cheong et al., 2023; Hahn & Klein, 2025). As a result, students may compute derivatives correctly but struggle to articulate the conceptual significance of gradients or interpret the geometric meaning of tangent planes.

Mathematics education researchers have highlighted the importance of instructional strategies that foster conceptual understanding alongside procedural fluency. Conceptual growth involves transforming operational procedures into structural objects, enabling students to reason about mathematical entities more flexibly (Andiani et al., 2024, 2025; E. D. Dubinsky & Mcdonald, 2001; E. Dubinsky & Mcdonald, 2001; Matindike, 2025; Shvarts et al., 2024; Trigueros et al., 2024; Villabona et al., 2024). In this regard, technology enhanced learning environments, particularly dynamic visualization tools such as GeoGebra, have been shown to support students' conceptual development by linking symbolic computations with graphical and interactive representations (Baye et al., 2021; Gurmu et al., 2024; Suparman et al., 2024). Such tools serve as mediating devices that can bridge students' fragmented understanding of multivariable functions, gradients, and tangent planes (Borji et al., 2022; Cheong et al., 2024; Herrera et al., 2024; Schoenherr et al., 2024). In particular, the doctoral thesis *Local Instruction Theory Perbandingan Trigonometri dengan Pendekatan Matematika Realistik untuk Mengembangkan Kemampuan Representasi Matematis* (Andiani et al., 2025) demonstrates that an LIT-based RME approach with realistic contexts (*bayangan*, *kemiringan tangga*, *posisi mata terhadap objek*) significantly enhances students' mathematical representation abilities in the trigonometric ratios compared to purely procedural learning.

Several studies have demonstrated the benefits of technology in calculus instruction. For example, Schoenherr found that interactive visualizations improved students' reasoning about multivariable functions (Cheong et al., 2023; Herrera et al., 2024; Rao et al., 2023; Schoenherr et al., 2024), while Karabay (2025) highlighted the role of scaffolding in managing cognitive load during problem solving (Faber et al., 2024; Karabay & Meşe, 2025; Li et al., 2023; van Nooijen et al., 2024). However, much of the existing research emphasizes quantitative outcomes, such as test performance, with limited attention to qualitative aspects of conceptual change or the dynamics of classroom interactions. Additionally, few studies have examined how iterative teaching experiment designs can trace students' conceptual difficulties and the pathways of conceptual change over time.

The novelty of this study lies in combining a teaching experiment approach with retrospective analysis to explore students' conceptual difficulties and subsequent changes in understanding gradients and tangent planes through GeoGebra-assisted learning. Teaching experiments enable iterative refinement of instructional interventions while closely observing students' reasoning (Kelly & Lesh, 2000). Retrospective analysis allows systematic identification of shifts in conceptual understanding across multiple data sources.

Previous research by Andiani et al.,(2020) demonstrated that students face challenges in solving geometric problems on competency-based assessments, highlighting difficulties in understanding and reasoning mathematically (Andiani et

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al., 2020). Furthermore, Andiani et al., (2025) found that realistic mathematics education approaches can improve students' mathematical representation skills, essential in multivariable calculus contexts (Andiani et al., 2025). These findings suggest that explicit scaffolding and visualization tools are crucial for fostering conceptual understanding.

Therefore, this research focuses on exploring students' conceptual difficulties and conceptual change in multivariable calculus through a qualitative teaching experiment approach supported by GeoGebra. The main aim is to describe how students develop an understanding of gradients and tangent planes when exposed to visualization-based instruction and to analyze how this process bridges the gap between procedural fluency and conceptual comprehension. Recent studies have confirmed that GeoGebra-assisted visualizations can significantly enhance students' conceptual understanding and engagement in multivariable calculus and related mathematical topics (Abdurrahman Wahid Pekalongan et al., n.d.; Bedada & Machaba, 2022; Cheong et al., 2024).

METHOD

This study employed a qualitative descriptive design through a teaching experiment and retrospective analysis to investigate students' conceptual difficulties and conceptual change in multivariable calculus. The researchers chose this design because it allowed them to explore students' thinking processes and capture how their understanding evolved during the learning activities.

The participants were eight undergraduate students enrolled in a multivariable calculus course at a mathematics education program in West Java, Indonesia. Participants were selected purposively based on their prior exposure to basic calculus concepts and their availability to participate in the entire teaching experiment cycle. The small group size was intended to enable intensive observation of students' learning processes.

The instruments used in this study included learning tasks in the form of problem sets related to the concept of limits in two variables and GeoGebra-based activities designed to promote visualization and conceptual exploration. The researchers employed observation sheets to document classroom interactions and student strategies, while semi-structured interview guidelines were used to explore students' reasoning, misconceptions, and conceptual changes in greater depth.

The procedures consisted of two stages of teaching experiments followed by retrospective analysis. Each teaching experiment involved introducing learning tasks, classroom interaction supported by GeoGebra visualization, and reflection activities. The researchers collected data through diagnostic tests, classroom transcripts, students' written work, and reflective notes.

Data analysis was conducted through coding students' responses, identifying recurring misconceptions, and comparing test results, classroom observations, and students' task products. Triangulation between these data sources was applied to ensure the credibility and validity of the findings. The researchers conducted the analysis iteratively, allowing them to refine their interpretations and connect the results with the theoretical framework on conceptual understanding in multivariable calculus.

RESULT AND DISCUSSION

Students' Initial Conceptual Difficulties

At the beginning of the task, most students successfully computed the partial derivatives of the potential function $V(x,y) = x^2 + 2y^2$, yielding the gradient $\nabla V(x,y) = (2x, 4y)$. When evaluated at the point (1,2), this became $\nabla V(1,2) = (2,8)$. However, their written responses tended to stop at the numerical level, showing little evidence of conceptual interpretation. This finding resonates with Borji (2022), who noted that many students demonstrate strong procedural skills while lacking structural understanding in multivariable calculus (Borji et al., 2022).

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A recurring pattern emerged where students simply reported, *"gradient* = (2,8)*, finished."* Such utterances illustrate what Sfard (2008) describes as *operational thinking* treating mathematical objects as results of procedures rather than entities with independent meaning (Sfard, 1991). Students in this stage failed to recognize that the gradient is more than just a pair of numbers; it is a vector with both magnitude and direction.

When pressed for further interpretation, some students attempted to describe the result in vague terms, such as "the slope in x is two and in y is 8," but without elaborating on its vectorial nature. This suggests that symbolic results were fragmented rather than integrated into a coherent conceptual framework. Cheong et al. (2023) also reported similar tendencies in their study, where students often compartmentalized partial derivatives without connecting them to vectorial interpretation (Cheong et al., 2023).

This difficulty is particularly important in vector calculus, where gradients play a central role in understanding directional change, optimization, and physical phenomena. Without conceptual grounding, students cannot connect gradients to applied contexts, such as interpreting potential fields in physics or optimization problems in engineering. Medina Herrera et al. (2021) highlight that such gaps hinder the transition from abstract calculation to practical application (Herrera et al., 2024).

A closer analysis of written work confirmed this gap. While algebraic manipulations were accurate, annotations explaining the meaning of the results were rare. Students did not spontaneously refer to the gradient as the direction of steepest ascent or discuss its physical implications in the context of a potential function. Students did not spontaneously refer to the gradient as the direction of steepest ascent or discuss its physical implications in the context of a potential function, as confirmed by Abreu et al. (2021), who emphasize that symbolic fluency alone is insufficient without scaffolding that links symbols to concepts (Abreu-Mendoza et al., 2021).

In retrospective discussions, some students admitted being unsure how to proceed after obtaining the ordered pair. They expressed uncertainty about substituting values for the original function, drawing a graph, or leaving the result. Such hesitation illustrates the lack of an internalized schema for interpreting gradient vectors.

These findings underscore the necessity of carefully designed instructional interventions. As Tall (2008) argues, advanced mathematical thinking requires students to move beyond computational comfort zones and integrate multiple representations (Tall, 2008). Here, the initial failure to articulate conceptual meaning highlights the importance of explicitly embedding interpretation tasks in calculus instruction.

Table 1 summarizes students' initial approaches to illustrate the distribution of responses. Categories include purely procedural responses (e.g., numerical computation only), partial interpretations (mentioning "slope" without directionality), and meaningful interpretations (rare at this stage).

Table 1. Categories of student responses to the gradient task at (1,2)

Response Type	**Example Student Statement**	**Frequency (N=20)**
Procedural only	"Gradient = (2,8), finished"	3
Partial interpretation	"Slope in x is 2, in y is 8"	0

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Response Type	Example Student Statement	Frequency (N=20)
Full interpretation	"The vector (2,8) shows the direction of steepest increase at (1,2); its rate of increase per unit length is $2\sqrt{17}$."	5

The predominance of procedural responses highlights the extent to which conceptual understanding was initially lacking. Such a distribution sets the stage for examining how scaffolding and visualization can support a deeper grasp of gradient meaning.

Overall, this first phase demonstrates that students could compute but not interpret. Without explicit prompts or visual tools, they remained anchored in procedural reasoning, which validates earlier findings by Borji (2022) and Cheong et al. (2023) that procedural fluency does not automatically translate into conceptual mastery (Borji et al., 2022; Cheong et al., 2023).

Transition from Symbolic Computation to Geometric Meaning

After the researchers introduced scaffolding strategies, students moved beyond numerical computation toward geometric meaning. The teacher encouraged them to see the gradient as a vector pointing toward the steepest ascent. This process of guided reinvention mirrors Sfard's (2008) description of *reification*, where operational processes gradually become conceived as structural objects (Sfard, 1991).

For example, after further questioning, a student remarked, *"So, the arrow shows the fastest way up."* What the student said illustrates a pivotal shift from perceiving the gradient as an inert ordered pair to recognizing it as a directional indicator. Rochaminah et al. (2025) emphasize that such discursive transitions are critical for advancing from procedural competence to conceptual understanding (Rochaminah et al., 2025).

The case of (2,8) at (1,2) was especially instructive. The researchers prompted students to interpret not only the numerical values but also the relative weights of each component. They concluded that the vector points northeast, with a stronger bias toward the y-axis due to the larger coefficient. This interpretation aligns with Cheong et al. (2023), who found that relative component magnitudes often served as an entry point for students in grasping geometric meaning (Cheong et al., 2023).

Discussions also emphasized physical interpretations. The researchers asked students to imagine the potential function representing an electric field. In this analogy, the gradient indicated the direction in which a test charge would experience the greatest force. This contextualization supported Medina Herrera et al. (2021), who argue that situating symbolic results in applied domains enhances engagement and meaning making (Herrera et al., 2024).

At this stage, evidence of conceptual change was visible in classroom transcripts. One student said, *"The 2 and 8 mean the function increases faster in y than in x."* Such statements go beyond restating numerical results and reveal an emerging awareness of the gradient's functional role.

This development illustrates Tall's (2008) assertion that students benefit from explicit opportunities to link symbolic expressions with graphical and contextual interpretations (Tall, 2008). The researchers facilitated the connection through structured questioning and prompts that required students to articulate their thinking.

GeoGebra was later employed to reinforce this connection. Visualizing the vector (2,8) at (1,2) provided immediate geometric confirmation of analytic calculations. Figure 1 shows a screenshot of the vector superimposed on the graph of the potential function. This use of technology resonates with Medina Herrera et al. (2021), who emphasize the value of dynamic software in bridging representational gaps (Herrera et al., 2024).

