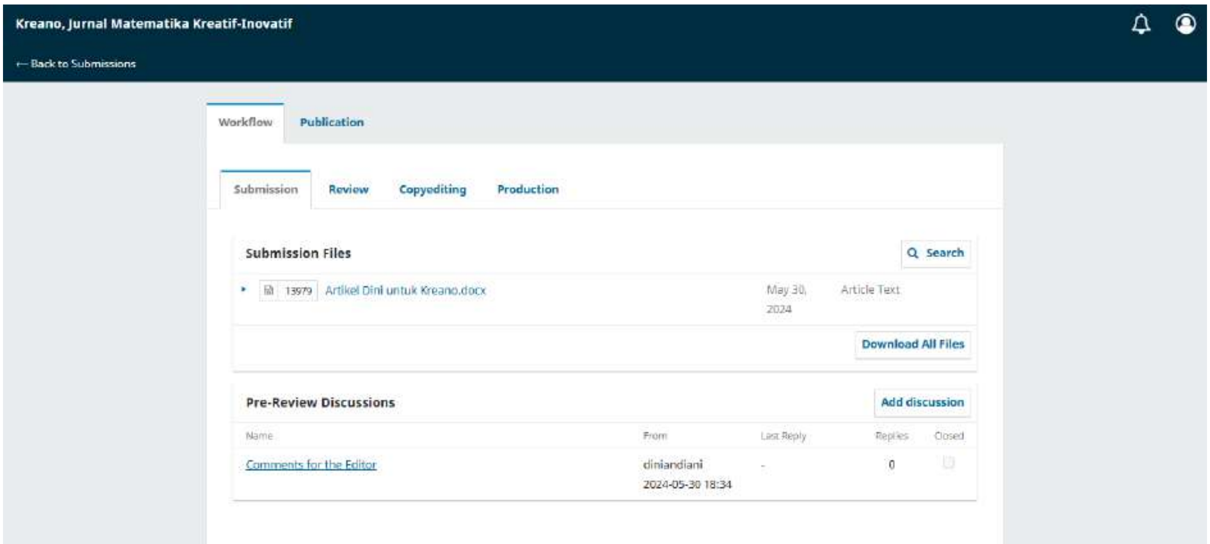
BUKTI KORESPONDENSI

ARTIKEL JURNAL INTERNASIONAL BEREPUTASI

Judul Artikel	:	**Development of a Local Instruction Theory for Trigonometric Ratios**
Jurnal	:	Kreano, Jurnal Matematika Kreatif-Inovatif
Penulis	:	Dini Andiani, Elah Nurlaelah, and Darhim

No.	Perihal	Tanggal
1.	Bukti konfirmasi submit artikel dan artikel yang disubmit	30 Mei 2024
2.	Bukti konfirmasi review dan hasil review pertama	21 November 2024
3.	Bukti konfirmasi submit revisi pertama, respon kepada reviewer, dan artikel yang diresubmit	24 November 2024
4.	Bukti konfirmasi artikel accepted	26 November 2024
5.	Bukti konfirmasi artikel published online	19 Desember 2024

1. Bukti Konfirmasi Submit Artikel dan Artikel Yang Disubmit (30 Mei 2024)



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Development of A Local Instruction Theory for Trigonometric Ratios

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Abstract

Several students view the concept of trigonometric ratios as intricate. This work aims to create the Local Instruction Theory (LIT) for instructing trigonometric ratios utilizing Realistic Mathematic Education (RME). Researchers developed a Learning Instruction Theory (LIT) to aid students in constructing a robust comprehension of the fundamental principles of trigonometric ratios. They accomplished this by identifying a highly efficient learning trajectory. The study approach utilizes a specific research design. The development of the Hypothetical Learning Trajectory (HLT) is now underway to improve students' comprehension of trigonometric ratios. The development of local instruction theory for trigonometric ratios involves three stages: formulating an initial plan, carrying out teaching experiments, and doing retrospective analysis. Through qualitative analysis of teaching experiments, it has been determined that incorporating instructional materials on trigonometric ratios in LIT that align with RME identities effectively assists students in developing fundamental concepts in trigonometric ratios.

Keywords: Local Instruction Theory; Hypothetical Learning Trajectory; Trigonometric Ratios; RME

Abstrak

Beberapa siswa menganggap materi perbandingan trigonometri cukup sulit. Penelitian ini bertujuan untuk mengembangkan Local Instruction Theory (LIT) untuk pembelajaran perbandingan trigonometri dengan menggunakan Realistic Mathematic Education (RME). Dalam upaya membantu siswa membangun konsep dasar pada materi perbandingan trigonometri, peneliti mengembangkan alur LIT dengan menemukan jalur pembelajaran yang efektif. Pencapaian tujuan penelitian menggunakan desain penelitian. Serangkaian kegiatan mengembangkan Hypothetical Learning Trajectory (HLT) sehingga siswa sekolah menengah atas (SHS) memiliki pemahaman yang lebih baik tentang perbandingan trigonometri. Pengembangan Local Instruction Theory untuk Perbandingan Trigonometri meliputi tiga tahap yaitu mengembangkan desain awal, melakukan percobaan pengajaran, dan melaksanakan analisis retrospektif. Siswa mampu membangun pemahaman tentang perbandingan trigonometri selama proses pembelajaran berlangsung. Berdasarkan analisis kualitatif eksperimen pengajaran, penggunaan bahan ajar LIT perbandingan trigonometri yang mengakomodir identitas RME, mampu memandu siswa dalam membangun konsep dasar pada materi perbandingan trigonometri.

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INTRODUCTION

The topic of trigonometric ratios is fundamental in the application of advanced applied sciences and pure sciences. Various fields of study require the use of trigonometry as an essential basis for applying their concepts. For example, the utilization of trigonometric ratios supports the proof of Kepler's first law (Simha, 2021). The importance of understanding trigonometric ratios lies in its role as fundamental knowledge for comprehending pre-calculus and calculus, such as algebra, geometry, and the ability to create graphs (Weber, 2005). Trigonometric comparison is utilized in calculating the width or height of various shapes, such as a bridge, household tools, and bicycles (Urrutia et al., 2019). Indeed, students see the topic of trigonometric ratios as challenging (Blackett & Tall, 1991; Weber, 2005; Yiğit Koyunkaya, 2016; Karthikeyan, 2017; Urrutia et al., 2019; Nordlander, 2022). During the initial stage of learning, students encounter difficulties in connecting triangle images to numerical relationships, as well as problems in manipulating the involved symbols (Blackett & Tall, 1991), and many students are unable to overcome these difficulties. While studying right triangles, students struggled to grasp the concept of trigonometric functions, such as sine, cosine, and tangent, which are defined as ratios (Weber, 2005a, 2008). According to Yiğit (2016), students exhibit a lack of flexibility when solving trigonometric comparison problems (Yiğit Koyunkaya, 2016). Students expressed that trigonometry was a subject that was highly stimulating and conceptual in comparison to other areas of mathematics (Karthikeyan, 2017). According to Sandip's research, trigonometry is a challenging subject that is disconnected from everyday life (Urrutia et al., 2019). Typically, students struggle to distinguish the front side, side view, and slanted side of a right triangle despite the fact that this distinction greatly aids their comprehension of trigonometric ratios. The topic of trigonometric comparison is considered abstract for students (Karthikeyan, 2017).

From the preliminary study and literature review conducted by the researcher on textbooks, instructional videos, analysis of students' learning difficulties, and curriculum analysis, several aspects have caught the researcher's attention. One of these aspects is the emergence of real-world contexts in trigonometry materials in textbooks. Various examples have begun to present contextual situations, but teachers still need to learn how students might construct formal concepts from the presented contextual case. Batanero dan Diaz (2012) argue that the existing textbooks and curriculum documents need to sufficiently support learning, as the presented concepts are narrow in scope . (Batanero & Díaz, 2012).

The typical pattern of learning that occurs in the delivery of contextual mathematics is the initial introduction and presentation of example problems, followed by the completion of practice exercises using general formulas. The typical pattern of learning still represents the teaching culture in carrying out the learning process. According to Weber (2005), mathematics learning in schools still follows the conventional sequence of explanation, example provision, and problem-solving practice. The learning process via direct application of general formulas can prevent students from developing their mathematical concepts independently (Weber, 2005b). Their thinking patterns need to be trained, resulting in a lack of familiarity with how these formulas are derived.

The application of real-life models to formal mathematical concepts has yet to be documented in supporting learning materials. The teacher must possess the ability to elucidate the models

generated by students during the learning process, which can subsequently facilitate the comprehension of formal mathematics. Turmudi (2018) states that students should have a strong desire in the learning process to independently discover concepts, provided that teachers provide facilities, teaching materials, and learning resources (Turmudi, 2012).

According to Sandip's research, trigonometry is a challenging subject that is disconnected from everyday life (Adhikari & Subedi, 2021; Asomah et al., 2022; Dhungana et al., 2023). To bridge this gap, Realistic Mathematics Education serves as an alternative to teaching trigonometric ratios. Realistic Mathematics Education (RME) is a teaching and learning method in math education that follows the philosophy of Hans Freudenthal, who sees math as a human endeavor. (Freudenthal, 1973; Gravemeijer & Terwel, 2000; Treffers, 1987; Van den Heuvel-Panhuizen & Drijvers, 2020).

Realistic Mathematics Education is characterized by students experiencing a concrete learning process that is aligned with real-life situations and contextualized according to their everyday lives (Yuanita et al., 2018). Students undergo an informal process of constructing their understanding based on what they see, observe, and comprehend about their surrounding environment, be it at home, school, or wherever they learn (Meika, 2018).

