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EXPLORING STUDENTS' CONCEPTUAL DIFFICULTIES AND CONCEPTUAL CHANGE IN MULTIVARIABLE CALCULUS: A TEACHING-EXPERIMENT AND RETROSPECTIVE ANALYSIS

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ABSTRACT

This study investigates the effectiveness of GeoGebra-assisted learning in enhancing students' conceptual understanding of multivariable calculus through a teaching experiment design. The research involved undergraduate students enrolled in a multivariable calculus course and was carried out in two iterative stages. Data were collected using diagnostic tests, observation sheets, and reflective notes, while analysis included coding of students' responses, identification of misconceptions, and triangulation of test results, classroom transcripts, and task products.

The findings revealed that, at the outset, students could perform symbolic computations of partial derivatives but often stopped at numerical results without interpreting them as components of a gradient vector. They also experienced difficulties visualizing the geometric meaning of the gradient and its relation to the tangent plane. After introducting of GeoGebra-based visualizations and guided scaffolding, students began to describe gradient components as rates of change and connected them to the direction of steepest ascent. Observations showed that students engaged more and participated actively in collaborative discussions using dynamic visualization tools.

The study demonstrates that GeoGebra-assisted learning, implemented within a teaching experiment framework, can effectively bridge the gap between procedural fluency and conceptual understanding in multivariable calculus. The study contributes to mathematics education by highlighting the role of technological integration and reflective instructional design in addressing students' persistent misconceptions about gradients and tangent planes.

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INTRODUCTION

Multivariable calculus is a core subject in undergraduate mathematics and science programs, playing a crucial role in developing students' higher-order thinking, spatial reasoning, and modeling skills. Despite its importance, many students face persistent difficulties when transitioning from single-variable to multivariable contexts, particularly in understanding abstract concepts such as partial derivatives, gradients, and tangent planes (Borji et al., 2024; Hahn & Klein, 2025; Kashefi et al., 2011; Rodríguez-Nieto & Moll, 2025). These difficulties often stem from an overreliance on procedural computation, with limited ability to integrate algebraic procedures with geometric interpretation (Borji et al., 2024; Cheong et al., 2023; Hahn & Klein, 2025). As a result, students may compute derivatives correctly but struggle to articulate the conceptual significance of gradients or interpret the geometric meaning of tangent planes.

Mathematics education researchers have highlighted the importance of instructional strategies that foster conceptual understanding alongside procedural fluency. Conceptual growth involves transforming operational procedures into structural objects, enabling students to reason about mathematical entities more flexibly (Andiani et al., 2024, 2025; E. D. Dubinsky & Mcdonald, 2001; E. Dubinsky & Mcdonald, 2001; Matindike, 2025; Shvarts et al., 2024; Trigueros et al., 2024; Villabona et al., 2024). In this regard, technology enhanced learning environments, particularly dynamic visualization tools such as GeoGebra, have been shown to support students' conceptual development by linking symbolic computations with graphical and interactive representations (Baye et al., 2021; Gurmu et al., 2024; Suparman et al., 2024). Such tools serve as mediating devices that can bridge students' fragmented understanding of multivariable functions, gradients, and tangent planes (Borji et al., 2022; Cheong et al., 2024; Herrera et al., 2024; Schoenherr et al., 2024). In particular, the doctoral thesis *Local Instruction Theory Perbandingan Trigonometri dengan Pendekatan Matematika Realistik untuk Mengembangkan Kemampuan Representasi Matematis* (Andiani et al., 2025) demonstrates that an LIT-based RME approach with realistic contexts (*bayangan*, *kemiringan tangga*, *posisi mata terhadap objek*) significantly enhances students' mathematical representation abilities in the trigonometric ratios compared to purely procedural learning.

Several studies have demonstrated the benefits of technology in calculus instruction. For example, Schoenherr found that interactive visualizations improved students' reasoning about multivariable functions (Cheong et al., 2023; Herrera et al., 2024; Rao et al., 2023; Schoenherr et al., 2024), while Karabay (2025) highlighted the role of scaffolding in managing cognitive load during problem solving (Faber et al., 2024; Karabay & Meşe, 2025; Li et al., 2023; van Nooijen et al., 2024). However, much of the existing research emphasizes quantitative outcomes, such as test performance, with limited attention to qualitative aspects of conceptual change or the dynamics of classroom interactions. Additionally, few studies have examined how iterative teaching experiment designs can trace students' conceptual difficulties and the pathways of conceptual change over time.