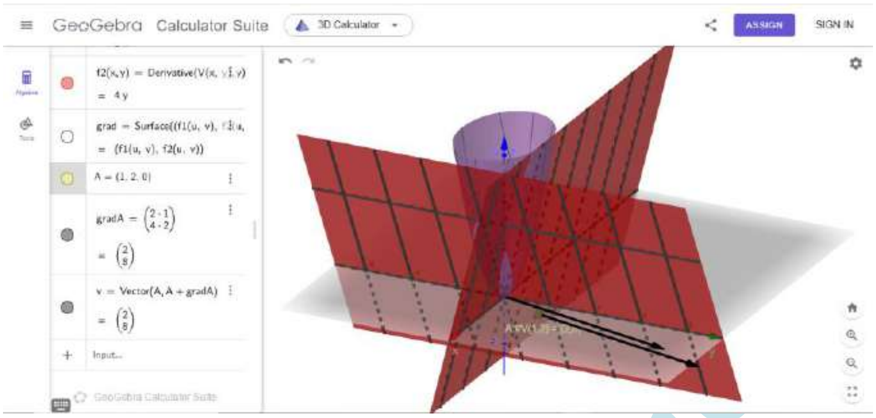

Figure 1. GeoGebra visualization of the gradient vector (2,8) at point (1,2)


Figure 2. Students exploring mathematics concepts with GeoGebra on mobile phones

Such visual aids confirmed symbolic computations and expanded students' reasoning. The visual of a steep arrow pointing northeast made the abstract idea of “steepest ascent” concrete. Sung et al. (2025) highlight that such multimodal engagements promote deeper conceptualization (Sung & Nathan, 2025; Yeh et al., 2025).

By the end of this phase, students were increasingly able to articulate the meaning of gradients in symbolic and geometric terms. Increased students demonstrate the potential of scaffolding and visualization in fostering representational integration.

The transition from symbolic to geometric reasoning marked a critical conceptual breakthrough. Students no longer viewed the gradient as a final numerical product but as an active object that connects algebraic, graphical, and physical dimensions.

Difficulties in Visualizing Gradient Vectors

Despite notable progress, visualization remained a challenge for many students. Several could describe the meaning of (2,8) but hesitated to represent it graphically. Some drew arrows incorrectly, while others avoided sketching altogether. This difficulty echoes Moreno (2021), who reported persistent struggles with translating symbolic gradients into graphical form (Moreno-Arotzena et al., 2021).

One common error involved placing the vector at the origin instead of at the point (1,2). The error of the activity suggests a limited understanding of gradient vectors as *anchored* at specific points on the surface. Similar misconceptions were identified by Cheong et al. (2023), who observed that students often treated gradient vectors as free-floating arrows (Cheong et al., 2023).

Another difficulty was scaling. Students who attempted sketches sometimes drew the components with incorrect proportions, e.g., equal lengths for 2 and 8. This indicates partial recognition of directionality but incomplete attention to magnitude. Bollen et al. (2017) caution that without attention to scale, students risk losing the quantitative meaning of vectors (Bollen et al., 2017; Dubey & Barniol, 2023; Sabah, 2023).

To address these issues, the class used GeoGebra's vector tools to plot (2,8) accurately at (1,2). Figure 3 illustrates how this technology provided precise positioning and scaling. Such scaffolding resonates with Medina Herrera et al. (2021), who show that dynamic software reduces cognitive load and reinforces representational accuracy (Herrera et al., 2024).

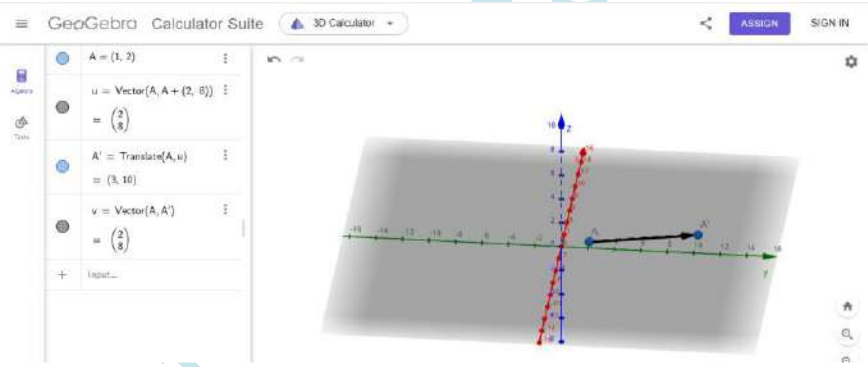

Figure 3. GeoGebra plot of gradient vector (2,8) anchored at (1,2)

Transcript evidence also showed that visualization helped students reconcile earlier errors. After seeing the GeoGebra output, one student said, *"Oh, the arrow is not from zero, but from (1,2)."* The student said that visualization supported correcting misconceptions and facilitated representational alignment. Nevertheless, reliance on technological scaffolding revealed another challenge: students struggled to visualize vectors without software support. Students' struggles to visualize vectors without software support suggest that internalization of geometric reasoning requires repeated practice, beyond one-time demonstrations. Sfard (1991) warns that without reification, students may remain dependent on external representations (Sfard, 1991).

Class discussions further emphasized the need to interpret vector length. The researchers guided students to compute the magnitude $\sqrt{2^2 + 8^2} = \sqrt{68}$, reinforcing that the gradient encodes direction and rate of change. This mistakes addressed earlier gaps where students only equated the gradient with slope, without recognizing its multidimensional meaning.

By integrating magnitude into interpretation, students began to view the gradient as a holistic object with geometric and physical significance. Tall (2002) notes that such integration is essential for moving from fragmented knowledge to connected understanding (Tall, 2002, 2004). Ultimately, while difficulties in visualization persisted, the combination of symbolic computation, guided scaffolding, and dynamic software fostered meaningful progress. The retrospective analysis suggests that systematic support can gradually shift students from procedural only reasoning to rich conceptualization of gradients as vectors.

CONCLUSION

This study investigated students' understanding of gradient vectors in the context of the potential function $V(x, y) = x^2 + 2y^2$. The results indicate that, although most students could correctly compute partial derivatives and obtain $\nabla V(x, y) = (2x, 4y)$, many initially stopped at procedural manipulation without conceptual articulation. For instance, when students responded with 'gradient = (4,13), done,' it revealed that they had not yet interpreted the numerical result as a directional vector with meaning in the context of surface growth. Through guided scaffolding and the use of GeoGebra visualizations, students gradually transitioned from symbolic computation to a structural understanding of the gradient vector as representing the direction of steepest ascent and its magnitude as the rate of change per unit distance (Sfard, 1991; Tall, 2002, 2004).

The implications of this study suggest that integrating dynamic visualization tools such as GeoGebra can play a crucial role in bridging the gap between procedural fluency and conceptual understanding in multivariable calculus. By anchoring gradient vectors at specific points, students observed the direction and connected the symbolic form $\nabla V(1,2) = (2,8)$ with its geometric representation. This approach facilitated the reification process described by Sfard (2008), helping students to perceive the gradient vector as both a computational output and a conceptual object (Sfard, 1991). The findings are consistent with previous studies highlighting the need for explicit tasks that link algebraic representations with visual and contextual meaning (Borji et al., 2022; Cheong et al., 2023).

Nevertheless, the study has some limitations. The number of participants was relatively small, and the analysis focused primarily on a single problem involving gradient interpretation. Broader exploration across different surfaces and longitudinal data, would strengthen the conclusions and provide deeper insights into how students develop an integrated understanding of gradients in multivariable contexts. Future research could expand on these findings by designing tasks that explicitly connect gradient concepts with applications in physics and engineering, thereby enhancing the relevance and transferability of students' learning (Herrera et al., 2024).

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3. Expand the practical implications for instructional design.
4. Strengthen the limitations with methodological reflection.
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AUTHOR CONTRIBUTIONS

Author One contributed to the conceptualization, methodology, formal analysis, and writing of the original draft.

Author Two was responsible for data curation, investigation, and writing, review & editing. **Author Three** contributed to software, supervision, validation, project administration, and funding acquisition.

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EXPLORING STUDENTS' CONCEPTUAL DIFFICULTIES AND CONCEPTUAL CHANGE IN MULTIVARIABLE CALCULUS: A TEACHING-EXPERIMENT AND RETROSPECTIVE ANALYSIS

Commented [A1]: Judul terlalu panjang
boleh digunakan jdul berikut
GeoGebra-Assisted Teaching Experiment on Students' Conceptual Change in Multivariable Calculus

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ABSTRACT

This study investigates the effectiveness of GeoGebra-assisted learning in enhancing students' conceptual understanding of multivariable calculus through a teaching experiment design. The research involved undergraduate students enrolled in a multivariable calculus course and was carried out in two iterative stages. Data were collected using diagnostic tests, observation sheets, and reflective notes, while analysis included coding of students' responses, identification of misconceptions, and triangulation of test results, classroom transcripts, and task products.

The findings revealed that, at the outset, students could perform symbolic computations of partial derivatives but often stopped at numerical results without interpreting them as components of a gradient vector. They also experienced difficulties visualizing the geometric meaning of the gradient and its relation to the tangent plane. After introducting of GeoGebra-based visualizations and guided scaffolding, students began to describe gradient components as rates of change and connected them to the direction of steepest ascent. Observations showed that students engaged more and participated actively in collaborative discussions using dynamic visualization tools.

The study demonstrates that GeoGebra-assisted learning, implemented within a teaching experiment framework, can effectively bridge the gap between procedural fluency and conceptual understanding in multivariable calculus. The study contributes to mathematics education by highlighting the role of technological integration and reflective instructional design in addressing students' persistent misconceptions about gradients and tangent planes.

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Corresponding Author:

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INTRODUCTION

Multivariable calculus is a core subject in undergraduate mathematics and science programs, playing a crucial role in developing students' higher-order thinking, spatial reasoning, and modeling skills. Despite its importance, many students face persistent difficulties when transitioning from single-variable to multivariable contexts, particularly in understanding abstract concepts such as partial derivatives, gradients, and tangent planes (Borji et al., 2024; Hahn & Klein, 2025; Kashefi et al., 2011; Rodríguez-Nieto & Moll, 2025). These difficulties often stem from an overreliance on procedural computation, with limited ability to integrate algebraic procedures with geometric interpretation (Borji et al., 2024; Cheong et al., 2023; Hahn & Klein, 2025). As a result, students may compute derivatives correctly but struggle to articulate the conceptual significance of gradients or interpret the geometric meaning of tangent planes.

Mathematics education researchers have highlighted the importance of instructional strategies that foster conceptual understanding alongside procedural fluency. Conceptual growth involves transforming operational procedures into structural objects, enabling students to reason about mathematical entities more flexibly (Andiani et al., 2024, 2025; E. D. Dubinsky & Mcdonald, 2001; E. Dubinsky & Mcdonald, 2001; Matindike, 2025; Shvarts et al., 2024; Trigueros et al., 2024; Villabona et al., 2024). In this regard, technology enhanced learning environments, particularly dynamic visualization tools such as GeoGebra, have been shown to support students' conceptual development by linking symbolic computations with graphical and interactive representations (Baye et al., 2021; Gurmu et al., 2024; Suparman et al., 2024). Such tools serve as mediating devices that can bridge students' fragmented understanding of multivariable functions, gradients, and tangent planes (Borji et al., 2022; Cheong et al., 2024; Herrera et al., 2024; Schoenherr et al., 2024). ~~In particular, the doctoral thesis *Local Instruction Theory Perbandingan Trigonometri dengan Pendekatan Matematika Realistik untuk Mengembangkan Kemampuan Representasi Matematis* (Andiani et al., 2025) demonstrates that an LIT-based RME approach with realistic contexts (*bayangan*, *kemiringan tangga*, *posisi mata terhadap objek*) significantly enhances students' mathematical representation abilities in the trigonometric ratios compared to purely procedural learning~~.