The three primary ideas of Realistic Mathematics Education are guided reinvention and progressive mathematizing, didactical phenomenology, and self-developed models. (Gravemeijer, 1994). During the teaching and learning process, these three principles are elaborated into five characteristics of RME, namely: 1) Building and solidifying; 2) degree and frameworks; 3) contemplation and specific task; 4) societal environment and communication; and 5) organizing and intertwining (Gravemeijer, 1994; Jzn & de, 1987). The significance of the Realistic Mathematics Education approach resides in its ability to empower students to develop concepts through interactive teaching and learning processes. The teachers has a crucial role in enabling students to explore concepts using both administrative and motivational methods (Meika, 2018).

The mistakes made by previous students should not be repeated in the following years. Penyediaan materi pembelajaran adalah salah satu solusi yang tepat untuk memulihkan keadaan menjadi lebih baik. The focus of learning will be further narrowed down if the instructional materials used are appropriate. In this teaching material, steps are needed that can facilitate students' learning paths. According to Gravemeijer, to improve mathematics education, it is necessary to implement an instructional design that facilitates learning and enables students to progress from their current level of thinking to a more sophisticated level of mathematical thinking (Gravemeijer, 2004).

The Local Instruction Theory establishes the learning process by incorporating a learning trajectory. Local Instruction Theory (LIT) is a comprehensive theory that includes both the process of acquiring knowledge on a specific topic and the methods used to facilitate that learning (Liljekvist et al., 2017). (Gravemeijer, 2004) states that Local Instruction Theory is an exact theory that can assist students in developing mathematical thinking on a particular topic or material through detailed and gradual instructional mediumResearchers will develop a systematic learning path, also known as a learning trajectory, within the process of teaching and learning. This design is the one Gravemeijer mentioned.

LIT is used to describe the appropriate learning route or learning activities. LIT not only offers how learning takes place but also facilitates a range of specific learning activities that may work. The existence of LIT provides a basis for teachers to consider adaptations that may be needed in implementing new approaches. In addition, LIT offers a framework in which teachers can develop hypotheses about learning trajectories. LIT is tailored to the specific circumstances of the particular

class and teacher. Teachers need to build HLT (Hypothetical Learning Trajectories) that are suitable for their own situations while using LIT as a foundational reference (Gravemeijer, 2004).

Gravemeijer proposes a research design for the development of local instruction theory. The research design consists of three phases: 1) preliminary design (developing initial design), 2) teaching experiment (doing teaching experiment), and 3) retrospective analysis (performing retrospective analysis) (Gravemeijer, 2004). Preliminary design is the outcome of an initial learning plan aimed at implementing ideas about the development of instructional materials for LIT (Meika, 2018). This design is expected to support students in the construction of mathematical thinking from an informal form to a formal one. The construction of mathematical thinking is achieved through a guided reinvention process facilitated by mathematizing. Mathematizing refers to several ways of managing learning activities to demonstrate mathematical characteristics, such as generality, certainty, accuracy, and agility (Loc & Hao, 2016). The purpose of the teaching experiment is to evaluate the effectiveness of the instructional materials for LIT. Retrospective analysis is employed to examine the complete dataset of the teaching experiment outcomes.

HLT is applied in every learning activity and has a crucial role in designing learning activities for students. The execution of learning is indissociable from the learning trajectory, encompassing a deliberate progression of learning materials and idea maps that students traverse during the learning process (Prahmana, 2016). Next, in the subsequent stage, the trigonometric ratios LIT is produced.

According to the given definition, it is crucial to create and design instructional resources that can reduce students' challenges in developing cognitive processes through Realistic Mathematics Education in order to build comprehension of trigonometric concepts. The desired teaching materials are obtained through the LIT design. The LIT design is used to address the difficulties experienced by students and to enhance their mathematical thinking process, particularly their understanding of trigonometric ratios through Realistic Mathematics Education.

METHOD

A design research method based on Gravemeijer's approach is the research methodology used. This research method is focused on local instruction theory, or LIT, which was developed by researchers in collaboration with teachers to achieve better learning quality (Gravemeijer & van Eerde, 2009). A study paradigm known as "design research" tries to create a series of tasks and comprehend the workings of the learning process. This study aims to develop and construct a Local Instruction Theory (LIT) on the subject of trigonometric ratios, with a focus on Realistic Mathematics Education (RME).

The research implementation is carried out in two stages, with each stage referring to the steps of design research (Gravemeijer, 2004) namely: The three main stages of the project include 1) formulating the initial concept, 2) executing the educational experiment, and 3) carrying out a retrospective study. The subject of the study during the teaching experiment phase consisted of 25 tenth-grade students from SMAN 7, Bandung. An analysis was conducted on the activities performed in the initial phase, and subsequently, recommendations were given to enhance the activities in the second phase. The findings from the data analysis of the second phase provide the final recommendations of this research.

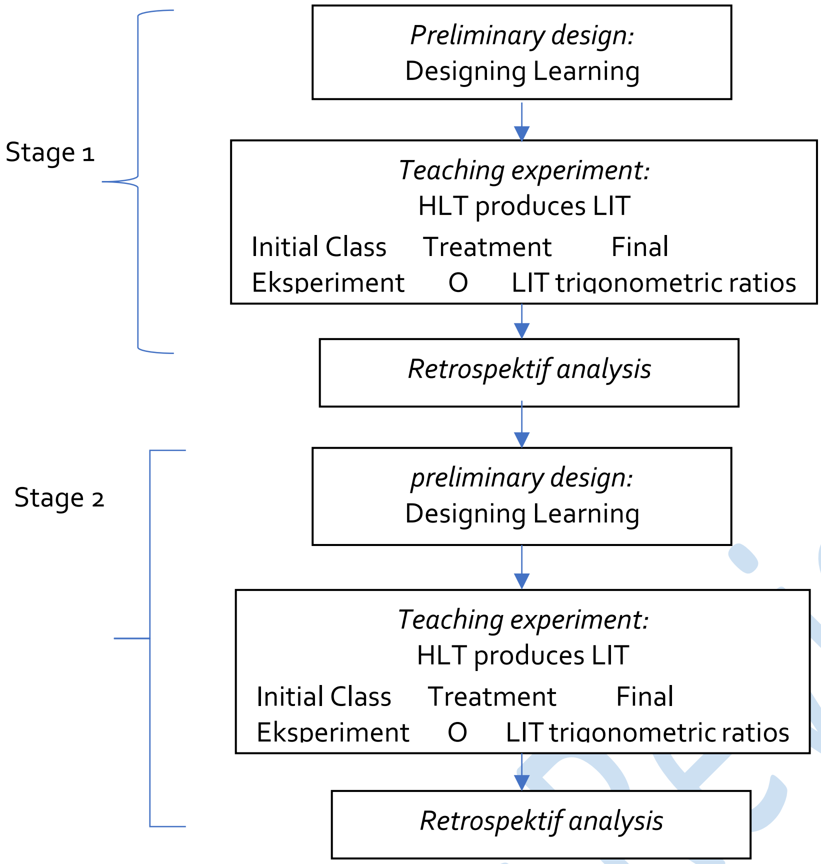


Figure 1. Research Design

RESULTS AND DISCUSSION

Results

The research is carried out in three stages: 1) formulating the fundamental design, 2) executing the teaching experiment, and 3) implementing the retrospective analysis.

Elaborating on the Initial Design (*preliminary design*)

During this phase, the researcher assembles the educational materials and finalizes the instructional content. To meet the study aims, various actions are conducted, including (1) identifying the research subjects based on their diverse cognitive abilities. The researchers selected students from class X of SMAN 7 as appropriate research subjects; (2) examining and analyzing the curriculum to identify the compatibility of trigonometric comparison material with the curriculum currently being used in the school; (3) analyzing trigonometric comparison material to obtain an overview that "the achievement of Phase E-learning based on the derived geometry domain becomes the learning objective," in accordance with the research objectives. Next, the researcher developed a teaching material design in the form of a descriptive outline of trigonometric comparisons, also known as "Local Instruction Theory Descriptive Teaching Material (UBA-LIT)". The Realistic Mathematics Education (RME) technique enhances the learning process by employing diverse learning materials, including teaching modules and research tools. The study instrument is designed to incorporate the specific attributes of Realistic Mathematics Education (RME) and the

curriculum used by the school. Additionally, the researchers analyze and assess the learning assistance tools. The result of the evaluation is referred to as the prototype. In the next stage, the research conducts an expert review, in which four experts validate the prototype based on its structure, content, and language.

Once the expert review procedure is finished, students who have been given trigonometric comparison information are tested on the prototype to get data on the readability and comprehension of the instructional material. The researchers engage in communication with the students during the testing process to anticipate any corrections or feedback regarding the instructional materials being tested by the students. The students write their input or suggestions on the prepared sheet. Students provide support for the discovery of formal concepts related to UBA-LIT through their comments. The researchers made improvements to the UBA-LIT after receiving feedback. After conducting expert validity testing and gathering comments from students, it was ultimately determined that the prototype design of the instructional material product generated by the researcher is valid at this level. The updated iteration of the prototype is referred to as the second prototype. Following that, the second prototype underwent testing in teaching experiment 1, including a total of 30 students from SMAN 7 Kota Bandung.