The novelty of this study lies in combining a teaching experiment approach with retrospective analysis to explore students' conceptual difficulties and subsequent changes in understanding gradients and tangent planes through GeoGebra-assisted learning. Teaching experiments enable iterative refinement of instructional interventions while closely observing students' reasoning (Kelly & Lesh, 2000). Retrospective analysis allows systematic identification of shifts in conceptual understanding across multiple data sources.

Previous research by Andiani et al.,(2020) demonstrated that students face challenges in solving geometric problems on competency-based assessments, highlighting difficulties in understanding and reasoning mathematically (Andiani et al., 2020). Furthermore, Andiani et al., (2025) found that realistic mathematics education approaches can improve students' mathematical representation skills, essential in multivariable calculus contexts (Andiani et al., 2025). These findings suggest that explicit scaffolding and visualization tools are crucial for fostering conceptual understanding.

Therefore, this research focuses on exploring students' conceptual difficulties and conceptual change in multivariable calculus through a qualitative teaching experiment approach supported by GeoGebra. The main aim is to describe how students develop an understanding of gradients and tangent planes when exposed to visualization-based instruction and to analyze how this process bridges the gap between procedural fluency and conceptual comprehension. Recent studies have confirmed that GeoGebra-assisted visualizations can significantly enhance students' conceptual understanding and engagement in multivariable calculus and related mathematical topics (Abdurrahman Wahid Pekalongan et al., n.d.; Bedada & Machaba, 2022; Cheong et al., 2024).

METHOD

This study employed a qualitative descriptive design through a teaching experiment and retrospective analysis to investigate students' conceptual difficulties and conceptual change in multivariable calculus. The researchers chose this design because it allowed them to explore students' thinking processes and capture how their understanding evolved during the learning activities.

The participants were eight undergraduate students enrolled in a multivariable calculus course at a mathematics education program in West Java, Indonesia. Participants were selected purposively based on their prior exposure to basic calculus concepts and their availability to participate in the entire teaching experiment cycle. The small group size was intended to enable intensive observation of students' learning processes.

The instruments used in this study included learning tasks in the form of problem sets related to the concept of limits in two variables and GeoGebra-based activities designed to promote visualization and conceptual exploration. The researchers employed observation sheets to document classroom interactions and student strategies, while semi-structured interview guidelines were used to explore students' reasoning, misconceptions, and conceptual changes in greater depth.

The procedures consisted of two stages of teaching experiments followed by retrospective analysis. Each teaching experiment involved introducing learning tasks, classroom interaction supported by GeoGebra visualization, and reflection activities. The researchers collected data through diagnostic tests, classroom transcripts, students' written work, and reflective notes.

Data analysis was conducted through coding students' responses, identifying recurring misconceptions, and comparing test results, classroom observations, and students' task products. Triangulation between these data sources was applied to ensure the credibility and validity of the findings. The researchers conducted the analysis iteratively, allowing them to refine their interpretations and connect the results with the theoretical framework on conceptual understanding in multivariable calculus.

RESULT AND DISCUSSION

Students' Initial Conceptual Difficulties

At the beginning of the task, most students successfully computed the partial derivatives of the potential function $V(x, y) = x^2 + 2y^2$, yielding the gradient $\nabla V(x, y) = (2x, 4y)$. When evaluated at the point (1,2), this became $\nabla V(1,2) = (2,8)$. However, their written responses tended to stop at the numerical level, showing little evidence of conceptual interpretation. This finding resonates with Borji (2022), who noted that many students demonstrate strong procedural skills while lacking structural understanding in multivariable calculus (Borji et al., 2022).

A recurring pattern emerged where students simply reported, *"gradient* = (2,8)*, finished."* Such utterances illustrate what Sfard (2008) describes as *operational thinking* treating mathematical objects as results of procedures rather than entities with independent meaning (Sfard, 1991). Students in this stage failed to recognize that the gradient is more than just a pair of numbers; it is a vector with both magnitude and direction.

When pressed for further interpretation, some students attempted to describe the result in vague terms, such as "the slope in x is two and in y is 8," but without elaborating on its vectorial nature. This suggests that symbolic results were fragmented rather than integrated into a coherent conceptual framework. Cheong et al. (2023) also reported similar tendencies in their study, where students often compartmentalized partial derivatives without connecting them to vectorial interpretation (Cheong et al., 2023).