Commented [A4]: konteks berbeda (RME dan trigonometri), yang kurang relevan langsung dengan topik kalkulus multivariabel—perlu penjelasan keterkaitannya

Several studies have demonstrated the benefits of technology in calculus instruction. For example, Schoenherr found that interactive visualizations improved students' reasoning about multivariable functions (Cheong et al., 2023; Herrera et al., 2024; Rao et al., 2023; Schoenherr et al., 2024), while Karabay (2025) highlighted the role of scaffolding in managing cognitive load during problem solving (Faber et al., 2024; Karabay & Meşe, 2025; Li et al., 2023; van Nooijen et al., 2024). However, much of the existing research emphasizes quantitative outcomes, such as test performance, with limited attention to qualitative aspects of conceptual change or the dynamics of classroom interactions. Additionally, few studies have examined how iterative teaching experiment designs can trace students' conceptual difficulties and the pathways of conceptual change over time.

The novelty of this study lies in combining a teaching experiment approach with retrospective analysis to explore students' conceptual difficulties and subsequent changes in understanding gradients and tangent planes through GeoGebra-assisted learning. Teaching experiments enable iterative refinement of instructional interventions while closely observing students' reasoning (Kelly & Lesh, 2000). Retrospective analysis allows systematic identification of shifts in conceptual understanding across multiple data sources.

Previous research by Andiani et al.,(2020) demonstrated that students face challenges in solving geometric problems on competency-based assessments, highlighting difficulties in understanding and reasoning mathematically (Andiani et al., 2020). Furthermore, Andiani et al., (2025) found that realistic mathematics education approaches can improve students' mathematical representation skills, essential in multivariable calculus contexts (Andiani et al., 2025). These findings suggest that explicit scaffolding and visualization tools are crucial for fostering conceptual understanding.

Therefore, this research focuses on exploring students' conceptual difficulties and conceptual change in multivariable calculus through a qualitative teaching experiment approach supported by GeoGebra. The main aim is to describe how students develop an understanding of gradients and tangent planes when exposed to visualization-based instruction and to analyze how this process bridges the gap between procedural fluency and conceptual comprehension. Recent studies have confirmed that GeoGebra-assisted visualizations can significantly enhance students' conceptual understanding and engagement in multivariable calculus and related mathematical topics (Abdurrahman Wahid Pekalongan et al., n.d.; Bedada & Machaba, 2022; Cheong et al., 2024).

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METHOD

This study employed a qualitative descriptive design through a teaching experiment and retrospective analysis to investigate students' conceptual difficulties and conceptual change in multivariable calculus. The researchers chose this design because it allowed them to explore students' thinking processes and capture how their understanding evolved during the learning activities.

The participants were eight undergraduate students enrolled in a multivariable calculus course at a mathematics education program in West Java, Indonesia. Participants were selected purposively based on their prior exposure to basic calculus concepts and their availability to participate in the entire teaching experiment cycle. The small group size was intended to enable intensive observation of students' learning processes.

The instruments used in this study included learning tasks in the form of problem sets related to the concept of limits in two variables and GeoGebra-based activities designed to promote visualization and conceptual exploration. The researchers employed observation sheets to document classroom interactions and student strategies, while semi-structured interview guidelines were used to explore students' reasoning, misconceptions, and conceptual changes in greater depth.

The procedures consisted of two stages of teaching experiments followed by retrospective analysis. Each teaching experiment involved introducing learning tasks, classroom interaction supported by GeoGebra visualization, and reflection activities. The researchers collected data through diagnostic tests, classroom transcripts, students' written work, and reflective notes.

Data analysis was conducted through coding students' responses, identifying recurring misconceptions, and comparing test results, classroom observations, and students' task products. Triangulation between these data sources was applied to ensure the credibility and validity of the findings. The researchers conducted the analysis iteratively, allowing them to refine their interpretations and connect the results with the theoretical framework on conceptual understanding in multivariable calculus.

RESULT AND DISCUSSION

Students' Initial Conceptual Difficulties

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At the beginning of the task, most students successfully computed the partial derivatives of the potential function $V(x,y) = x^2 + 2y^2$, yielding the gradient $\nabla V(x,y) = (2x, 4y)$. When evaluated at the point (1,2), this became $\nabla V(1,2) = (2,8)$. However, their written responses tended to stop at the numerical level, showing little evidence of conceptual interpretation. This finding resonates with Borji (2022), who noted that many students demonstrate strong procedural skills while lacking structural understanding in multivariable calculus (Borji et al., 2022).

A recurring pattern emerged where students simply reported, *"gradient* = (2,8), *finished."* Such utterances illustrate what Sfard (2008) describes as *operational thinking* treating mathematical objects as results of procedures rather than entities with independent meaning (Sfard, 1991). Students in this stage failed to recognize that the gradient is more than just a pair of numbers; it is a vector with both magnitude and direction.

When pressed for further interpretation, some students attempted to describe the result in vague terms, such as "the slope in x is two and in y is 8," but without elaborating on its vectorial nature. This suggests that symbolic results were fragmented rather than integrated into a coherent conceptual framework. Cheong et al. (2023) also reported similar tendencies in their study, where students often compartmentalized partial derivatives without connecting them to vectorial interpretation (Cheong et al., 2023).

This difficulty is particularly important in vector calculus, where gradients play a central role in understanding directional change, optimization, and physical phenomena. Without conceptual grounding, students cannot connect gradients to applied contexts, such as interpreting potential fields in physics or optimization problems in engineering. Medina Herrera et al. (2021) highlight that such gaps hinder the transition from abstract calculation to practical application (Herrera et al., 2024).

A closer analysis of written work confirmed this gap. While algebraic manipulations were accurate, annotations explaining the meaning of the results were rare. Students did not spontaneously refer to the gradient as the direction of steepest ascent or discuss its physical implications in the context of a potential function. Students did not spontaneously refer to the gradient as the direction of steepest ascent or discuss its physical implications in the context of a potential function, as confirmed by Abreu et al. (2021), who emphasize that symbolic fluency alone is insufficient without scaffolding that links symbols to concepts (Abreu-Mendoza et al., 2021).

In retrospective discussions, some students admitted being unsure how to proceed after obtaining the ordered pair. They expressed uncertainty about substituting values for the original function, drawing a graph, or leaving the result. Such hesitation illustrates the lack of an internalized schema for interpreting gradient vectors.

These findings underscore the necessity of carefully designed instructional interventions. As Tall (2008) argues, advanced mathematical thinking requires students to move beyond computational comfort zones and integrate multiple representations (Tall, 2008). Here, the initial failure to articulate conceptual meaning highlights the importance of explicitly embedding interpretation tasks in calculus instruction.

Table 1 summarizes students' initial approaches to illustrate the distribution of responses. Categories include purely procedural responses (e.g., numerical computation only), partial interpretations (mentioning "slope" without directionality), and meaningful interpretations (rare at this stage).

Table 1. Categories of student responses to the gradient task at (1,2)

Response Type	**Example Student Statement**	**Frequency (N=20)**
Procedural only	"Gradient = (2,8), finished"	3
Partial interpretation	"Slope in x is 2, in y is 8"	0
Full interpretation	"The vector (2,8) shows the direction of steepest increase at (1,2); its rate of increase per unit length is $2\sqrt{17}$."	5

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The predominance of procedural responses highlights the extent to which conceptual understanding was initially lacking. Such a distribution sets the stage for examining how scaffolding and visualization can support a deeper grasp of gradient meaning.

Overall, this first phase demonstrates that students could compute but not interpret. Without explicit prompts or visual tools, they remained anchored in procedural reasoning, which validates earlier findings by Borji (2022) and Cheong et al. (2023) that procedural fluency does not automatically translate into conceptual mastery (Borji et al., 2022; Cheong et al., 2023).

Transition from Symbolic Computation to Geometric Meaning

After the researchers introduced scaffolding strategies, students moved beyond numerical computation toward geometric meaning. The teacher encouraged them to see the gradient as a vector pointing toward the steepest ascent. This process of guided reinvention mirrors Sfard's (2008) description of *reification*, where operational processes gradually become conceived as structural objects (Sfard, 1991).

For example, after further questioning, a student remarked, *"So, the arrow shows the fastest way up."* What the student said illustrates a pivotal shift from perceiving the gradient as an inert ordered pair to recognizing it as a directional indicator. Rochaminah et al. (2025) emphasize that such discursive transitions are critical for advancing from procedural competence to conceptual understanding (Rochaminah et al., 2025).

The case of (2,8) at (1,2) was especially instructive. The researchers prompted students to interpret not only the numerical values but also the relative weights of each component. They concluded that the vector points northeast, with a stronger bias toward the y-axis due to the larger coefficient. This interpretation aligns with Cheong et al. (2023), who found that relative component magnitudes often served as an entry point for students in grasping geometric meaning (Cheong et al., 2023).

Discussions also emphasized physical interpretations. The researchers asked students to imagine the potential function representing an electric field. In this analogy, the gradient indicated the direction in which a test charge would experience the greatest force. This contextualization supported Medina Herrera et al. (2021), who argue that situating symbolic results in applied domains enhances engagement and meaning making (Herrera et al., 2024).

At this stage, evidence of conceptual change was visible in classroom transcripts. One student said, *"The 2 and 8 mean the function increases faster in y than in x."* Such statements go beyond restating numerical results and reveal an emerging awareness of the gradient's functional role.

This development illustrates Tall's (2008) assertion that students benefit from explicit opportunities to link symbolic expressions with graphical and contextual interpretations (Tall, 2008). The researchers facilitated the connection through structured questioning and prompts that required students to articulate their thinking.

GeoGebra was later employed to reinforce this connection. Visualizing the vector (2,8) at (1,2) provided immediate geometric confirmation of analytic calculations. Figure 1 shows a screenshot of the vector superimposed on the graph of the potential function. This use of technology resonates with Medina Herrera et al. (2021), who emphasize the value of dynamic software in bridging representational gaps (Herrera et al., 2024).

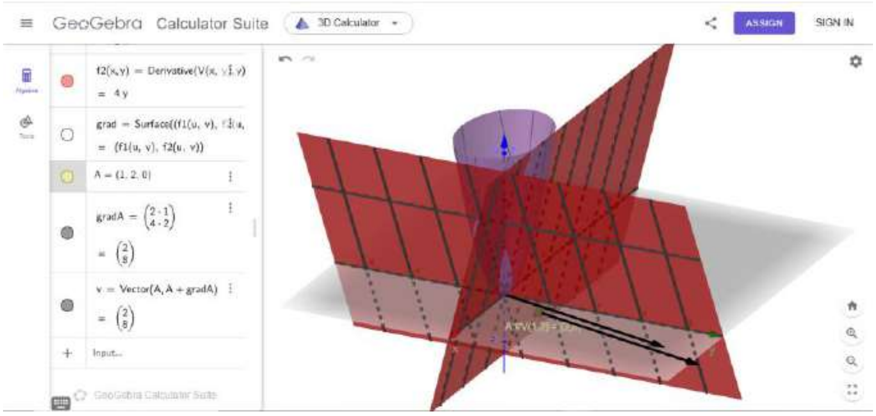

Figure 1. GeoGebra visualization of the gradient vector (2,8) at point (1,2)

Figure 2. Students exploring mathematics concepts with GeoGebra on mobile phones

Such visual aids confirmed symbolic computations and expanded students' reasoning. The visual of a steep arrow pointing northeast made the abstract idea of "steepest ascent" concrete. Sung et al. (2025) highlight that such multimodal engagements promote deeper conceptualization (Sung & Nathan, 2025; Yeh et al., 2025).

By the end of this phase, students were increasingly able to articulate the meaning of gradients in symbolic and geometric terms. Increased students demonstrate the potential of scaffolding and visualization in fostering representational integration.

The transition from symbolic to geometric reasoning marked a critical conceptual breakthrough. Students no longer viewed the gradient as a final numerical product but as an active object that connects algebraic, graphical, and physical dimensions.