From a design research standpoint, the primary objective of the initial design is to develop a Local Instruction Theory (LIT) that can be visually portrayed as a portrayal of the process of acquiring and improving knowledge through teaching experiments (Gravemeijer & Cobb, 2006). During the preliminary study, the researcher designed the learning process activities based on a literature review. This set of educational exercises covers the beliefs held by students while they are building knowledge and the methods that will be used, which will be presented as an initial challenge, including advanced thinking skills. The contents of HLT are subject to change. The activities in HLT can be adapted and customized based on the circumstances and parameters of the ongoing educational experiment. HLT has a curriculum that students will adhere to when studying trigonometric ratios. The HLT curriculum comprises learning objectives, activity descriptions, and student thought conjectures that occur during the learning process. During the teaching experiment, HLT functions as a reference for teachers and researchers to determine the teaching emphasis, observations, and interviews. Moreover, HLT functions as a point of reference for the examination or analysis of a dataset in the context of a teaching experiment. Hypotethical Learning Trajectory (HLT) used in the teaching experiment is displayed in Table 1.

Tabel 1. HLT illustration for learning the concept of trigonometry comparisons

Activity	Learning Objectives	Activity Description	Conjecture
Students Demonstrate the activity of identifying by body height.	The students are able to identify the front, side, and diagonal positions by referring to specific angles of a right-angled	- The teacher instructed three students to engage in a practical exercise involving the formation of a right-angled	- Some students may be able to illustrate a sketch of a right triangle from the demonstration activity. - Some students may quickly recognize the

	triangle.	triangle using a table base, a measuring tape, and their height. - Each group carefully observes the demonstration and discusses the position of the corner, front, side, and slant.	positions of the corner, front side, side side, and hypotenuse.
Shows high shadow intensity activity	The students can articulate the trigonometric ratios (sin θ, cos θ, and tan θ) by employing the notion of ratios within a right triangle.	- The teacher instructs the students to face away from the light until a shadow appears in front of the standing student. The sketch formed is a right-angled triangle. - Students can determine the front side, side view, and diagonal side of a right triangle sketch based on the determining angle. Students will be *guided* in *rediscovering the concept* of trigonometric ratios.	- Some students may use tables as a tool to solve problems. - Certain students may employ finger manipulation methods to solve difficulties. - Some students may utilize measuring instruments to solve problems. - Some students may use the number parity method to solve problems. - Certain students may possess the ability to represent this activity in a visual manner. They initiated the construction of a model and engaged in horizontal and vertical mathematization in order to uncover the notion of trigonometric ratios through the shown examples.

			- - Some students may be able to relate the demonstration process to the concept of trigonometric ratios.
Problem-Solving	Students are able to solve real-life problems related to right triangles using trigonometric ratios	Students use trigonometric ratios to solve real-life problems by performing calculations.	- Certain students possess the ability to perceive the relationship between sine and cosine values in mathematical calculations and are capable of streamlining the calculation process. - Some students may be able to remember or repeat knowledge they have previously acquired to solve problems. - Some students may use tables, finger methods, odd-even tricks, or measuring tools to solve problems. - Some students may directly apply trigonometric ratio concepts.

Teaching Experiment

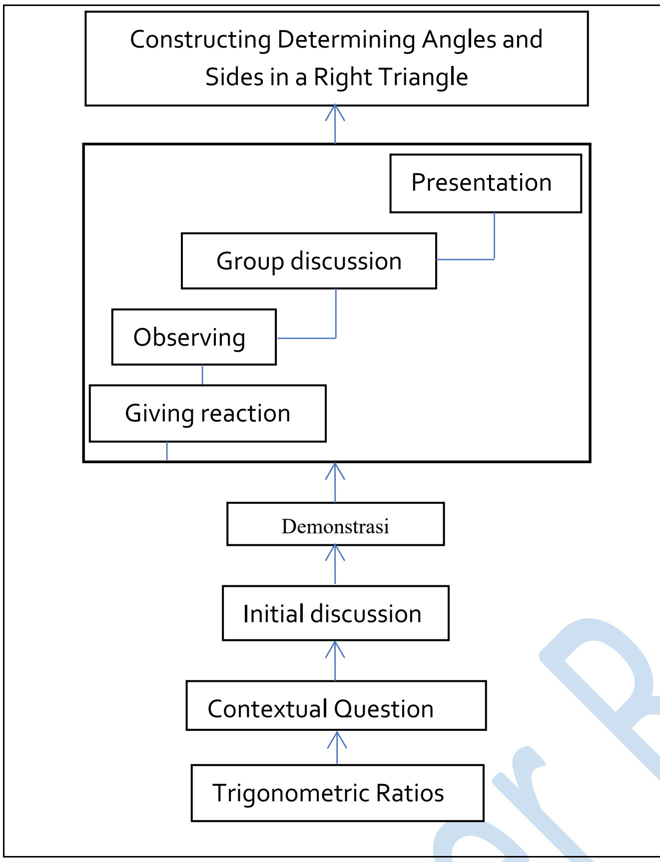
The primary objective of a teaching experiment is to systematically evaluate and enhance original ideas regarding Local Instruction Theory (LIT) while also striving to gain a more profound comprehension of the underlying mechanisms and functioning of LIT (Gravemeijer & Cobb, 2006).

Activity 1

The teacher requested three students to perform a demonstration using the provided tables/chairs and ropes.

The teacher requested one of the students to fasten the rope to the base of the table or chair provided. A student holds the end of the neat rope and moves back 1 meter from the base of the table/chair. After that, the student who is holding the end of the rapia rope shifts his grip to the end of the head. Other students are asked to draw sketches of the shapes that are occurring. The other students observed the demonstration requested by the teacher, depicted the requested sketch, and identified the positions of the angles, front side, side view, and oblique side of the formed triangle sketch. All students strive to comprehend the right-angled triangle sketch and identify the positions

of angles, front side, side view, and hypotenuse on the sketch. The teacher provides a comprehensive understanding through the following activities.



Model siswa mendemonstrasikan kegiatan mengidentifikasi melalui tinggi badan dalam pembelajaran perbandingan trigonometri

Figure 2. Demonstration of the flow of LIT and height in learning comparative trigonometry

Activity 2

The teacher instructs the class to define the trigonometric ratios (sin θ, cos θ, and tan θ) with a demonstration involving the height of a shadow. This activity guides students in formally discovering the concept of trigonometric comparisons. The students were able to perform this activity proficiently after acquiring the idea of identifying angles and sides in the previous activity. The presented issues are contextual and aligned with the student's level of understanding. The LIT design is structured from the simplest to the most complex in order to provide a general solution. In this activity, the teacher instructs the students to face away from the light source so that a shadow appears in front of the standing students. The results of the student's practice are represented in the form of an image or sketch of a right-angled triangle. This sketch facilitates students in identifying the positions of angles, front side, side view, and inclined side of a right triangle based on the predetermined angle. Creating a sketch of a right triangle and determining angles helps students mathematize vertically and find formal solutions to define trigonometric ratios.

Figure 3 illustrates various outcomes of students' process variations in discovering formal mathematics from the concept of trigonometric ratios. The students' work begins with a demonstration of a contextual problem, allowing them to find solutions using various methods based on their prior knowledge. During the learning process, students are guided to discover formal mathematics, which is a process of mathematization based on the presented problems

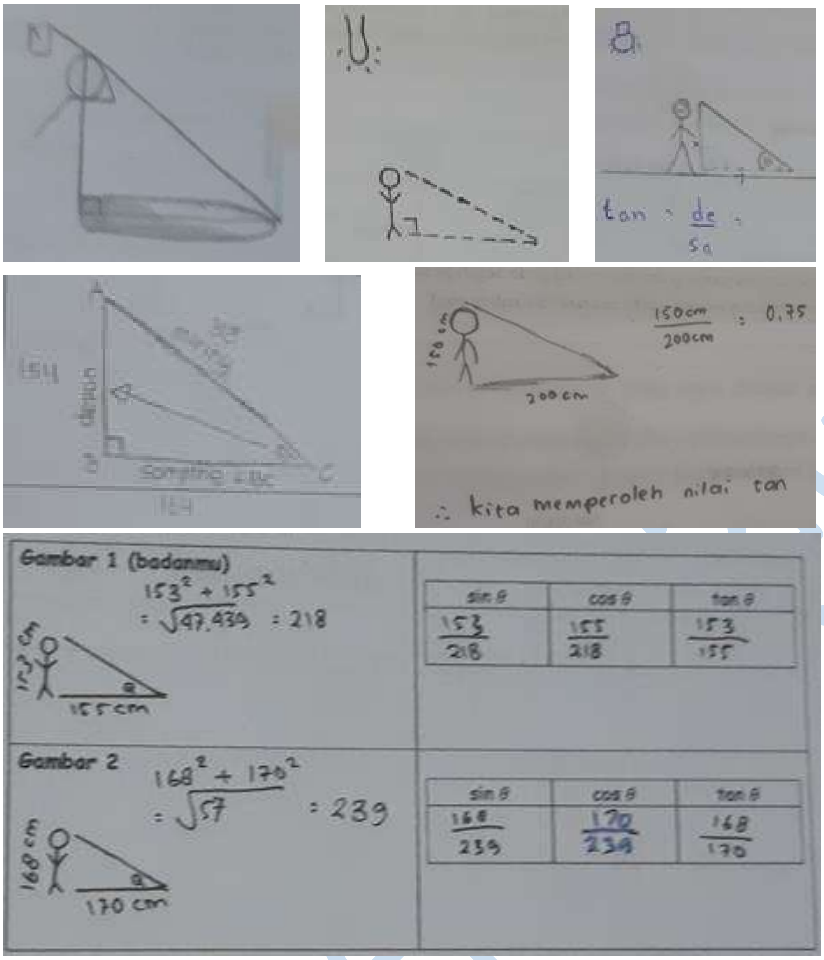


Figure 3. The vertical mathematical process of students in finding formal solutions to trigonometric ratios