This difficulty is particularly important in vector calculus, where gradients play a central role in understanding directional change, optimization, and physical phenomena. Without conceptual grounding, students cannot connect gradients to applied contexts, such as interpreting potential fields in physics or optimization problems in engineering. Medina Herrera et al. (2021) highlight that such gaps hinder the transition from abstract calculation to practical application (Herrera et al., 2024).

A closer analysis of written work confirmed this gap. While algebraic manipulations were accurate, annotations explaining the meaning of the results were rare. Students did not spontaneously refer to the gradient as the direction of steepest ascent or discuss its physical implications in the context of a potential function. Students did not spontaneously refer to the gradient as the direction of steepest ascent or discuss its physical implications in the context of a potential function, as confirmed by Abreu et al. (2021), who emphasize that symbolic fluency alone is insufficient without scaffolding that links symbols to concepts (Abreu-Mendoza et al., 2021).

In retrospective discussions, some students admitted being unsure how to proceed after obtaining the ordered pair. They expressed uncertainty about substituting values for the original function, drawing a graph, or leaving the result. Such hesitation illustrates the lack of an internalized schema for interpreting gradient vectors.

These findings underscore the necessity of carefully designed instructional interventions. As Tall (2008) argues, advanced mathematical thinking requires students to move beyond computational comfort zones and integrate multiple representations (Tall, 2008). Here, the initial failure to articulate conceptual meaning highlights the importance of explicitly embedding interpretation tasks in calculus instruction.

Table 1 summarizes students' initial approaches to illustrate the distribution of responses. Categories include purely procedural responses (e.g., numerical computation only), partial interpretations (mentioning "slope" without directionality), and meaningful interpretations (rare at this stage).

Table 1. Categories of student responses to the gradient task at (1,2)

Response Type	**Example Student Statement**	**Frequency (N=20)**
Procedural only	"Gradient = (2,8), finished"	3
Partial interpretation	"Slope in x is 2, in y is 8"	0

Response Type	Example Student Statement	Frequency (N=20)
Full interpretation	"The vector (2,8) shows the direction of steepest increase at (1,2); its rate of increase per unit length is $2\sqrt{17}$."	5

The predominance of procedural responses highlights the extent to which conceptual understanding was initially lacking. Such a distribution sets the stage for examining how scaffolding and visualization can support a deeper grasp of gradient meaning.

Overall, this first phase demonstrates that students could compute but not interpret. Without explicit prompts or visual tools, they remained anchored in procedural reasoning, which validates earlier findings by Borji (2022) and Cheong et al. (2023) that procedural fluency does not automatically translate into conceptual mastery (Borji et al., 2022; Cheong et al., 2023).

Transition from Symbolic Computation to Geometric Meaning

After the researchers introduced scaffolding strategies, students moved beyond numerical computation toward geometric meaning. The teacher encouraged them to see the gradient as a vector pointing toward the steepest ascent. This process of guided reinvention mirrors Sfard's (2008) description of *reification*, where operational processes gradually become conceived as structural objects (Sfard, 1991).

For example, after further questioning, a student remarked, *"So, the arrow shows the fastest way up."* What the student said illustrates a pivotal shift from perceiving the gradient as an inert ordered pair to recognizing it as a directional indicator. Rochaminah et al. (2025) emphasize that such discursive transitions are critical for advancing from procedural competence to conceptual understanding (Rochaminah et al., 2025).

The case of (2,8) at (1,2) was especially instructive. The researchers prompted students to interpret not only the numerical values but also the relative weights of each component. They concluded that the vector points northeast, with a stronger bias toward the *y*-axis due to the larger coefficient. This interpretation aligns with Cheong et al. (2023), who found that relative component magnitudes often served as an entry point for students in grasping geometric meaning (Cheong et al., 2023).

Discussions also emphasized physical interpretations. The researchers asked students to imagine the potential function representing an electric field. In this analogy, the gradient indicated the direction in which a test charge would experience the greatest force. This contextualization supported Medina Herrera et al. (2021), who argue that situating symbolic results in applied domains enhances engagement and meaning making (Herrera et al., 2024).

At this stage, evidence of conceptual change was visible in classroom transcripts. One student said, *"The 2 and 8 mean the function increases faster in y than in x."* Such statements go beyond restating numerical results and reveal an emerging awareness of the gradient's functional role.

This development illustrates Tall's (2008) assertion that students benefit from explicit opportunities to link symbolic expressions with graphical and contextual interpretations (Tall, 2008). The researchers facilitated the connection through structured questioning and prompts that required students to articulate their thinking.