Difficulties in Visualizing Gradient Vectors

Despite notable progress, visualization remained a challenge for many students. Several could describe the meaning of (2,8) but hesitated to represent it graphically. Some drew arrows incorrectly, while others avoided sketching altogether. This difficulty echoes Moreno (2021), who reported persistent struggles with translating symbolic gradients into graphical form (Moreno-Arotzena et al., 2021).

One common error involved placing the vector at the origin instead of at the point (1,2). The error of the activity suggests a limited understanding of gradient vectors as *anchored* at specific points on the surface. Similar misconceptions were identified by Cheong et al. (2023), who observed that students often treated gradient vectors as free-floating arrows (Cheong et al., 2023).

Another difficulty was scaling. Students who attempted sketches sometimes drew the components with incorrect proportions, e.g., equal lengths for 2 and 8. This indicates partial recognition of directionality but incomplete attention to magnitude. Bollen et al. (2017) caution that without attention to scale, students risk losing the quantitative meaning of vectors (Bollen et al., 2017; Dubey & Barniol, 2023; Sabah, 2023).

To address these issues, the class used GeoGebra's vector tools to plot (2,8) accurately at (1,2). Figure 3 illustrates how this technology provided precise positioning and scaling. Such scaffolding resonates with Medina Herrera et al. (2021), who show that dynamic software reduces cognitive load and reinforces representational accuracy (Herrera et al., 2024).

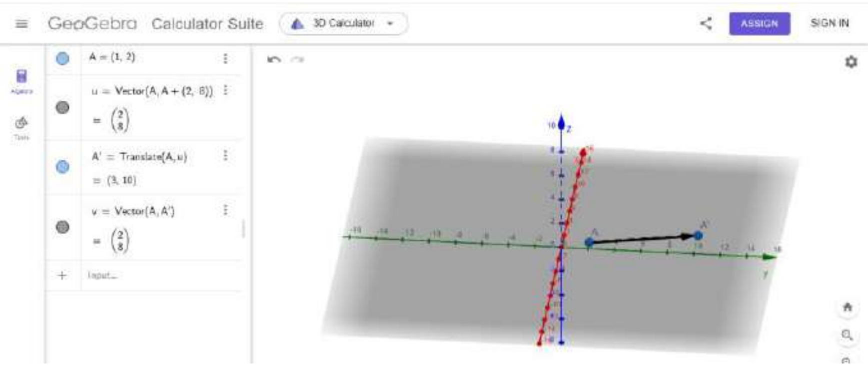

Figure 3. GeoGebra plot of gradient vector (2,8) anchored at (1,2)

Transcript evidence also showed that visualization helped students reconcile earlier errors. After seeing the GeoGebra output, one student said, *"Oh, the arrow is not from zero, but from (1,2)."* The student said that visualization supported correcting misconceptions and facilitated representational alignment. Nevertheless, reliance on technological scaffolding revealed another challenge: students struggled to visualize vectors without software support. Students' struggles to visualize vectors without software support suggest that internalization of geometric reasoning requires repeated practice, beyond one-time demonstrations. Sfard (1991) warns that without reification, students may remain dependent on external representations (Sfard, 1991).

Class discussions further emphasized the need to interpret vector length. The researchers guided students to compute the magnitude $\sqrt{2^2 + 8^2} = \sqrt{68}$, reinforcing that the gradient encodes direction and rate of change. This mistakes addressed earlier gaps where students only equated the gradient with slope, without recognizing its multidimensional meaning.

By integrating magnitude into interpretation, students began to view the gradient as a holistic object with geometric and physical significance. Tall (2002) notes that such integration is essential for moving from fragmented knowledge to connected understanding (Tall, 2002, 2004). Ultimately, while difficulties in visualization persisted, the combination of symbolic computation, guided scaffolding, and dynamic software fostered meaningful progress. The retrospective analysis suggests that systematic support can gradually shift students from procedural only reasoning to rich conceptualization of gradients as vectors.

CONCLUSION

This study investigated students' understanding of gradient vectors in the context of the potential function $V(x, y) = x^2 + 2y^2$. The results indicate that, although most students could correctly compute partial derivatives and obtain $\nabla V(x, y) = (2x, 4y)$, many initially stopped at procedural manipulation without conceptual articulation. For instance, when students responded with 'gradient = (4,13), done,' it revealed that they had not yet interpreted the numerical result as a directional vector with meaning in the context of surface growth. Through guided scaffolding and the use of GeoGebra visualizations, students gradually transitioned from symbolic computation to a structural understanding of the gradient vector as representing the direction of steepest ascent and its magnitude as the rate of change per unit distance (Sfard, 1991; Tall, 2002, 2004).

The implications of this study suggest that integrating dynamic visualization tools such as GeoGebra can play a crucial role in bridging the gap between procedural fluency and conceptual understanding in multivariable calculus. By anchoring gradient vectors at specific points, students observed the direction and connected the symbolic form $\nabla V(1,2) = (2,8)$ with its geometric representation. This approach facilitated the reification process described by Sfard (2008), helping students to perceive the gradient vector as both a computational output and a conceptual object (Sfard, 1991). The findings are consistent with previous studies highlighting the need for explicit tasks that link algebraic representations with visual and contextual meaning (Borji et al., 2022; Cheong et al., 2023).

Nevertheless, the study has some limitations. The number of participants was relatively small, and the analysis focused primarily on a single problem involving gradient interpretation. Broader exploration across different surfaces and longitudinal data, would strengthen the conclusions and provide deeper insights into how students develop an integrated understanding of gradients in multivariable contexts. Future research could expand on these findings by designing tasks that explicitly connect gradient concepts with applications in physics and engineering, thereby enhancing the relevance and transferability of students' learning (Herrera et al., 2024).

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AUTHOR CONTRIBUTIONS

Author One contributed to the conceptualization, methodology, formal analysis, and writing of the original draft. **Author Two** was responsible for data curation, investigation, and writing, review & editing. **Author Three** contributed to software, supervision, validation, project administration, and funding acquisition.

Commented [A8]: Pembahasan sering mengulang ide dari literatur tanpa mengaitkan secara eksplisit dengan data empiris. Lakukan analisis kritis temuan penelitian dalam pembahasan

Commented [A9]: Terlalu panjang, fokus pada tujuan penelitian saja

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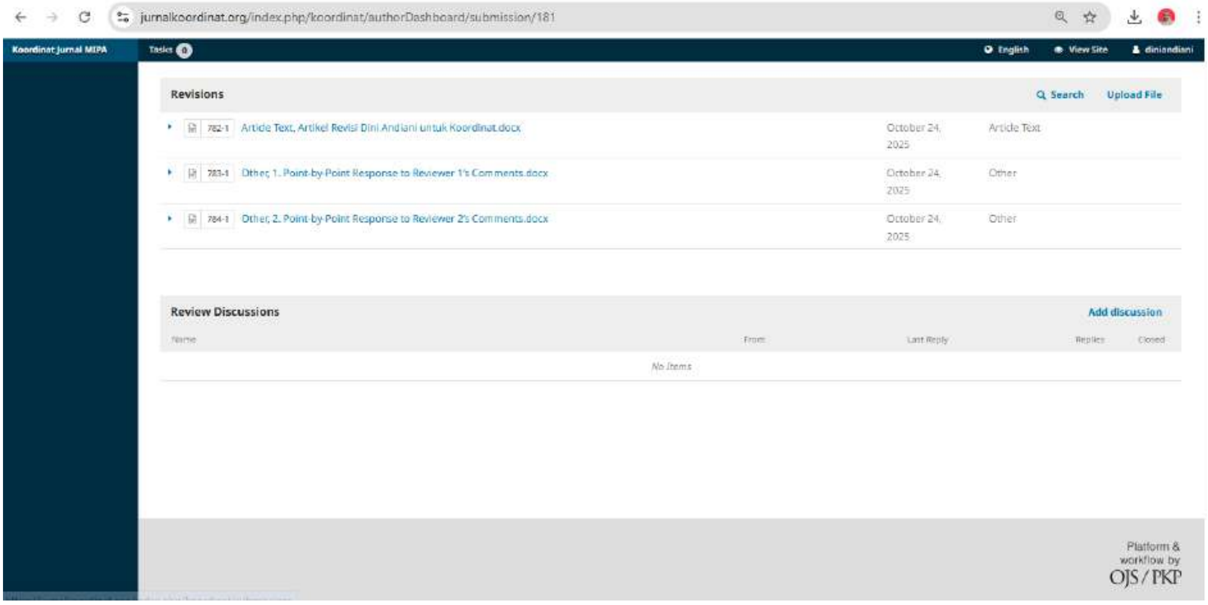
3. Bukti Konfirmasi Submit Revisi Pertama, Respon Kepada Reviewer, dan Artikel Yang Diresubmit

(24 Oktober 2025)



Bukti Konfirmasi Submit Revisi Pertama dan Respon kepada Reviewer pada 24 Oktober 2025 melalui OJS
https://jurnalkoordinat.org/index.php/koordinat/authorDashboard/submission/181

Point-by-Point Response to Reviewer 1's Comments

TANGGAPAN PENULIS TERHADAP KOMENTAR REVIEWER 1

(Point-by-Point Response to Reviewers' Comments)

Judul Artikel: *GeoGebra-Assisted Teaching Experiment on Students' Conceptual Change in Multivariable Calculus*
Penulis: Dini Andiani, Siti Dwi Rahayu, Cecep Suwanda
Nomor Artikel: A-181
Jurnal: *Koordinat Jurnal MIPA*

No	Komentar Reviewer	Tanggapan Penulis	Tindakan Revisi / Lokasi
1	Judul terlalu panjang, boleh digunakan judul berikut: *GeoGebra-Assisted Teaching Experiment on Students' Conceptual Change in Multivariable Calculus.*	Terima kasih atas sarannya. Judul telah direvisi agar lebih ringkas dan langsung menggambarkan fokus penelitian.	Judul di halaman pertama telah diperbarui.
2	Tuliskan subjek penelitian.	Telah ditambahkan penjelasan subjek penelitian: *"The study involved eight undergraduate mathematics students enrolled at a university located in West Java, Indonesia."*	Bagian Abstrak dan Metode diperjelas.
3	Paragraf hasil terlalu panjang dan kompleks, tuliskan secara lebih sederhana.	Kalimat telah disederhanakan agar lebih mudah dipahami: *"The findings showed that students could calculate partial derivatives but did not understand their meaning as gradient vectors. They also had trouble visualizing the gradient and its connection to the tangent plane. After using GeoGebra and receiving guidance, students began to see the gradient as a rate of change and the direction of steepest ascent. They became more engaged in class discussions. This study shows that GeoGebra-assisted learning helps students connect procedural skills with conceptual understanding in multivariable calculus."*	Bagian Abstrak disederhanakan.
4	Konteks tesis RME dan trigonometri kurang relevan langsung dengan topik kalkulus multivariabel—perlu penjelasan keterkaitannya.	Telah ditambahkan penjelasan keterkaitan prinsip RME (*guided reinvention*) yang menginspirasi desain pembelajaran dalam penelitian ini.	Bagian Pendahuluan paragraf ke-5 direvisi.