Activity 3

Activity 3 begins with the teacher presenting problems based on everyday life situations. Everyday scenarios were employed to enable students to apply the notion of trigonometric ratios in practical circumstances. After the activity is finished, the teacher conducts a test as a final evaluation. One of the trigonometric comparison problems presented is as follows:

> "Perhatikan gambar tempat tidur berikut. What are the geometric shapes that occur between the stairs, bed, and floor? Sketch a model. (1) *Does the shape formed constitute a right triangle? If so, mark one corner with a right angle*. (2) *If the angle between the stairs and the floor is unknown, and the distance from the bed to the stairs is 10 cm. Place the obtained data into the image. Subsequently, calculate the distance from the floor to the end of the staircase, which forms a right-angled triangle, using the tangent ratio.* ($\tan 37^0 = 0{,}75$)"

Figure 4. Image of a bed as a model for the use of trigonometric ratios

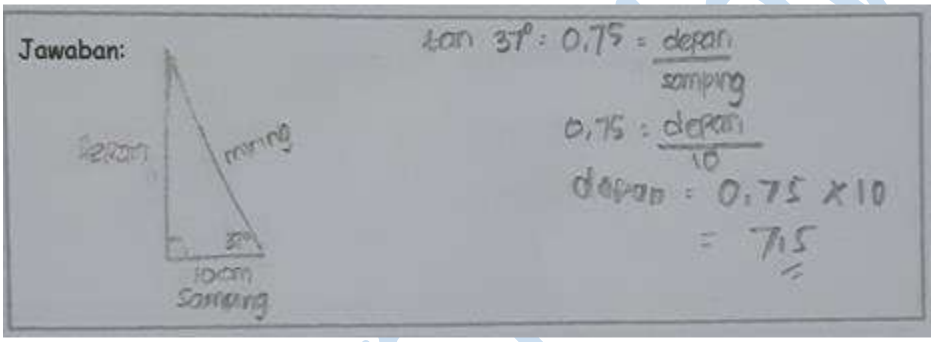


Figure 5. One of the student's responses was to solve the story problem

Figure 5 is an example of a student's response using the concept of trigonometric ratios to solve a word problem. The student solves the problem using the formal solution of trigonometric ratios that was previously constructed.

Analisis Retrospektif

Researchers employ Hypothetical Learning Trajectory (HLT) as a first guide and reference when assessing a dataset during the data analysis stage of a teaching experiment. The HLT created is compared to the classroom learning process. The analysis includes descriptions of anticipated occurrences during the learning process and examples of behaviors that will occur among students. The conclusions drawn from the analysis will answer the research questions. A good design is not the main objective of the study but rather the reason for the study's success, which is the main objective (Gravemeijer & Cobb, 2006).

The many activities carried out during the learning process are represented in the flowchart of LIT before the learning process on the topic of trigonometric ratios. The flowchart is presented in Figure 6. The image shows the differences in stages or steps during the learning process. The difference is evident in the red-colored box. Before the learning process begins, the design aims to facilitate activities, experiments, observations, group discussions, and presentations as processes that run parallel. Students will go through these stages simultaneously.

Meanwhile, after the learning process takes place, a revision of the initial design is made. The stage of differentiation involves an initial discussion process with students prior to the demonstration (*Figure 7*). This stage provides students with an introduction to understand the

concept of trigonometric ratios in relation to the prior knowledge they have already constructed. This stage is considered crucial because linking it to the previously acquired knowledge is necessary for students to comprehend the fundamental concepts of trigonometric ratios.

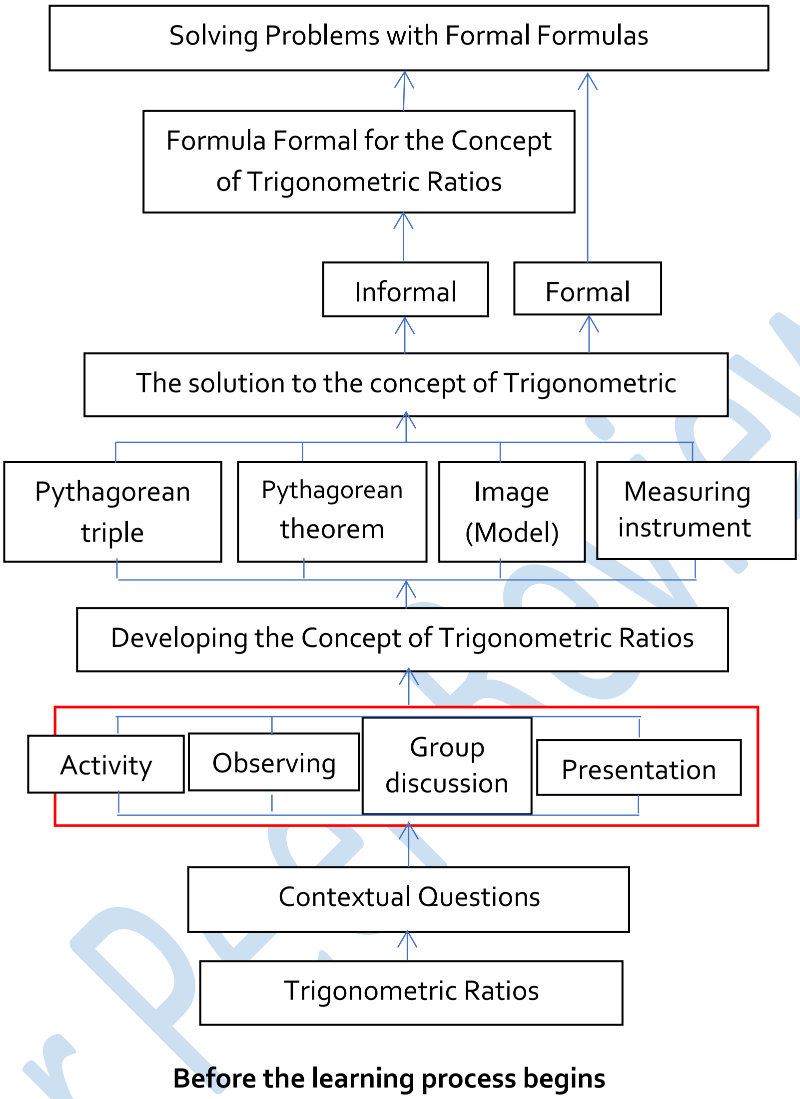
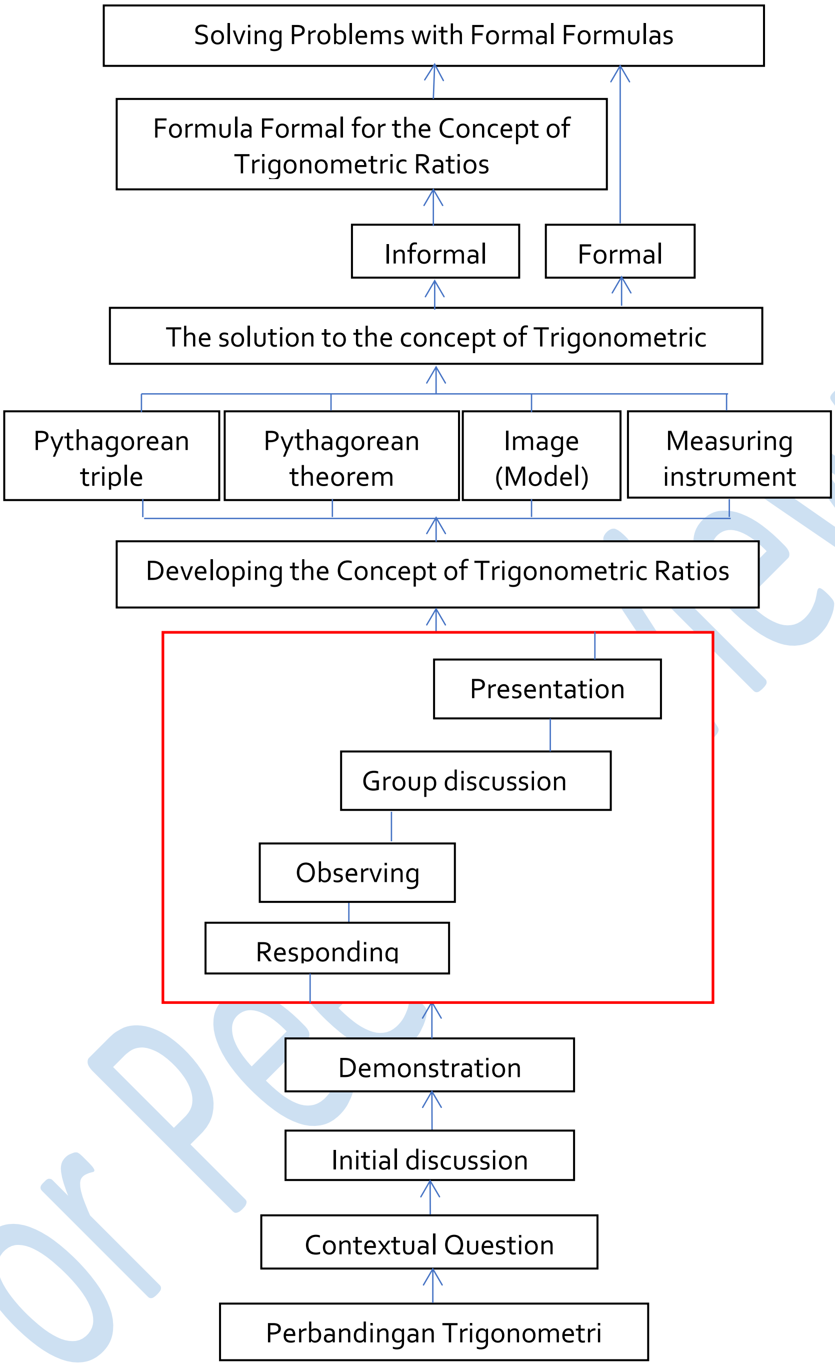


Figure 6. Flowchart of LIT on the topic of Trigonometric Ratios before the lesson

The next difference is in the "responding" stage, which students go through after conducting a demonstration and before the "observing" stage. The act of "responding" is the first step for students to participate actively in the learning process. The students will provide their responses or feedback to the demonstration presented. Their knowledge will guide and stimulate the students to continue the learning process via the "observing" stage.