GeoGebra was later employed to reinforce this connection. Visualizing the vector (2,8) at (1,2) provided immediate geometric confirmation of analytic calculations. Figure 1 shows a screenshot of the vector superimposed on the graph of the potential function. This use of technology resonates with Medina Herrera et al. (2021), who emphasize the value of dynamic software in bridging representational gaps (Herrera et al., 2024).

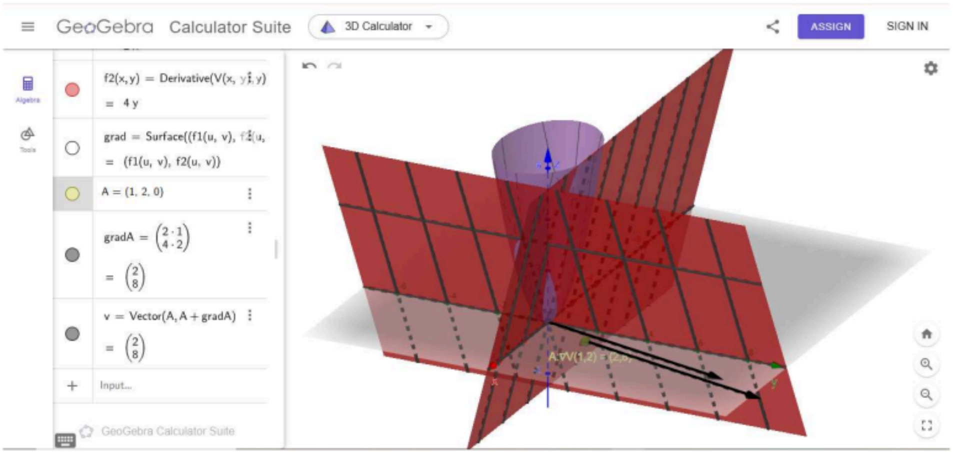

Figure 1. GeoGebra visualization of the gradient vector (2,8) at point (1,2)

Figure 2. Students exploring mathematics concepts with GeoGebra on mobile phones

Such visual aids confirmed symbolic computations and expanded students' reasoning. The visual of a steep arrow pointing northeast made the abstract idea of "steepest ascent" concrete. Sung et al. (2025) highlight that such multimodal engagements promote deeper conceptualization (Sung & Nathan, 2025; Yeh et al., 2025).

By the end of this phase, students were increasingly able to articulate the meaning of gradients in symbolic and geometric terms. Increased students demonstrate the potential of scaffolding and visualization in fostering representational integration.

The transition from symbolic to geometric reasoning marked a critical conceptual breakthrough. Students no longer viewed the gradient as a final numerical product but as an active object that connects algebraic, graphical, and physical dimensions.

Difficulties in Visualizing Gradient Vectors

Despite notable progress, visualization remained a challenge for many students. Several could describe the meaning of (2,8) but hesitated to represent it graphically. Some drew arrows incorrectly, while others avoided sketching altogether. This difficulty echoes Moreno (2021), who reported persistent struggles with translating symbolic gradients into graphical form (Moreno-Arotzena et al., 2021).

One common error involved placing the vector at the origin instead of at the point (1,2). The error of the activity suggests a limited understanding of gradient vectors as *anchored* at specific points on the surface. Similar misconceptions were identified by Cheong et al. (2023), who observed that students often treated gradient vectors as free-floating arrows (Cheong et al., 2023).

Another difficulty was scaling. Students who attempted sketches sometimes drew the components with incorrect proportions, e.g., equal lengths for 2 and 8. This indicates partial recognition of directionality but incomplete attention to magnitude. Bollen et al. (2017) caution that without attention to scale, students risk losing the quantitative meaning of vectors (Bollen et al., 2017; Dubey & Barniol, 2023; Sabah, 2023).

To address these issues, the class used GeoGebra's vector tools to plot (2,8) accurately at (1,2). Figure 3 illustrates how this technology provided precise positioning and scaling. Such scaffolding resonates with Medina Herrera et al. (2021), who show that dynamic software reduces cognitive load and reinforces representational accuracy (Herrera et al., 2024).