Point-by-Point Response to Reviewer 1's Comments

No	Komentar Reviewer	Tanggapan Penulis	Tindakan Revisi / Lokasi
5	Sampaikan manfaat penelitian.	Ditambahkan pernyataan kontribusi penelitian: *"The findings of this study are expected to contribute both theoretically and practically. Theoretically, it may deepen understanding of how visualization and guided reinvention promote conceptual change in higher mathematics. Practically, it can inform the development of effective, technology-enhanced learning materials and instructional strategies for calculus education."*	Paragraf terakhir pendahuluan diperluas.
6	Perlu lebih banyak bukti konkret (visual jawaban siswa atau transkrip wawancara).	Telah ditambahkan kutipan langsung dari jawaban siswa dan cuplikan transkrip wawancara sebagai bukti pendukung.	Bagian Hasil diperkuat dengan data empiris.
7	Jumlah subjek 8 atau 20?	Terima kasih atas koreksinya. Jumlah partisipan disesuaikan dengan data aktual, yaitu 8 mahasiswa.	Tabel 1 dan teks diperbarui agar konsisten.
8	Pembahasan sering mengulang ide dari literatur tanpa mengaitkan secara eksplisit dengan data empiris.	Bagian pembahasan telah direvisi agar menghubungkan temuan empiris dengan teori secara lebih eksplisit dan mengurangi pengulangan literatur.	Bagian Pembahasan diperbarui secara substansial.
9	Kesimpulan terlalu panjang, fokus pada tujuan penelitian saja.	Kesimpulan telah diperpendek dan difokuskan pada pencapaian tujuan dan hasil utama penelitian.	Bagian Kesimpulan disederhanakan.

Point-by-Point Response to Reviewer 2's Comments

TANGGAPAN PENULIS TERHADAP KOMENTAR REVIEWER 2

(Point-by-Point Response to Reviewers' Comments)

Judul Artikel: *GeoGebra-Assisted Teaching Experiment on Students' Conceptual Change in Multivariable Calculus*
Penulis: Dini Andiani, Siti Dwi Rahayu, Cecep Suwanda
Nomor Artikel: A-181
Jurnal: *Koordinat Jurnal MIPA*

No.	Reviewer's Comment	Author's Response	Revision Made / Location in Manuscript
1	*Option 1: Revise the title to match the abstract… or Option 2: Revise the abstract to match the title.*	We revised the title and abstract for consistency. The title now reads: "GeoGebra-Assisted Teaching Experiment on Students' Conceptual Change in Multivariable Calculus." The abstract was rewritten using the IMRaD structure and now emphasizes the conceptual change framework and process of understanding, not only the effectiveness of GeoGebra.	Title page and Abstract section, p.1
2	The abstract must use the IMRAD method (Introduction, Method, Results, and Discussion/Conclusion).	The abstract has been revised to follow the IMRaD structure. It now begins with a clear introduction explaining the research context and purpose, followed by a concise description of the method, the main findings, and a concluding statement highlighting both theoretical and practical implications. The revision maintains a natural paragraph format in accordance with the *Koordinat Journal* style while ensuring logical IMRaD order.	Revised in the Abstract section, page 1
3	There is an inconsistency between the title and the objectives, namely, from the title it appears that what is wanted to be known is related to "conceptual difficulties" and "conceptual changes" with the "teaching experiment" method. However, the objectives stated in the abstract are related to "the effectiveness of GeoGebra" and "increased understanding of gradients and tangent planes after GeoGebra intervention".	The inconsistency has been resolved. The title has been revised to clearly reflect the study's focus on students' conceptual change within a GeoGebra-assisted teaching experiment, rather than testing effectiveness. The revised title is: *"GeoGebra-Assisted Teaching Experiment on Students' Conceptual Change in Multivariable Calculus."* Corresponding adjustments were also made in the abstract and research	*Title (page 1); Abstract section (page 1); Research Objective paragraph in Introduction*

Point-by-Point Response to Reviewer 2's Comments

No.	Reviewer's Comment	Author's Response	Revision Made / Location in Manuscript
	Please follow up, whether the title is changed or the research objectives and results are corrected according to the title.	objectives to ensure alignment with the title.	
4	*Which semester and academic year of students were used in the research? And how many were there?*	Added details specifying that participants were second-year mathematics education students (Semester IV, Academic Year 2023/2024) with n = 8 participants in the main teaching experiment.	Method section
5	How about "Conceptual difficulties", "conceptual change" and "Retrospective analysisi"? In my opinion, the appropriate keywords are "conceptual difficulties", "conceptual change", "multivariable calculus", "teaching experiment", and "retrospective analysis".	Thank you for the constructive suggestion. The keywords have been revised to better align with the study's title, objectives, and theoretical framework. The updated keywords now include: *GeoGebra-assisted learning; teaching experiment; conceptual change; conceptual difficulties; multivariable calculus.*	Keywords section (after Abstract)
6	1. Please add a data-based phenomenon or problem (there is no strong empirical data yet). 2. Please add "Direct link to the conceptual change framework." 3. For the final paragraph, it would be better not to end with a reference. It would be better to focus on the purpose or formulation.	1. The Introduction was expanded to include data-based evidence of students' persistent difficulties in understanding multivariable calculus, supported by previous diagnostic data and literature (Borji et al., 2024; Hahn & Klein, 2025). 2. A direct explanation of how the study adopts the Conceptual Change framework (Vosniadou, 2008) was added to clarify its theoretical basis and relevance to multivariable calculus learning. 3. The final paragraph of the Introduction was rewritten to end with the research objective, not a citation.	1. Introduction. 2. Introduction, final paragraph. 3. Introduction, last paragraph
7	1. The abstract does not explain the use of interviews, while the method does include	1. The abstract has been revised to mention the use of semi-structured	**1. Abstract:** Paragraph 1 → Added phrase:

Point-by-Point Response to Reviewer 2's Comments

No.	Reviewer's Comment	Author's Response	Revision Made / Location in Manuscript
	interviews. Please be consistent. 2. Please include guidelines or sample questions for the interview.	interviews to ensure consistency with the Method section. 2. In addition, the Method section has been expanded to describe the purpose, timing, and focus of the interviews, including sample question themes (e.g., students' interpretation of gradients, their connection between symbolic and graphical representations, and how GeoGebra supported conceptual understanding).	"… data were collected through diagnostic tests, classroom observations, students' written work, reflective notes, and semi-structured interviews …" **2. Method:** Paragraph after instrument description → Added explanation of interview purpose, timing, and example question themes.
8	add a chart or diagram of the research procedure.	A research procedure diagram has been added to illustrate the overall process of the study, showing two iterative cycles that include *preliminary design*, *teaching experiment*, and *retrospective analysis* in each cycle.	Method section
9	*Please add references to support the use of qualitative descriptive design, teaching experiment, and retrospective analysis.*	References have been added to justify the methodological framework. The paragraph now cites *Gravemeijer & Cobb (2006)*, *Kelly & Lesh (2000)*, and *Bakker (2018)* to strengthen the rationale for employing a teaching experiment and retrospective analysis in design-based educational research.	Method section, first paragraph
10	1. Add the results of student interviews, because the research method includes semi-structured interviews. 2. please clarify regarding "conceptual change" 3. Use more informative captions and image descriptions.	1. The results of student interviews have been explicitly incorporated in the *Results and Discussion* section to illustrate students' conceptual change. Excerpts from semi-structured interviews (e.g., Participant S3) show shifts in students' reasoning before and after the GeoGebra intervention.	*Results and Discussion*

Point-by-Point Response to Reviewer 2's Comments

No.	Reviewer's Comment	Author's Response	Revision Made / Location in Manuscript
	4. add critical analysis, not just literature confirmation. 5. where the results of "coding" are based on the explanation of the research method related to "data analysis" 6. please add research limitations	2. A clear explanation of conceptual change has been added in the *Results and Discussion* section. The paragraph explicitly defines conceptual change according to Vosniadou et al. (2008) and links it to the observed shift in students' reasoning from symbolic to geometric understanding. 3. All figures were revised to include more informative captions and contextual descriptions. Each caption now clarifies what the figure represents, its relation to the corresponding task or student response, and how it supports the discussion of conceptual change. For example, Figure 3 now explicitly states that it shows the gradient vector (2,8) anchored at (1,2) and illustrates students' transition from symbolic computation to geometric visualization. 4. The *Results and Discussion* section was revised to include a critical analysis connecting empirical findings on gradient understanding with theoretical perspectives on conceptual change. The revised paragraph contrasts current findings with prior studies, highlighting that students' conceptual change emerged through guided reinvention and reflection, not solely from GeoGebra use. 5. We have added a new subsection titled *"Overview of Data Coding and Categories"* to explicitly describe how the results of data coding were derived from the analysis process. This subsection outlines the coding procedure, the major categories that emerged from the data, and how these	

Point-by-Point Response to Reviewer 2's Comments

No.	Reviewer's Comment	Author's Response	Revision Made / Location in Manuscript
11		categories guided the presentation and interpretation of findings. A transition sentence was also added to link the coding results to the subsequent discussion of students' conceptual difficulties. 6. A new paragraph describing research limitations has been added at the end of the *Results and Discussion* section. It addresses limitations in sample size, duration, and the scope of technological tools, and includes suggestions for future studies.	
11	As a reader, I'm confused. It says n=20, but it's listed in tables 5 and 3 (a total of 8). Please provide a complete explanation (the research method also lists 8. Where does the 20 come from?)	We thank the reviewer for pointing out this inconsistency. The value "N = 20" in Table 1 was a typographical error. The correct sample size for both the diagnostic data presented in tables and the main teaching experiment is $n = 8$, as reported in the Methods section. We have corrected all occurrences of "N = 20" to $n = 8$ (including Table 1 caption and any related text) to ensure consistency throughout the manuscript.	Methods section; Results section
12	1. Reiterate the initial difficulties to connect them to the title. 2. Add contributions to the theory of conceptual change. 3. Expand the practical implications for instructional design. 4. Strengthen the limitations with methodological reflection. 5. Add further research options (longitudinal/multiple contexts).	1. The revised conclusion explicitly restates the students' initial conceptual difficulties in interpreting gradients and tangent planes, emphasizing their procedural fluency but lack of conceptual understanding. This directly reconnects the conclusion to the study's title and central research problem. 2. The revision explicitly explains how the study extends the Conceptual Change framework by identifying the mechanisms supporting transformation, guided reinvention, metacognitive reflection, and visualization.	Conclusion

Point-by-Point Response to Reviewer 2's Comments

No.	Reviewer's Comment	Author's Response	Revision Made / Location in Manuscript
		3. A new sentence was added to explain pedagogical implications for technology-assisted calculus instruction, focusing on scaffolding and representational understanding. 4. The revised version elaborates on methodological limitations, including sample size, duration, and technological focus, showing awareness of research boundaries. 5. Future directions were expanded to include longitudinal and multicontext research for broader validation of conceptual change patterns.	
13	In the conclusion there is no need to include references	All references have been removed from the Conclusion section to comply with the reviewer's suggestion. The paragraph was revised to focus only on summarizing findings, theoretical contributions, and practical implications without citation.	Conclusion section, entire paragraph (final revision).

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GEOGEBRA-ASSISTED TEACHING EXPERIMENT ON STUDENTS' CONCEPTUAL CHANGE IN MULTIVARIABLE CALCULUS

Dini Andiani[1*], Siti Dwi Rahayu[2], Cecep Suwanda[3]

[1, 2, 3] Department of Mathematics, Faculty of Mathematics and Natural Sciences, Universitas Bale Bandung, West Java, Indonesia

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ABSTRACT

This study explores students' conceptual change in understanding gradients and tangent planes through a GeoGebra-assisted teaching experiment in a multivariable calculus course. The study involved eight second year undergraduate mathematics students enrolled at the Mathematics Department of a university in West Java, Indonesia. Data were collected through diagnostic tests, classroom observations, students' written work, and reflective notes, and semi sructured interviews, then analyzed qualitatively using coding, triangulation, and retrospective analysis.

The findings revealed that, at the outset, students were able to perform symbolic differentiation of partial derivatives but struggled to interpret the gradient vector conceptually or relate it to the tangent plane. Through guided use of GeoGebra, students gradually reconceptualized the gradient as representing both the direction and rate of steepest ascent, demonstrating a shift from procedural to structural understanding.