Before the learning process begins, the steps "activity/experiment," "observe," "group discussion," and "presentation" have the same level of suitabilityThere is no text provided. However, the post-learning phases in these four things exhibit distinct variations. The "responding" stage initiates the steps of learning. Next, the process begins with the "observation" stage, followed by the "group discussion" step, and concludes with the "presentation" step. The students go through the four steps of the learning process in different stages.



After the learning process is complete

Figure 7. Flowchart of LIT on the topic of Trigonometric Ratios after the lesson

The appropriate use of the LIT framework, as shown in Figure 7, allows students to construct an understanding of the concept of trigonometric ratios through everyday problems. They are able to identify trigonometric ratios and solve real-life issues accordingly, as a result of analyzing the HLT and conducting teaching experiments. Based on the results of the student's work using the LIT design for trigonometric comparison, the number of students capable of completing the learning activities is determined. The students are becoming accustomed to the learning process, which involves determining mathematics formally based on pre-established models. The utilization of

Pythagorean triples, the Pythagorean theorem, models (diagrams), and measuring tools can aid in achieving vertical mathematization. The students may effectively solve common problems by utilizing trigonometric ratios.

Discussion

The findings reveal that there are still students who lack the prerequisite knowledge that should have been acquired based on the current curriculum. The lack of prerequisite knowledge is caused by external factors (Kurniasih et al., 2020; Puspita et al., 2023) such as mental unpreparedness in receiving knowledge, didactic design of teachers, and limitations in teaching materials. Teachers are still engaged in the process of teaching by providing students with rote memorization to remember trigonometric comparison formulas. Introducing the formula of trigonometric ratios without providing a geometric explanation leads to a lack of motivation for students to construct their understanding of trigonometric ratios (Andika et al., 2017; Urrutia et al., 2019). Learning methods that are teacher-centered and rely on memorization only provide knowledge about trigonometric comparisons to students for a short time. Still, this knowledge only lasts for a short time (Orhun, 2002). For example, the teacher introduces the concept of the sine of an angle in a right triangle as a ratio between the lengths of the opposite side and the hypotenuse. The idea of the sine of an angle in a right triangle as a ratio between the lengths of the opposite side and the hypotenuse can be found by rotating the right triangle to any position (Blackett & Tall, 1991).

Another important aspect for teachers is to enhance their comprehensive understanding of trigonometry so that they can connect and present different trigonometry concepts, which can construct students' knowledge of trigonometry (Rittle-Johnson & Schneider, 2014). Effective interactions between teachers and students facilitate students to actively ask questions, explain, and confirm the understanding they are developing. According to Dhungana's research, effective communication between teachers and students can significantly contribute to understanding trigonometry (Dhungana et al., 2023)

The majority of instructional materials provided to students focus on trigonometric functions (Fi, 2003). On the other hand, the material on trigonometric comparison is fundamental in understanding the material on trigonometric functions. Trigonometry has a significant role in various fields, such as in teaching geometric conjectures and proofs (Fiallo & Gutiérrez, 2017; Nurhayati, 2017). Possible errors in the learning process may arise in procedural knowledge, conceptual information, or the interplay between these two types of knowledge (Dhungana et al., 2023).

Testing and interviews with students unveiled that students require assistance comprehending the principles of trigonometric ratios as a result of their inadequate grasp of triangles (Dhungana et al., 2023). The difficulty of determining the correct position of angles and sides should have been overcome as students acquire the necessary knowledge. The process of building understanding among students is essential for determining the correct position of angles and sides.

Research has also discovered that certain students find learning more relevant, particularly in the subject of trigonometric ratios, when contextual life problems are utilized. Integrating students' everyday experiences using objects and diagrams (models) makes learning meaningful and constructed knowledge long-lasting (Dhungana et al., 2023).

The use of LIT (Local Instruction Theory) through the Realistic Mathematics Education (RME) approach enhances students' ability to construct their understanding of the concept of trigonometric ratios. Enhances students' ability is evident from the student's work, which

demonstrates outcomes that align with the learning objectives.

Implication of Research

The design of this LIT Trigonometric Comparison utilizes the syntax of the Realistic Mathematics Education (RME) approach. The genesis of this LIT Trigonometric ratio is a conjecture on the learning trajectory using the Realistic Mathematics Education (RME) approach. Therefore, the designed learning trajectory needs further development in accordance with the intended content to be achieved. For other researchers who will utilize the learning trajectory, it is recommended to master various learning steps or various methods in designing problems. By mastering problem design, one can obtain a variety of problem-solving solutions, both formally and informally.

Limitation

This research was conducted in two stages, each of which followed the steps of design research (Gravemeijer, 2004) namely: 1) developing the preliminary design, 2) conducting the teaching experiment, and 3) performing the retrospective analysis. There are two reasons for choosing only two stages in this research: firstly, limited researcher time and resources. Secondly, the material is only conducted once a year.

CONCLUSION

The outcome of the learning trajectory in this study is a Comparative Trigonometry LIT developed through the Realistic Mathematics Education (RME) approach. This theory encompasses several steps that students must go through in constructing their knowledge related to trigonometric comparisons. These steps nonetheless adhere to the syntax of the Realistic Mathematics Education (RME) approach.

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2. Bukti Konfirmasi Review dan Hasil Review Pertama (22 November 2024)

[Kreano] Your submission has been reviewed and we encourage you to submit revisions

2024-11-22 05:56

Dear Dini Andiani,

Your submission Development of A Local Instruction Theory for Trigonometric Ratios has been reviewed and we would like to encourage you to submit revisions that address the reviewers' comments. An editor will review these revisions and if they address the concerns adequately, your submission may be accepted for publication.

The reviewers' comments are included at the bottom of this email. Please respond to each point in the reviewers' comments and identify what changes you have made. If you find any of the reviewer's comments to be unjustified or inappropriate, please explain your perspective.

When you have completed your revisions, you can upload revised documents along with your response to the reviewers' comments at your submission dashboard. If you have been logged out, you can login again with the username diniandiani.

If you have any questions, please contact me from your submission dashboard.

We look forward to receiving your revised submission.

Kind regards,

Executive Editor

The following comments were received from reviewers.

Reviewer 1:
Recommendation: Revisions Required

ABSTRACT

Minor Revision

Comment to Abstract

At the beginning of abstract, show us your GAP of research. In this abstract you just write a fact, not a GAP. And please add an implication of your research at the end of abstract.

INTRODUCTION

Minor Revision

Comment to Introduction

Some part need to be moved to Method.
Author, your keyword is LIT. so, in introduction you should explain what, who, when, why, where, and how LIT implemented. I can't find it.
You also should highlighted your research question, your research urgency.

Gravemeijer proposes a research design for the development of local instruction theory. --- This part better put it on method.

METHOD

Minor Revision

Comment to method

Why do you use design research?
How do you organize the participant of your research?
How do you maintain the validation of your research? Explain after the chart.

RESULT AND DISCUSSION

Minor Revision

Comment to Result and Discussion

Clarity and Focus:
• Some parts of the discussion are repetitive (e.g., the explanation of sine ratios). A more concise presentation would improve readability.
• The transition between problems and proposed solutions could be more seamless.

Details on LIT and RME:
• While the potential of LIT and RME is highlighted, the specifics of how these approaches were implemented could be expanded upon. For example:
• What specific real-life problems or models were used?
• How did students respond during the teaching experiment?

Addressing Limitations:
• The discussion mentions resource and time constraints as limitations but does not suggest how future research might address these issues.

Student Diversity:
• The research could explore variations in student experiences based on factors such as prior knowledge, learning styles, or socio-economic backgrounds.

CONCLUSSION

Good. No Revision

GENERAL COMMENT TO AUTHOR

Need revision. Some areas for Improvement has been put it on each part. see manuscript.

Reviewer 2:
Recommendation: Revisions Required

ABSTRACT

Good. No revision

Comment to Abstract

INTRODUCTION

Minor Revision

Comment to Introduction

You said that LIT is very related to teachers. In the introduction, I could not find, examples of LIT implementation by teachers and problems of its implementation where? How about LIT in mathematics? examples with mathematical materials need to be revealed in the introduction so that mathematics education becomes clearer. Why do teachers need to understand LIT? You need to describe this to form a research GAP in this article.