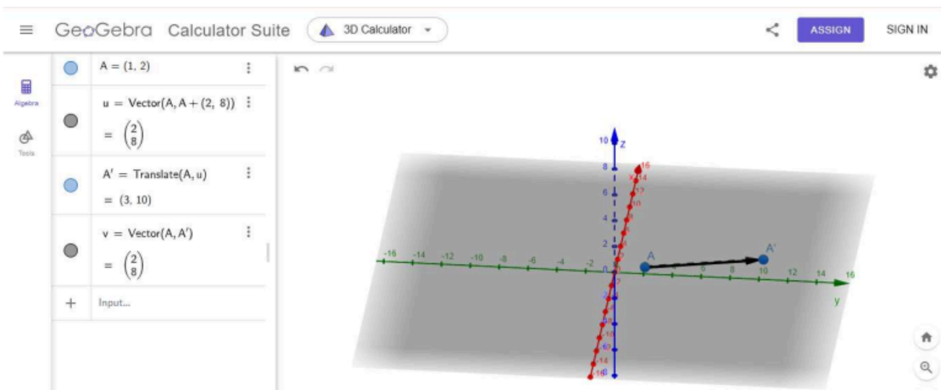

Figure 3. GeoGebra plot of gradient vector (2,8) anchored at (1,2)

Transcript evidence also showed that visualization helped students reconcile earlier errors. After seeing the GeoGebra output, one student said, *"Oh, the arrow is not from zero, but from (1,2)."* The student said that visualization supported correcting misconceptions and facilitated representational alignment. Nevertheless, reliance on technological scaffolding revealed another challenge: students struggled to visualize vectors without software support. Students' struggles to visualize vectors without software support suggest that internalization of geometric reasoning requires repeated practice, beyond one-time demonstrations. Sfard (1991) warns that without reification, students may remain dependent on external representations (Sfard, 1991).

Class discussions further emphasized the need to interpret vector length. The researchers guided students to compute the magnitude $\sqrt{2^2+8^2}=\sqrt{68}$, reinforcing that the gradient encodes direction and rate of change. This mistakes addressed earlier gaps where students only equated the gradient with slope, without recognizing its multidimensional meaning.

By integrating magnitude into interpretation, students began to view the gradient as a holistic object with geometric and physical significance. Tall (2002) notes that such integration is essential for moving from fragmented knowledge to connected understanding (Tall, 2002, 2004). Ultimately, while difficulties in visualization persisted, the combination of symbolic computation, guided scaffolding, and dynamic software fostered meaningful progress. The retrospective analysis suggests that systematic support can gradually shift students from procedural only reasoning to rich conceptualization of gradients as vectors.

CONCLUSION

This study investigated students' understanding of gradient vectors in the context of the potential function $V(x,y) = x^2 + 2y^2$. The results indicate that, although most students could correctly compute partial derivatives and obtain $\nabla V(x,y) = (2x, 4y)$, many initially stopped at procedural manipulation without conceptual articulation. For instance, when students responded with 'gradient = (4,13), done,' it revealed that they had not yet interpreted the numerical result as a directional vector with meaning in the context of surface growth. Through guided scaffolding and the use of GeoGebra visualizations, students gradually transitioned from symbolic computation to a structural understanding of the gradient vector as representing the direction of steepest ascent and its magnitude as the rate of change per unit distance (Sfard, 1991; Tall, 2002, 2004).

The implications of this study suggest that integrating dynamic visualization tools such as GeoGebra can play a crucial role in bridging the gap between procedural fluency and conceptual understanding in multivariable calculus. By anchoring gradient vectors at specific points, students observed the direction and connected the symbolic form $\nabla V(1,2) = (2,8)$ with its geometric representation. This approach facilitated the reification process described by Sfard (2008), helping students to perceive the gradient vector as both a computational output and a conceptual object (Sfard, 1991). The findings are consistent with previous studies highlighting the need for explicit tasks that link algebraic representations with visual and contextual meaning (Borji et al., 2022; Cheong et al., 2023).

Nevertheless, the study has some limitations. The number of participants was relatively small, and the analysis focused primarily on a single problem involving gradient interpretation. Broader exploration across different surfaces and longitudinal data, would strengthen the conclusions and provide deeper insights into how students develop an integrated understanding of gradients in multivariable contexts. Future research could expand on these findings by designing tasks that explicitly connect gradient concepts with applications in physics and engineering, thereby enhancing the relevance and transferability of students' learning (Herrera et al., 2024).

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AUTHOR CONTRIBUTIONS

Author One contributed to the conceptualization, methodology, formal analysis, and writing of the original draft.

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Author Two was responsible for data curation, investigation, and writing, review & editing. **Author Three** contributed to software, supervision, validation, project administration, and funding acquisition.

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