This study contributes to the theory of conceptual change by illustrating how dynamic visualization and guided reinvention can facilitate the transformation of students' procedural knowledge into coherent conceptual models. Practically, the findings provide insights for designing technology-assisted instruction to address persistent misconceptions in multivariable calculus learning.

Corresponding Author:
Dini Andiani, Department of Mathematics, Faculty of Mathematics and Natural Sciences, Universitas Bale Bandung, West Java, Indonesia
Email: diniandiani@unibba.ac.id

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INTRODUCTION

Multivariable calculus is a core subject in undergraduate mathematics and science programs, playing a crucial role in developing students' higher-order thinking, spatial reasoning, and modeling skills. Despite its importance, many students face persistent difficulties when transitioning from single-variable to multivariable contexts, particularly in understanding abstract concepts such as partial derivatives, gradients, and tangent planes (Borji et al., 2024; Hahn & Klein, 2025; Kashefi et al., 2011; Rodríguez-Nieto & Moll, 2025). These difficulties often stem from an overreliance on procedural computation, with limited ability to integrate algebraic procedures with geometric interpretation (Borji et al., 2024; Cheong et al., 2023; Hahn & Klein, 2025). As a result, students may compute derivatives correctly but struggle to articulate the conceptual significance of gradients or interpret the geometric meaning of tangent planes.

Recent diagnostic data collected from calculus students at the Mathematics Department of a university in West Java, Indonesia, further confirmed these persistent difficulties. In a preliminary assessment involving gradient and tangent plane tasks, only three out of eight students (37.5%) were able to correctly interpret the geometric meaning of a gradient vector, and none could connect the symbolic expression $\nabla V(x, y)$ with its graphical representation. Most participants performed symbolic differentiation correctly but failed to describe the gradient as representing the direction and rate of maximum change. These findings align with previous research reporting similar misconceptions, where students often treat the gradient as a mere algebraic output rather than a geometric or physical quantity (Borji et al., 2024; Hahn & Klein, 2025; Rodríguez-Nieto & Moll, 2025). Such consistent evidence underscores the need for instructional designs that integrate visualization and conceptual scaffolding to promote deeper understanding.Mathematics education researchers have highlighted the importance of instructional strategies that foster conceptual understanding alongside procedural fluency. Conceptual growth involves transforming operational procedures into structural objects, enabling students to reason about mathematical entities more flexibly (Andiani et al., 2024, 2025; E. D. Dubinsky & Mcdonald, 2001; E. Dubinsky & Mcdonald, 2001; Matindike, 2025; Shvarts et al., 2024; Trigueros et al., 2024; Villabona et al., 2024). In this regard, technology enhanced learning environments, particularly dynamic visualization tools such as GeoGebra, have been shown to support students' conceptual development by linking symbolic computations with graphical and interactive representations (Baye et al., 2021; Gurmu et al., 2024; Suparman et al., 2024). Such tools serve as mediating devices that can bridge students' fragmented understanding of multivariable functions, gradients, and tangent planes (Borji et al., 2022; Cheong et al., 2024; Herrera et al., 2024; Schoenherr et al., 2024). Click or tap here to enter text.

Several studies have demonstrated the benefits of technology in calculus instruction. For example, Schoenherr found that interactive visualizations improved students' reasoning about multivariable functions (Cheong et al., 2023; Herrera et al., 2024; Rao et al., 2023; Schoenherr et al., 2024), while Karabay (2025) highlighted the role of scaffolding in managing cognitive load during problem solving (Faber et al., 2024; Karabay & Meşe, 2025; Li et al., 2023; van Nooijen et al., 2024). However, much of the existing research emphasizes quantitative outcomes, such as test performance, with limited attention to qualitative aspects of conceptual change or the dynamics of classroom interactions. Additionally, few studies have examined how iterative teaching experiment designs can trace students' conceptual difficulties and the pathways of conceptual change over time.

The novelty of this study lies in combining a teaching experiment approach with retrospective analysis to explore students' conceptual difficulties and subsequent changes in understanding gradients and tangent planes through GeoGebra-assisted learning. Teaching experiments enable iterative refinement of instructional interventions while closely observing students' reasoning (Kelly & Lesh, 2000). Retrospective analysis allows systematic identification of shifts in conceptual understanding across multiple data sources.

In designing the instructional intervention, this study was also inspired by the core principle of Realistic Mathematics Education (RME), particularly guided reinvention, which emphasizes that students should be supported in reconstructing mathematical concepts through exploration and teacher-guided reflection rather than direct instruction (Freudenthal, 1991; Gravemeijer & Cobb, 2006). Within the GeoGebra-based environment, guided reinvention is realized through dynamic visualization and scaffolding that allow students to gradually develop conceptual understanding of gradients and tangent planes from their own explorations. This approach aligns with the goal of promoting conceptual change, as students actively reorganize their prior procedural knowledge into deeper structural understanding. Previous research by Andiani et al., (2020) demonstrated that students face challenges in solving geometric problems on competency-based assessments, highlighting difficulties in understanding and reasoning mathematically (Andiani et al., 2020). Furthermore, Andiani et al., (2025) found that realistic mathematics education approaches can improve students' mathematical representation skills, essential in multivariable calculus contexts (Andiani et al., 2025). These findings suggest that explicit scaffolding and visualization tools are crucial for fostering conceptual understanding.

Therefore, this research focuses on exploring students' conceptual difficulties and conceptual change in multivariable calculus through a qualitative teaching experiment approach supported by GeoGebra. The main aim is to describe how students develop an understanding of gradients and tangent planes when exposed to visualization-based instruction and to analyze how this process bridges the gap between procedural fluency and conceptual comprehension.

The findings of this study are expected to contribute both theoretically and practically. Theoretically, it may deepen understanding of how visualization and guided reinvention promote conceptual change in higher mathematics. Practically, it can inform the development of effective, technology-enhanced learning materials and instructional strategies for calculus education.

This study is theoretically grounded in the Conceptual Change framework (Vosniadou et al., 2008) which explains how learners restructure their prior knowledge when confronted with cognitive conflict or anomalous data. Within this perspective, conceptual understanding develops through the gradual reorganization of students' existing mental models toward more coherent and scientifically accurate ones. In the context of multivariable calculus, this framework is particularly relevant because students often rely on procedural schemas, such as differentiating symbolically, without connecting them to geometric or physical meanings. The GeoGebra-assisted learning design in this study was intended to trigger such cognitive conflict and guide students toward reconceptualizing gradients and tangent planes as dynamic representations of rate and direction, thereby fostering meaningful conceptual change.

Recent studies have confirmed that GeoGebra-assisted visualizations can significantly enhance students' conceptual understanding and engagement in multivariable calculus and related mathematical topics (Bedada & Machaba, 2022; Cheong et al., 2024; Sari et al., 2024)[A1][D2]. Building on these insights, the present study focuses on examining how GeoGebra-assisted instruction can facilitate students' conceptual change in understanding gradients and tangent planes. Specifically, it aims to trace the progression of students' reasoning as they move from procedural manipulation toward conceptual comprehension within a dynamic learning environment.

METHOD

This study employed a qualitative descriptive design through a teaching experiment and retrospective analysis to investigate students' conceptual difficulties and conceptual change in multivariable calculus. The researchers chose this design because it allowed them to explore students' thinking processes and capture how their understanding evolved during the learning activities.

The participants were second year undergraduate students (semester IV, Academic Year 2023/2024) enrolled in a multivariable calculus course at the Mathematics Department of a university in West Java, Indonesia. A total of eight ($n =$ 8) students participated in the main teaching experiment phase, selected purposively based on their prior exposure to basic calculus concepts and their availability to engange in all instructional sessions. The small group size was intended to enable intensive observation of students' learning processes.

The main instruments used in this study was a diagnostic test consisting of a series of structured problems on the concept of limits of two variables and GeoGebra-based activities designed to promote visualization and conceptual exploration. This diagnostic test aimed to identify students' conceptual understanding and misconceptions related to gradients and tangent planes while simultaneously encouraging visual reasoning through dynamic representations. The researchers employed observation sheets to document classroom interactions and student strategies, while semi-structured interview guidelines were used to explore students' reasoning, misconceptions, and conceptual changes in greater depth.

Semi-structured interviews were conducted after each teaching experiment session to elicit students' reflections on their learning processes. The questions focused on how students interpreted the meaning of gradient vectors, related symbolic differentiation to graphical visualization, and perceived the role of GeoGebra in helping them reconstruct their understanding. These interviews provided qualitative evidence to support the interpretation of students' conceptual change.

The procedures consisted of two stages of teaching experiments followed by retrospective analysis. Each teaching experiment involved introducing learning tasks, classroom interaction supported by GeoGebra visualization, and reflection activities. The researchers collected data through diagnostic tests, classroom transcripts, students' written work, and reflective notes.

Data analysis was conducted through qualitative coding of students' responses, identifying recurring misconceptions, and comparison across diagnostic test results, classroom observations, and written work. Triangulation between these data sources was applied to ensure the credibility and validity of the findings. The analysis was conducted iteratively, allowing them to refinement of interpretations and alignment with the theoretical framework of conceptual change in multivariable calculus.

RESULT AND DISCUSSION

Overview of Data Coding and Categories

Data from diagnostic tests, written work, classroom observations, and semi-structured interviews were analyzed qualitatively through thematic coding. Three major categories of student reasoning were identified: (1) procedural responses, focused only on symbolic computation, (2) partial conceptual awareness, showing fragmented interpretations, and (3) integrated conceptual understanding, demonstrating coordinated algebraic and geometric reasoning. These categories guided the structure of the findings presented below and formed the analytical lens through which students' conceptual development was interpreted.

Building on these categories, the following section presents students' initial conceptual difficulties, illustrating the dominance of procedural reasoning before the GeoGebra-assisted intervention.

Students' Initial Conceptual Difficulties[A3][D4]

At the beginning of the task, most students successfully computed the partial derivatives of the potential function $V(x,y) = x^2 + 2y^2$, yielding the gradient $\nabla V(x,y) = (2x, 4y)$. When evaluated at the point (1,2), this became $\nabla V(1,2) = (2,8)$. However, their written responses tended to stop at the numerical level, showing little evidence of conceptual interpretation. This finding resonates with Borji (2022), who noted that many students demonstrate strong procedural skills while lacking structural understanding in multivariable calculus (Borji et al., 2022).

A recurring pattern emerged where students simply wrote, *"gradient* = (2,8), *finished."* Such utterances illustrate what Sfard (2008) describes as *operational thinking*, where mathematical objects are treated merely as the results of procedures. Click or tap here to enter text. Students in this stage failed to recognize that the gradient is not only a pair of numbers but a vector with both magnitude and direction. Figures 1 and 2 show examples of students' written responses. Both students correctly derived the gradient function $\nabla V(x,y) = (2x, 4y)$ and substituted values to obtain $\nabla V(1,2) = (2,8)$, yet neither provided any explanation of its meaning.

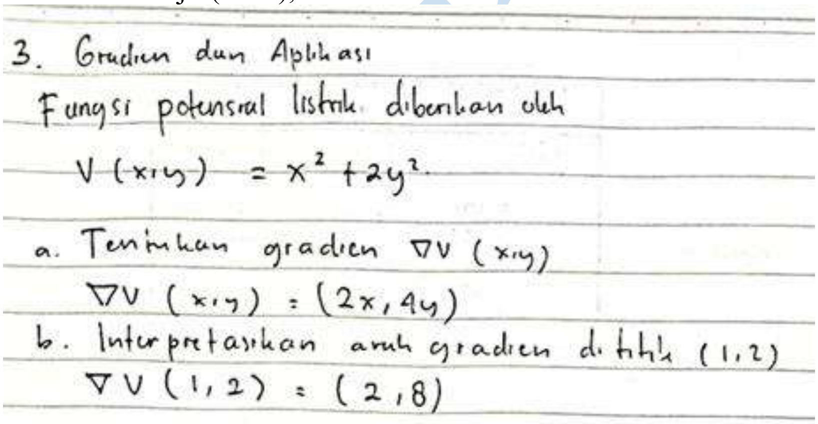

Figure 1. Example Of Student S1's Written Response To The Gradient Task $V(x,y) = x^2 + 2y^2$, Illustrating Purely Procedural Reasoning.