METHOD

Minor Revision

Comment to method

I still haven't got any information, why design research? why are the subjects students of SMA 7 Bandung, what's special about it? are there any teachers who have practiced LIT? academic reasons need to be conveyed in this section. How are the research data collected? how is it analyzed? how is the conclusion drawn? you must describe all of this in this section.

RESULT AND DISCUSSION

Minor Revision

Comment to Result and Discussion

In this discussion, you have just told the research facts. Let the research facts be in the Result section. In the discussion, you have not explained what new things you found. How did students respond to your LIT design? Then the discussion must be critical in the theoretical GAP. I have not found the big picture of your research either. You can describe it in the before-after research. What's new?

CONCLUSSION

Good. No Revision

GENERAL COMMENT TO AUTHOR

See manuscript

Development of A Local Instruction Theory for Trigonometric Ratios

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Abstract

Several students view the concept of trigonometric ratios as intricate. This work aims to create the Local Instruction Theory (LIT) for instructing trigonometric ratios utilizing Realistic Mathematic Education (RME). Researchers developed a Learning Instruction Theory (LIT) to aid students in constructing a robust comprehension of the fundamental principles of trigonometric ratios. They accomplished this by identifying a highly efficient learning trajectory. The study approach utilizes a specific research design. The development of the Hypothetical Learning Trajectory (HLT) is now underway to improve students' comprehension of trigonometric ratios. The development of local instruction theory for trigonometric ratios involves three stages: formulating an initial plan, carrying out teaching experiments, and doing retrospective analysis. Through qualitative analysis of teaching experiments, it has been determined that incorporating instructional materials on trigonometric ratios in LIT that align with RME identities effectively assists students in developing fundamental concepts in trigonometric ratios.

Keywords: Local Instruction Theory; Hypothetical Learning Trajectory; Trigonometric Ratios; RME

Abstrak

Beberapa siswa menganggap materi perbandingan trigonometri cukup sulit. Penelitian ini bertujuan untuk mengembangkan Local Instruction Theory (LIT) untuk pembelajaran perbandingan trigonometri dengan menggunakan Realistic Mathematic Education (RME). Dalam upaya membantu siswa membangun konsep dasar pada materi perbandingan trigonometri, peneliti mengembangkan alur LIT dengan menemukan jalur pembelajaran yang efektif. Pencapaian tujuan penelitian menggunakan desain penelitian. Serangkaian kegiatan mengembangkan Hypothetical Learning Trajectory (HLT) sehingga siswa sekolah menengah atas (SHS) memiliki pemahaman yang lebih baik tentang perbandingan trigonometri. Pengembangan Local Instruction Theory untuk Perbandingan Trigonometri meliputi tiga tahap yaitu mengembangkan desain awal, melakukan percobaan pengajaran, dan melaksanakan analisis retrospektif. Siswa mampu membangun pemahaman tentang perbandingan trigonometri selama proses pembelajaran berlangsung. Berdasarkan analisis kualitatif eksperimen pengajaran, penggunaan bahan ajar LIT perbandingan trigonometri yang mengakomodir identitas RME, mampu memandu siswa dalam membangun konsep dasar pada materi perbandingan trigonometri.

Mathematics Clasification: ---- must be filled ----

Please check Mathematics Clasification here:

https://mathscinet.ams.org/msc/msc2010.html?t=97-XX&s=&btn=Search&ls=s

Metadata of Author
First author: SCOPUS ID: - optional - | ORCID ID: - mandatory - | Google Scholar : VmAlN5YAAAAJ
Co-author: SCOPUS ID: - *57196237689* - | ORCID ID: - mandatory - | Google Scholar : Kc-9bRgAAAAJ

INTRODUCTION

The topic of trigonometric ratios is fundamental in the application of advanced applied sciences and pure sciences. Various fields of study require the use of trigonometry as an essential basis for applying their concepts. For example, the utilization of trigonometric ratios supports the proof of Kepler's first law (Simha, 2021). The importance of understanding trigonometric ratios lies in its role as fundamental knowledge for comprehending pre-calculus and calculus, such as algebra, geometry, and the ability to create graphs (Weber, 2005). Trigonometric comparison is utilized in calculating the width or height of various shapes, such as a bridge, household tools, and bicycles (Urrutia et al., 2019). Indeed, students see the topic of trigonometric ratios as challenging (Blackett & Tall, 1991; Weber, 2005; Yiğit Koyunkaya, 2016; Karthikeyan, 2017; Urrutia et al., 2019; Nordlander, 2022). During the initial stage of learning, students encounter difficulties in connecting triangle images to numerical relationships, as well as problems in manipulating the involved symbols (Blackett & Tall, 1991), and many students are unable to overcome these difficulties. While studying right triangles, students struggled to grasp the concept of trigonometric functions, such as sine, cosine, and tangent, which are defined as ratios (Weber, 2005a, 2008). According to Yiğit (2016), students exhibit a lack of flexibility when solving trigonometric comparison problems (Yiğit Koyunkaya, 2016). Students expressed that trigonometry was a subject that was highly stimulating and conceptual in comparison to other areas of mathematics (Karthikeyan, 2017). According to Sandip's research, trigonometry is a challenging subject that is disconnected from everyday life (Urrutia et al., 2019). Typically, students struggle to distinguish the front side, side view, and slanted side of a right triangle despite the fact that this distinction greatly aids their comprehension of trigonometric ratios. The topic of trigonometric comparison is considered abstract for students (Karthikeyan, 2017).

From the preliminary study and literature review conducted by the researcher on textbooks, instructional videos, analysis of students' learning difficulties, and curriculum analysis, several aspects have caught the researcher's attention. One of these aspects is the emergence of real-world contexts in trigonometry materials in textbooks. Various examples have begun to present contextual situations, but teachers still need to learn how students might construct formal concepts from the presented contextual case. Batanero dan Diaz (2012) argue that the existing textbooks and curriculum documents need to sufficiently support learning, as the presented concepts are narrow in scope . (Batanero & Díaz, 2012).

The typical pattern of learning that occurs in the delivery of contextual mathematics is the initial introduction and presentation of example problems, followed by the completion of practice exercises using general formulas. The typical pattern of learning still represents the teaching culture in carrying out the learning process. According to Weber (2005), mathematics learning in schools still follows the conventional sequence of explanation, example provision, and problem-solving practice. The learning process via direct application of general formulas can prevent students from developing their mathematical concepts independently (Weber, 2005b). Their thinking patterns need to be trained, resulting in a lack of familiarity with how these formulas are derived.

The application of real-life models to formal mathematical concepts has yet to be documented in supporting learning materials. The teacher must possess the ability to elucidate the models

Commented [Isti1]: You said that LIT is very related to teachers. In the introduction, I could not find, examples of LIT implementation by teachers and problems of its implementation where? How about LIT in mathematics? examples with mathematical materials need to be revealed in the introduction so that mathematics education becomes clearer. Why do teachers need to understand LIT? You need to describe this to form a research GAP in this article.

generated by students during the learning process, which can subsequently facilitate the comprehension of formal mathematics. Turmudi (2018) states that students should have a strong desire in the learning process to independently discover concepts, provided that teachers provide facilities, teaching materials, and learning resources (Turmudi, 2012).

According to Sandip's research, trigonometry is a challenging subject that is disconnected from everyday life (Adhikari & Subedi, 2021; Asomah et al., 2022; Dhungana et al., 2023). To bridge this gap, Realistic Mathematics Education serves as an alternative to teaching trigonometric ratios. Realistic Mathematics Education (RME) is a teaching and learning method in math education that follows the philosophy of Hans Freudenthal, who sees math as a human endeavor. (Freudenthal, 1973; Gravemeijer & Terwel, 2000; Treffers, 1987; Van den Heuvel-Panhuizen & Drijvers, 2020).

Realistic Mathematics Education is characterized by students experiencing a concrete learning process that is aligned with real-life situations and contextualized according to their everyday lives (Yuanita et al., 2018). Students undergo an informal process of constructing their understanding based on what they see, observe, and comprehend about their surrounding environment, be it at home, school, or wherever they learn (Meika, 2018).

The three primary ideas of Realistic Mathematics Education are guided reinvention and progressive mathematizing, didactical phenomenology, and self-developed models. (Gravemeijer, 1994). During the teaching and learning process, these three principles are elaborated into five characteristics of RME, namely: 1) Building and solidifying; 2) degree and frameworks; 3) contemplation and specific task; 4) societal environment and communication; and 5) organizing and intertwining (Gravemeijer, 1994; Jzn & de, 1987). The significance of the Realistic Mathematics Education approach resides in its ability to empower students to develop concepts through interactive teaching and learning processes. The teachers has a crucial role in enabling students to explore concepts using both administrative and motivational methods (Meika, 2018).

The mistakes made by previous students should not be repeated in the following years. Penyediaan materi pembelajaran adalah salah satu solusi yang tepat untuk memulihkan keadaan menjadi lebih baik. The focus of learning will be further narrowed down if the instructional materials used are appropriate. In this teaching material, steps are needed that can facilitate students' learning paths. According to Gravemeijer, to improve mathematics education, it is necessary to implement an instructional design that facilitates learning and enables students to progress from their current level of thinking to a more sophisticated level of mathematical thinking (Gravemeijer, 2004).