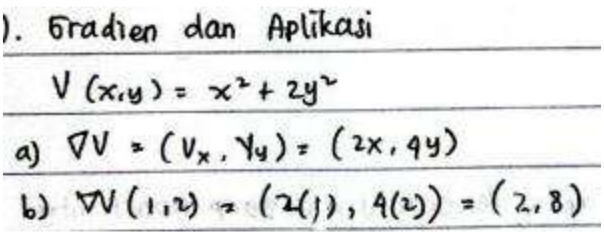

Figure 2. Example Of Student S2's Written Response To The Same Task, Showing Correct Computation But Lacking Conceptual Explanation.

When asked for further interpretation, some students vaguely described the result as "the slope in x is two and in y is 8," without recognizing its vectorial nature. This fragmentation mirrors Cheong et al. (2023), who found that many students treated partial derivatives as isolated algebraic procedures rather than components of a unified vector concept (Cheong et al., 2023). As Herera et al. (2024) srgue, such disconnections limit students' ability to transfer abstract computation to applied contexts, preventing them from developing a coherent understanding of multivariable relationships (Herrera et al., 2024) .Click or tap here to enter text.

This difficulty is particularly important in vector calculus, where gradients play a central role in understanding directional change, optimization, and physical phenomena. Without conceptual grounding, students cannot connect gradients to applied contexts, such as interpreting potential fields in physics or optimization problems in engineering. Medina Herrera et al. (2024) highlight that such gaps hinder the transition from abstract calculation to practical application (Herrera et al., 2024).

A closer analysis of written work confirmed this gap. While algebraic manipulations were accurate, annotations explaining the meaning of the results were rare. Students did not spontaneously refer to the gradient as the direction of steepest ascent or discuss its physical implications in the context of a potential function. Students did not spontaneously refer to the gradient as the direction of steepest ascent or discuss its physical implications in the context of a potential function, as confirmed by Abreu et al. (2021), who emphasize that symbolic fluency alone is insufficient without scaffolding that links symbols to concepts (Abreu-Mendoza et al., 2021).

In retrospective discussions, some students admitted being unsure how to proceed after obtaining the ordered pair. They expressed uncertainty about substituting values for the original function, drawing a graph, or leaving the result. Such hesitation illustrates the lack of an internalized schema for interpreting gradient vectors.

Excerpts from semi-structured interview (Participant S3):
Session 1 (before intervention):
Interviewer (I): *What does the pair (2, 8) mean to you?*
S3: "It's just the slope — so x is 2 and y is 8, that's it."
I: *Can you explain how to draw it on the surface?*
S3: "I don't know, maybe draw a line? I usually stop after computing numbers."

Session 2 (after GeoGebra intervention):
I: *Now that you used the GeoGebra applet, what does (2,8) represent?*
S3: "This is an arrow at (1,2), it shows which direction the surface goes up fastest. The 8 means it increases faster in y than x."

These excerpts clearly illustrate the change in conceptualization before and after the GeoGebra-based intervention. Initially, the student viewed the gradient as a mere numerical pair, but later began to interpret it as a vector that conveys direction and rate of change. These findings underscore the necessity of carefully designed instructional interventions. As Tall (2008) argues, advanced mathematical thinking requires students to move beyond computational comfort zones and integrate multiple representations (Tall, 2008). Here, the initial failure to articulate conceptual meaning highlights the importance of explicitly embedding interpretation tasks in calculus instruction.

Table 1 summarizes students' initial approaches to illustrate the distribution of responses. Categories include purely procedural responses (e.g., numerical computation only), partial interpretations (mentioning "slope" without directionality), and meaningful interpretations (rare at this stage).

Table 1. Categories Of Student Responses To The Gradient Task At (1,2)

Response Type	Example Student Statement	Frequency (N=8)[A5][D6]
Procedural only	“Gradient = (2,8), finished”	3
Partial interpretation	“Slope in x is 2, in y is 8”	0
Full interpretation	“The vector (2,8) shows the direction of steepest increase at (1,2); its rate of increase per unit length is $2\sqrt{17}$.”	5

The predominance of procedural responses highlights the extent to which conceptual understanding was initially lacking. Such a distribution sets the stage for examining how scaffolding and visualization can support a deeper grasp of gradient meaning.

Overall, this first phase demonstrates that students could compute but not interpret. Without explicit prompts or visual tools, they remained anchored in procedural reasoning, which validates earlier findings by Borji (2022) and Cheong et al. (2023) that procedural fluency does not automatically translate into conceptual mastery (Borji et al., 2022; Cheong et al., 2023).

Transition from Symbolic Computation to Geometric Meaning

After the researchers introduced scaffolding strategies, students moved beyond numerical computation toward geometric meaning. The teacher encouraged them to see the gradient as a vector pointing toward the steepest ascent. This process of guided reinvention mirrors Sfard’s (2008) description of *reification*, where operational processes gradually become conceived as structural objects (Sfard, 1991).

For example, after further questioning, a student remarked, *“So, the arrow shows the fastest way up.”* What the student said illustrates a pivotal shift from perceiving the gradient as an inert ordered pair to recognizing it as a directional indicator. Rochaminah et al. (2025) emphasize that such discursive transitions are critical for advancing from procedural competence to conceptual understanding (Rochaminah et al., 2025).

The case of (2,8) at (1,2) was especially instructive. The researchers prompted students to interpret not only the numerical values but also the relative weights of each component. They concluded that the vector points northeast, with a stronger bias toward the *y*-axis due to the larger coefficient. This interpretation aligns with Cheong et al. (2023), who found that relative component magnitudes often served as an entry point for students in grasping geometric meaning (Cheong et al., 2023).

Discussions also emphasized physical interpretations. The researchers asked students to imagine the potential function representing an electric field. In this analogy, the gradient indicated the direction in which a test charge would experience the greatest force. This contextualization supported Medina Herrera et al. (2021), who argue that situating symbolic results in applied domains enhances engagement and meaning making (Herrera et al., 2024).

At this stage, evidence of conceptual change was visible in classroom transcripts. One student said, *“The 2 and 8 mean the function increases faster in y than in x.”* Such statements go beyond restating numerical results and reveal an emerging awareness of the gradient’s functional role.

This development was further supported by interview data. When asked to explain what the gradient vector (2, 8) represents, the following exchange occurred: Researcher: “Can you explain what the numbers 2 and 8 mean in your result?” Student A: “It means the surface goes up

faster in the y direction than in x. So, the arrow should be longer toward y."
Researcher: "How do you know it's the direction of the fastest increase?"
Student A: "Because when we move in that direction, the value of the function increases most quickly. GeoGebra showed the arrow pointing that way."
This excerpt illustrates how visualization and guided questioning facilitated the student's shift from procedural to geometric reasoning, marking the early stages of conceptual change.

This development illustrates Tall's (2008) assertion that students benefit from explicit opportunities to link symbolic expressions with graphical and contextual interpretations (Tall, 2008). The researchers facilitated the connection through structured questioning and prompts that required students to articulate their thinking.

GeoGebra was later employed to reinforce this connection. Visualizing the vector (2,8) at (1,2) provided immediate geometric confirmation of analytic calculations. Figure 1 shows a screenshot of the vector superimposed on the graph of the potential function. This use of technology resonates with Medina Herrera et al. (2021), who emphasize the value of dynamic software in bridging representational gaps (Herrera et al., 2024).

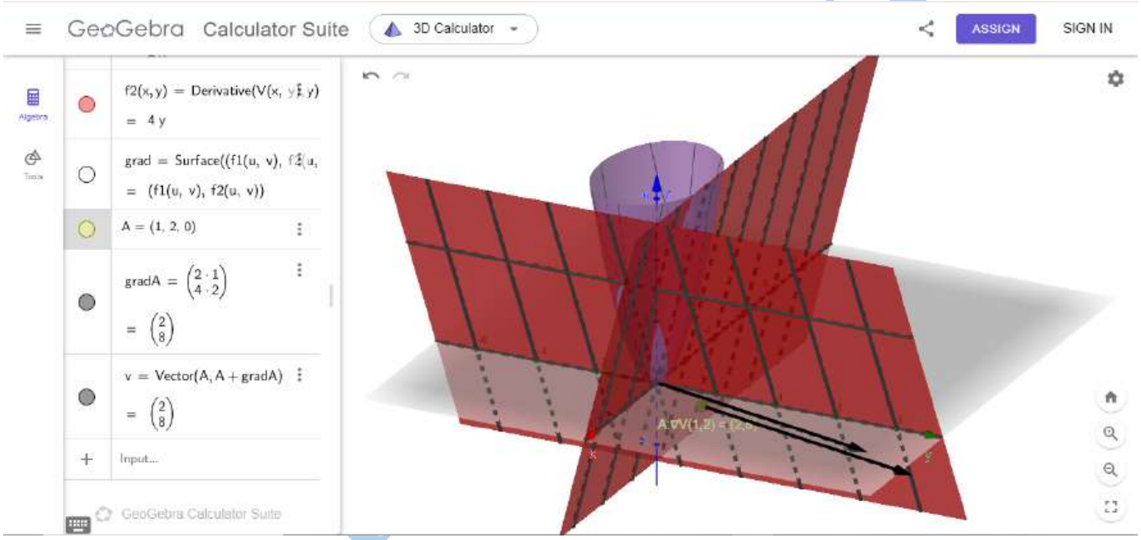

Figure 3. Geogebra Visualization Of The Gradient Vector (2,8) Anchored At Point (1,2) On The Surface $V(x,y) = x^2 + 2y^2$, Illustrating The Direction Of Steepest Ascent

Figure 4. Students Exploring Mathematics Concepts With Geogebra On Mobile Phones

Such visual aids confirmed symbolic computations and expanded students' reasoning. The visual of a steep arrow pointing northeast made the abstract idea of “steepest ascent” concrete. Sung et al. (2025) highlight that such multimodal engagements promote deeper conceptualization (Sung & Nathan, 2025; Yeh et al., 2025).

By the end of this phase, students were increasingly able to articulate the meaning of gradients in symbolic and geometric terms. Increased students demonstrate the potential of scaffolding and visualization in fostering representational integration.

The transition from symbolic to geometric reasoning marked a critical conceptual breakthrough. Students no longer viewed the gradient as a final numerical product but as an active object that connects algebraic, graphical, and physical dimensions. This evolution of students’ reasoning exemplifies conceptual change, defined as the restructuring of prior mental models into more coherent scientific understanding (Vosniadou et al., 2008). Students moved from perceiving gradients as isolated numeric results to viewing them as integrated geometric and physical entities.

Difficulties in Visualizing Gradient Vectors

Despite notable progress, visualization remained a challenge for many students. Several could describe the meaning of (2,8) but hesitated to represent it graphically. Some drew arrows incorrectly, while others avoided sketching altogether. This difficulty echoes Moreno (2021), who reported persistent struggles with translating symbolic gradients into graphical form (Moreno-Arotzena et al., 2021).

One common error involved placing the vector at the origin instead of at the point (1,2). The error of the activity suggests a limited understanding of gradient vectors as *anchored* at specific points on the surface. Similar misconceptions were identified by Cheong et al. (2023), who observed that students often treated gradient vectors as free-floating arrows (Cheong et al., 2023).

At this point, transcript evidence further illustrated this difficulty.

Researcher: “Where should the arrow start?”

Student B: “From zero, I think, because it’s a vector.”

Researcher: “What happens if you start it at (1,2) instead?”

Student B: “Oh, that makes more sense, it shows the slope at that point!”This short exchange demonstrates how guided scaffolding and visualization in GeoGebra helped students reconstruct meaning through interaction, a clear instancce of guided reinvention (Freudenthal, 1991; Gravemeijer & Cobb, 2006), where conceptual understanding emerges from exploration rather than direct instruction.

Another difficulty was scaling. Students who attempted sketches sometimes drew the components with incorrect proportions, e.g., equal lengths for 2 and 8. This indicates partial recognition of directionality but incomplete attention to magnitude. Bollen et al. (2017) caution that without attention to scale, students risk losing the quantitative meaning of vectors (Bollen et al., 2017; Dubey & Barniol, 2023; Sabah, 2023)(Bollen et al., 2017; Dubey & Barniol, 2023; Sabah, 2023). To address these issues, the class used GeoGebra's vector tools to plot (2,8) accurately at (1,2). Figure 3 illustrates how this technology provided precise positioning and scaling. Such scaffolding resonates with Medina Herrera et al. (2021), who show that dynamic software reduces cognitive load and reinforces representational accuracy (Herrera et al., 2024).

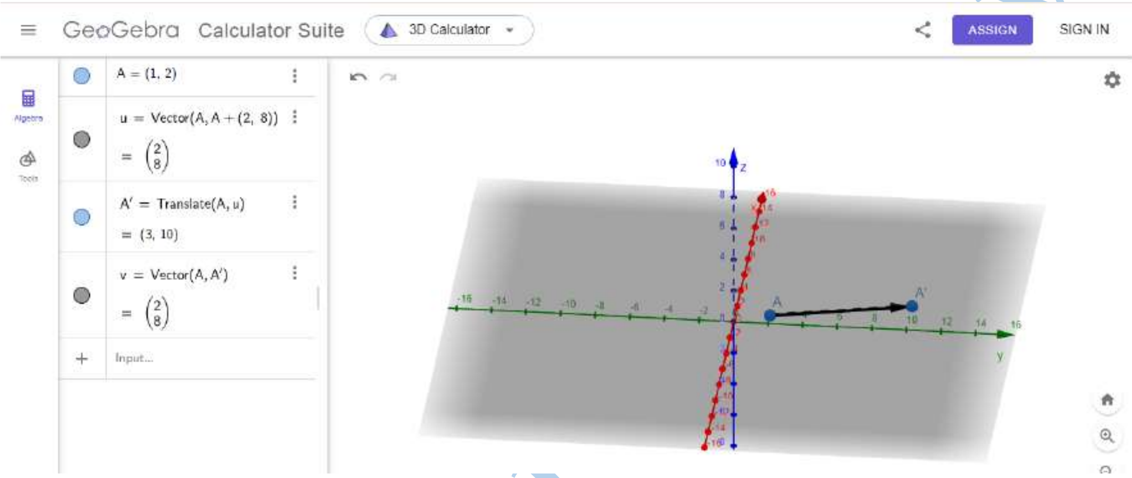

Figure 5. Geogebra Plot Of Gradient Vector (2,8) Anchored At (1,2)

Transcript evidence also showed that visualization helped students reconcile earlier errors. After seeing the GeoGebra output, one student said, *"Oh, the arrow is not from zero, but from (1,2)."* The student said that visualization supported correcting misconceptions and facilitated representational alignment. Nevertheless, reliance on technological scaffolding revealed another challenge: students struggled to visualize vectors without software support. Students' struggles to visualize vectors without software support suggest that internalization of geometric reasoning requires repeated practice, beyond one-time demonstrations. As Sfard (1991) warns that without reification, students may remain dependent on external representations (Sfard, 1991).

Class discussions further emphasized the need to interpret vector length. During this phase, students were guided to compute its magnitude, $\sqrt{2^2+8^2}=\sqrt{68}$, and to relate it to the rate of change of the surface. This activity addressed earlier gaps where students equated the gradient merely with "slope," without recognizing its multidimensional meaning. A classroom transcript captures this moment: Student A: *"Because when we move in that direction, the value of the function increases most quickly."*

This statement reflects students' growing awareness that magnitude encodes the rate of change, evidence of conceptual reorganization in progress. Integrating magnitude into interpretation led students to view the gradient as a holistic object with both geometric and physical significance. This shift corresponds to the reification process described by Sfard (1991), in which operational procedures become structural mathematical objects (Sfard, 1991).

Likewise, this progression aligns with Tall's (2002, 2004) framework of moving from fragmented knowledge toward connected understanding through multiple representationsClick or tap here to enter text. (Tall, 2002, 2004). In the context of this teaching experiment, GeoGebra functioned as a mediating tool supporting this transition. Although visualization difficulties persisted, the combination of symbolic computation, guided scaffolding, and dynamic software effectively mediated reasoning and supported the development of conceptual insight.

Overall, the retrospective analysis provides concrete empirical evidence that systematic scaffolding and dynamic visualization can transform procedural reasoning into a richer conceptual understanding of gradient vectors, illustrating guided reinvention and conceptual change in advanced calculus learning.

While these findings are consistent with previous studies (Cheong et al., 2023; Herrera et al., 2024), this research extends existing knowledge by uncovering the mechanisms underlying students' conceptual change in multivariable calculus. The data suggest that conceptual restructuring did not occur solely because of GeoGebra's visual affordances, but through cycles of guided reflection, questioning, and reinterpretation of prior misconceptions. In particular, the shift from viewing the gradient as a mere numerical output to understanding it as a geometric and physical entity illustrates the transformation of operational knowledge into structural understanding, a process central to conceptual change (Sfard, 1991; Vosniadou et al., 2008). This study therefore moves beyond confirming GeoGebra's effectiveness and contributes theoretically by demonstrating that meaningful conceptual change requires the interplay of visualization, guided reinvention, and metacognitive engagement, rather than visualization alone.

Despite these contributions, several limitations of the study should be acknowledged. The small number of participants ($n = 8$) restricts the generalizability of the findings to broader populations. The short duration of the teaching experiment may also limit insights into the long-term stability of students' conceptual change. Furthermore, the exclusive use of GeoGebra as the visualization tool might have shaped the learning dynamics in specific ways. Future research should therefore involve larger and more diverse participant groups, extend the intervention period, and compare different visualization or instructional frameworks to provide a more comprehensive understanding of how conceptual change evolves and endures in advanced mathematics learning.

CONCLUSION[A7][D8]

This study identified persistent conceptual difficulties among students in interpreting gradients and tangent planes. Initially, students demonstrated strong procedural fluency in computing partial derivatives but failed to connect symbolic results with geometric meaning. These findings confirm the central issue highlighted in the study's title and established a basis for examining conceptual change.

Through iterative GeoGebra-assisted instruction and guided reflection, students gradually reconstructed their understanding, shifting from viewing gradients as mere numerical results to recognizing them as vectors representing both direction and rate of change. This transformation exemplifies conceptual change as the reorganization of prior mental models into coherent conceptual structures. The study extends the Conceptual Change framework by showing that lasting conceptual transformation requires not only visualization but also guided reinvention, metacognitive engagement, and representational integration.

Practically, the findings suggest that instructional design in advanced calculus should integrate dynamic visualization with reflective discussion and scaffolding tasks to promote representational understanding rather than procedural repetition.

However, the study's limitations include its small participant group ($n = 8$), brief intervention period, and focus on a single digital platform (GeoGebra). Future research should adopt longitudinal and multicontext investigations to explore how conceptual change develops and stabilizes across varied technological and instructional environments.

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AUTHOR CONTRIBUTIONS

Author One contributed to the conceptualization, methodology, formal analysis, and writing of the original draft. **Author Two** was responsible for data curation, investigation, and writing, review & editing. **Author Three** contributed to software, supervision, validation, project administration, and funding acquisition.

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4. Bukti Konfirmasi Artikel Accepted (24 Oktober 2025)

[Koordinat] Editor Decision

2025-10-24 05:16 AM

Dini Andiani, Siti Dwi Rahayu, Cecep Suwanda:

We have reached a decision regarding your submission to Koordinat Jurnal MIPA, "EXPLORING STUDENTS’ CONCEPTUAL DIFFICULTIES AND CONCEPTUAL CHANGE IN MULTIVARIABLE CALCULUS: A TEACHING-EXPERIMENT AND RETROSPECTIVE ANALYSIS ".

Our decision is to: Accept Submission

Rafiq Badjeber
State Islamic University Datokarama Palu
rafiqbadjeber@uindatokarama.ac.id

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5. Bukti Konfirmasi Artikel Published Online

(13 Desember 2025)

[Koordinat] Editor Decision

2025-12-12 11:18 PM

Dini Andiani, Siti Dwi Rahayu, Cecep Suwanda:

The editing of your submission, "EXPLORING STUDENTS’ CONCEPTUAL DIFFICULTIES AND CONCEPTUAL CHANGE IN MULTIVARIABLE CALCULUS: A TEACHING-EXPERIMENT AND RETROSPECTIVE ANALYSIS ," is complete. We are now sending it to production.

Submission URL: https://www.jurnalkoordinat.org/index.php/koordinat/authorDashboard/submission/181

Nursupiamin Nursupiamin
Institut Agama Islam Negeri Palu
nursupiamin@gmail.com

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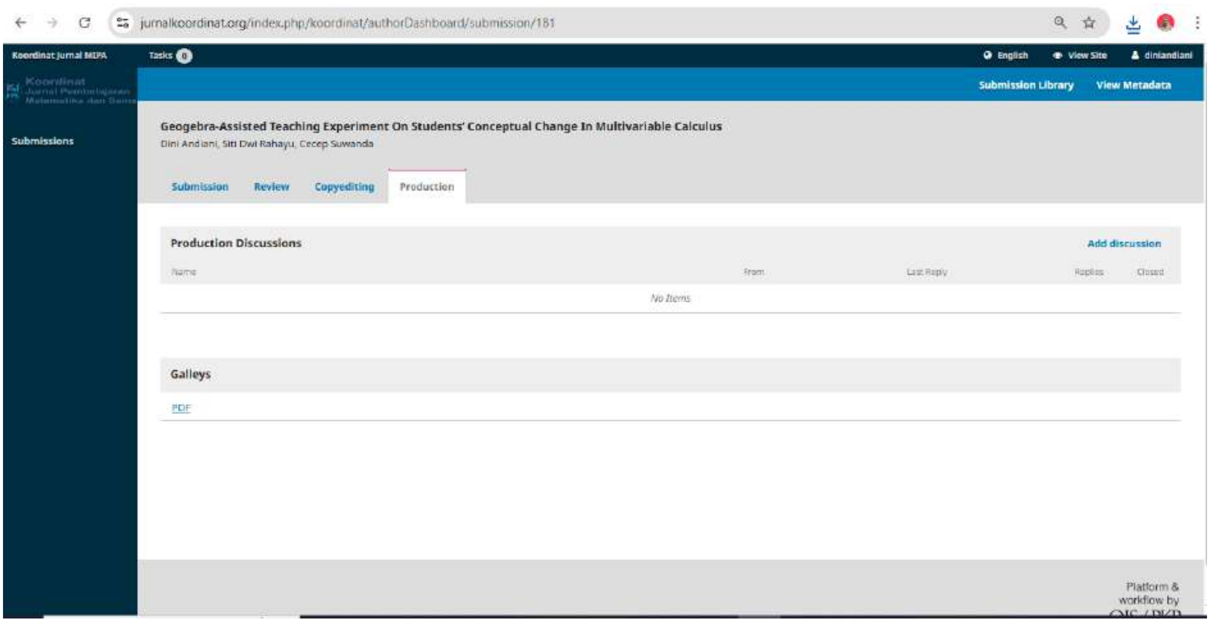

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