The Local Instruction Theory establishes the learning process by incorporating a learning trajectory. Local Instruction Theory (LIT) is a comprehensive theory that includes both the process of acquiring knowledge on a specific topic and the methods used to facilitate that learning (Liljekvist et al., 2017). (Gravemeijer, 2004) states that Local Instruction Theory is an exact theory that can assist students in developing mathematical thinking on a particular topic or material through detailed and gradual instructional mediumResearchers will develop a systematic learning path, also known as a learning trajectory, within the process of teaching and learning. This design is the one Gravemeijer mentioned.

LIT is used to describe the appropriate learning route or learning activities. LIT not only offers how learning takes place but also facilitates a range of specific learning activities that may work. The existence of LIT provides a basis for teachers to consider adaptations that may be needed in implementing new approaches. In addition, LIT offers a framework in which teachers can develop hypotheses about learning trajectories. LIT is tailored to the specific circumstances of the particular

class and teacher. Teachers need to build HLT (Hypothetical Learning Trajectories) that are suitable for their own situations while using LIT as a foundational reference (Gravemeijer, 2004).

Gravemeijer proposes a research design for the development of local instruction theory. The research design consists of three phases: 1) preliminary design (developing initial design), 2) teaching experiment (doing teaching experiment), and 3) retrospective analysis (performing retrospective analysis) (Gravemeijer, 2004). Preliminary design is the outcome of an initial learning plan aimed at implementing ideas about the development of instructional materials for LIT (Meika, 2018). This design is expected to support students in the construction of mathematical thinking from an informal form to a formal one. The construction of mathematical thinking is achieved through a guided reinvention process facilitated by mathematizing. Mathematizing refers to several ways of managing learning activities to demonstrate mathematical characteristics, such as generality, certainty, accuracy, and agility (Loc & Hao, 2016). The purpose of the teaching experiment is to evaluate the effectiveness of the instructional materials for LIT. Retrospective analysis is employed to examine the complete dataset of the teaching experiment outcomes.

HLT is applied in every learning activity and has a crucial role in designing learning activities for students. The execution of learning is indissociable from the learning trajectory, encompassing a deliberate progression of learning materials and idea maps that students traverse during the learning process (Prahmana, 2016). Next, in the subsequent stage, the trigonometric ratios LIT is produced.

According to the given definition, it is crucial to create and design instructional resources that can reduce students' challenges in developing cognitive processes through Realistic Mathematics Education in order to build comprehension of trigonometric concepts. The desired teaching materials are obtained through the LIT design. The LIT design is used to address the difficulties experienced by students and to enhance their mathematical thinking process, particularly their understanding of trigonometric ratios through Realistic Mathematics Education.

METHOD

A design research method based on Gravemeijer's approach is the research methodology used. This research method is focused on local instruction theory, or LIT, which was developed by researchers in collaboration with teachers to achieve better learning quality (Gravemeijer & van Eerde, 2009). A study paradigm known as "design research" tries to create a series of tasks and comprehend the workings of the learning process. This study aims to develop and construct a Local Instruction Theory (LIT) on the subject of trigonometric ratios, with a focus on Realistic Mathematics Education (RME).

The research implementation is carried out in two stages, with each stage referring to the steps of design research (Gravemeijer, 2004) namely: The three main stages of the project include 1) formulating the initial concept, 2) executing the educational experiment, and 3) carrying out a retrospective study. The subject of the study during the teaching experiment phase consisted of 25 tenth-grade students from SMAN 7, Bandung. An analysis was conducted on the activities performed in the initial phase, and subsequently, recommendations were given to enhance the activities in the second phase. The findings from the data analysis of the second phase provide the final recommendations of this research.

Commented [Isti2]: I still haven't got any information, why design research? why are the subjects students of SMA 7 Bandung, what's special about it? are there any teachers who have practiced LIT? academic reasons need to be conveyed in this section. How are the research data collected? how is it analyzed? how is the conclusion drawn? you must describe all of this in this section.

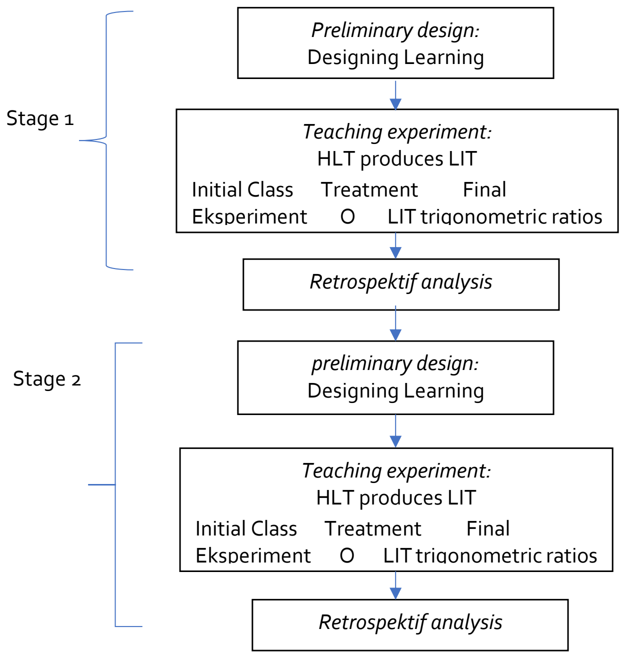


Figure 1. Research Design

RESULTS AND DISCUSSION

Results

The research is carried out in three stages: 1) formulating the fundamental design, 2) executing the teaching experiment, and 3) implementing the retrospective analysis.

Elaborating on the Initial Design (*preliminary design*)

During this phase, the researcher assembles the educational materials and finalizes the instructional content. To meet the study aims, various actions are conducted, including (1) identifying the research subjects based on their diverse cognitive abilities. The researchers selected students from class X of SMAN 7 as appropriate research subjects; (2) examining and analyzing the curriculum to identify the compatibility of trigonometric comparison material with the curriculum currently being used in the school; (3) analyzing trigonometric comparison material to obtain an overview that "the achievement of Phase E-learning based on the derived geometry domain becomes the learning objective," in accordance with the research objectives. Next, the researcher developed a teaching material design in the form of a descriptive outline of trigonometric comparisons, also known as "Local Instruction Theory Descriptive Teaching Material (UBA-LIT)". The Realistic Mathematics Education (RME) technique enhances the learning process by employing diverse learning materials, including teaching modules and research tools. The study instrument is designed to incorporate the specific attributes of Realistic Mathematics Education (RME) and the

curriculum used by the school. Additionally, the researchers analyze and assess the learning assistance tools. The result of the evaluation is referred to as the prototype. In the next stage, the research conducts an expert review, in which four experts validate the prototype based on its structure, content, and language.

Once the expert review procedure is finished, students who have been given trigonometric comparison information are tested on the prototype to get data on the readability and comprehension of the instructional material. The researchers engage in communication with the students during the testing process to anticipate any corrections or feedback regarding the instructional materials being tested by the students. The students write their input or suggestions on the prepared sheet. Students provide support for the discovery of formal concepts related to UBA-LIT through their comments. The researchers made improvements to the UBA-LIT after receiving feedback. After conducting expert validity testing and gathering comments from students, it was ultimately determined that the prototype design of the instructional material product generated by the researcher is valid at this level. The updated iteration of the prototype is referred to as the second prototype. Following that, the second prototype underwent testing in teaching experiment 1, including a total of 30 students from SMAN 7 Kota Bandung.

From a design research standpoint, the primary objective of the initial design is to develop a Local Instruction Theory (LIT) that can be visually portrayed as a portrayal of the process of acquiring and improving knowledge through teaching experiments (Gravemeijer & Cobb, 2006). During the preliminary study, the researcher designed the learning process activities based on a literature review. This set of educational exercises covers the beliefs held by students while they are building knowledge and the methods that will be used, which will be presented as an initial challenge, including advanced thinking skills. The contents of HLT are subject to change. The activities in HLT can be adapted and customized based on the circumstances and parameters of the ongoing educational experiment. HLT has a curriculum that students will adhere to when studying trigonometric ratios. The HLT curriculum comprises learning objectives, activity descriptions, and student thought conjectures that occur during the learning process. During the teaching experiment, HLT functions as a reference for teachers and researchers to determine the teaching emphasis, observations, and interviews. Moreover, HLT functions as a point of reference for the examination or analysis of a dataset in the context of a teaching experiment. Hypotethical Learning Trajectory (HLT) used in the teaching experiment is displayed in Table 1.

Tabel 1. HLT illustration for learning the concept of trigonometry comparisons

Activity	Learning Objectives	Activity Description	Conjecture
Students Demonstrate the activity of identifying by body height.	The students are able to identify the front, side, and diagonal positions by referring to specific angles of a right-angled	- The teacher instructed three students to engage in a practical exercise involving the formation of a right-angled	- Some students may be able to illustrate a sketch of a right triangle from the demonstration activity. - Some students may quickly recognize the

	triangle.	triangle using a table base, a measuring tape, and their height. - Each group carefully observes the demonstration and discusses the position of the corner, front, side, and slant.	positions of the corner, front side, side side, and hypotenuse.
Shows high shadow intensity activity	The students can articulate the trigonometric ratios (sin θ, cos θ, and tan θ) by employing the notion of ratios within a right triangle.	- The teacher instructs the students to face away from the light until a shadow appears in front of the standing student. The sketch formed is a right-angled triangle. - Students can determine the front side, side view, and diagonal side of a right triangle sketch based on the determining angle. Students will be *guided* in *rediscovering the concept* of trigonometric ratios.	- Some students may use tables as a tool to solve problems. - Certain students may employ finger manipulation methods to solve difficulties. - Some students may utilize measuring instruments to solve problems. - Some students may use the number parity method to solve problems. - Certain students may possess the ability to represent this activity in a visual manner. They initiated the construction of a model and engaged in horizontal and vertical mathematization in order to uncover the notion of trigonometric ratios through the shown examples.

			- - Some students may be able to relate the demonstration process to the concept of trigonometric ratios.
Problem-Solving	Students are able to solve real-life problems related to right triangles using trigonometric ratios	Students use trigonometric ratios to solve real-life problems by performing calculations.	- Certain students possess the ability to perceive the relationship between sine and cosine values in mathematical calculations and are capable of streamlining the calculation process. - Some students may be able to remember or repeat knowledge they have previously acquired to solve problems. - Some students may use tables, finger methods, odd-even tricks, or measuring tools to solve problems. - Some students may directly apply trigonometric ratio concepts.

Teaching Experiment

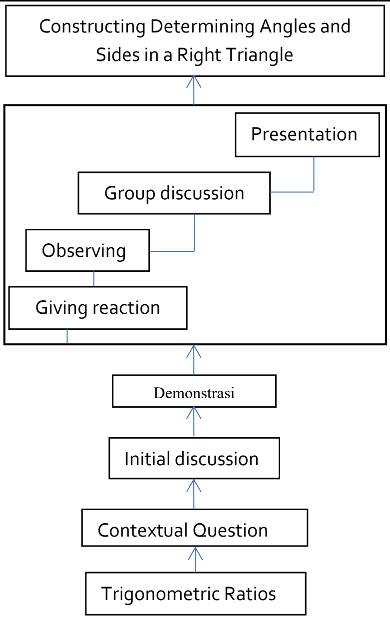
The primary objective of a teaching experiment is to systematically evaluate and enhance original ideas regarding Local Instruction Theory (LIT) while also striving to gain a more profound comprehension of the underlying mechanisms and functioning of LIT (Gravemeijer & Cobb, 2006).

Activity 1

The teacher requested three students to perform a demonstration using the provided tables/chairs and ropes.

The teacher requested one of the students to fasten the rope to the base of the table or chair provided. A student holds the end of the neat rope and moves back 1 meter from the base of the table/chair. After that, the student who is holding the end of the rapia rope shifts his grip to the end of the head. Other students are asked to draw sketches of the shapes that are occurring. The other students observed the demonstration requested by the teacher, depicted the requested sketch, and identified the positions of the angles, front side, side view, and oblique side of the formed triangle sketch. All students strive to comprehend the right-angled triangle sketch and identify the positions

of angles, front side, side view, and hypotenuse on the sketch. The teacher provides a comprehensive understanding through the following activities.



Model siswa mendemonstrasikan kegiatan mengidentifikasi melalui tinggi badan dalam pembelajaran perbandingan trigonometri

Figure 2. Demonstration of the flow of LIT and height in learning comparative trigonometry

Activity 2

The teacher instructs the class to define the trigonometric ratios (sin θ, cos θ, and tan θ) with a demonstration involving the height of a shadow. This activity guides students in formally discovering the concept of trigonometric comparisons. The students were able to perform this activity proficiently after acquiring the idea of identifying angles and sides in the previous activity. The presented issues are contextual and aligned with the student's level of understanding. The LIT design is structured from the simplest to the most complex in order to provide a general solution. In this activity, the teacher instructs the students to face away from the light source so that a shadow appears in front of the standing students. The results of the student's practice are represented in the form of an image or sketch of a right-angled triangle. This sketch facilitates students in identifying the positions of angles, front side, side view, and inclined side of a right triangle based on the predetermined angle. Creating a sketch of a right triangle and determining angles helps students mathematize vertically and find formal solutions to define trigonometric ratios.

Figure 3 illustrates various outcomes of students' process variations in discovering formal mathematics from the concept of trigonometric ratios. The students' work begins with a demonstration of a contextual problem, allowing them to find solutions using various methods based on their prior knowledge. During the learning process, students are guided to discover formal mathematics, which is a process of mathematization based on the presented problems

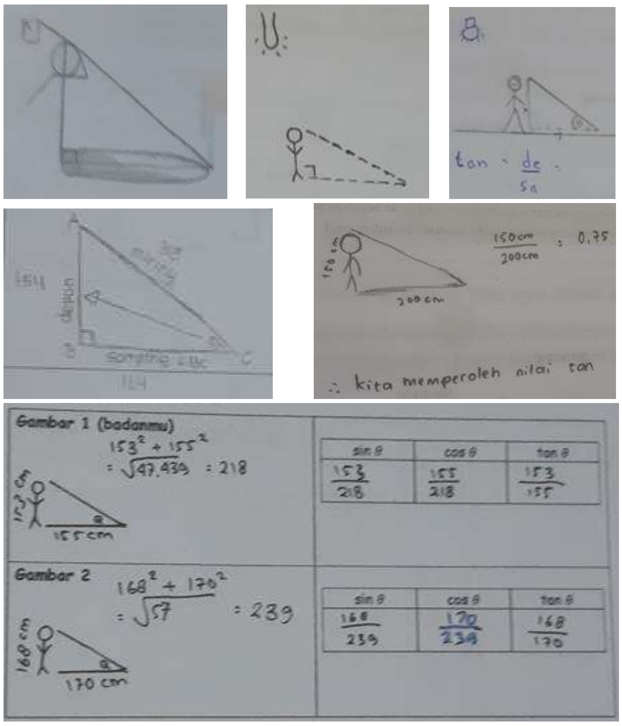


Figure 3. The vertical mathematical process of students in finding formal solutions to trigonometric ratios

Activity 3

Activity 3 begins with the teacher presenting problems based on everyday life situations. Everyday scenarios were employed to enable students to apply the notion of trigonometric ratios in practical circumstances. After the activity is finished, the teacher conducts a test as a final evaluation. One of the trigonometric comparison problems presented is as follows:

> "Perhatikan gambar tempat tidur berikut. What are the geometric shapes that occur between the stairs, bed, and floor? Sketch a model. (1) *Does the shape formed constitute a right triangle? If so, mark one corner with a right angle*. (2) *If the angle between the stairs and the floor is unknown, and the distance from the bed to the stairs is 10 cm. Place the obtained data into the image. Subsequently, calculate the distance from the floor to the end of the staircase, which forms a right-angled triangle, using the tangent ratio. (*$\tan 37^0 = 0{,}75$)*"*

Figure 4. Image of a bed as a model for the use of trigonometric ratios

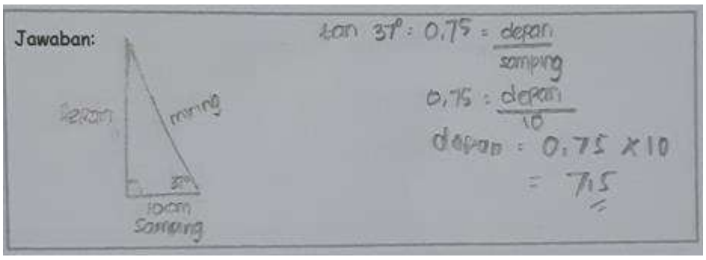


Figure 5. One of the student's responses was to solve the story problem

Figure 5 is an example of a student's response using the concept of trigonometric ratios to solve a word problem. The student solves the problem using the formal solution of trigonometric ratios that was previously constructed.

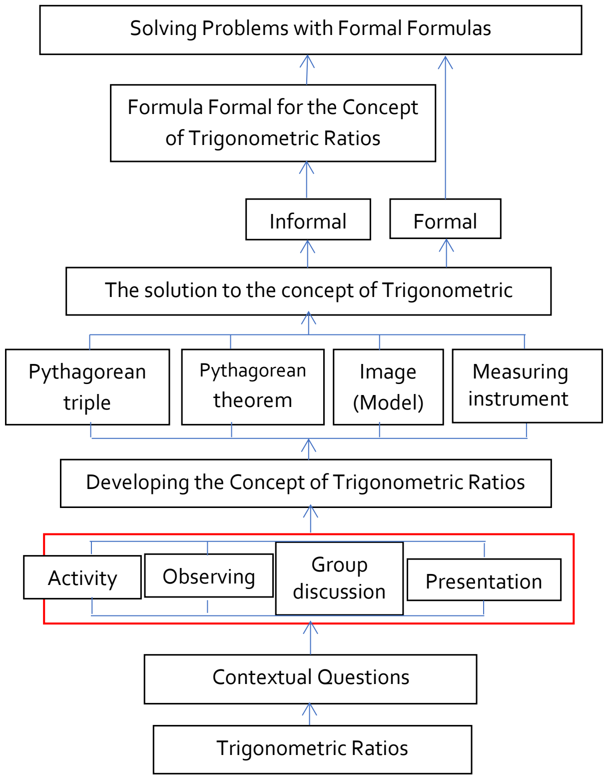
Analisis Retrospektif

Researchers employ Hypothetical Learning Trajectory (HLT) as a first guide and reference when assessing a dataset during the data analysis stage of a teaching experiment. The HLT created is compared to the classroom learning process. The analysis includes descriptions of anticipated occurrences during the learning process and examples of behaviors that will occur among students. The conclusions drawn from the analysis will answer the research questions. A good design is not the main objective of the study but rather the reason for the study's success, which is the main objective (Gravemeijer & Cobb, 2006).

The many activities carried out during the learning process are represented in the flowchart of LIT before the learning process on the topic of trigonometric ratios. The flowchart is presented in Figure 6. The image shows the differences in stages or steps during the learning process. The difference is evident in the red-colored box. Before the learning process begins, the design aims to facilitate activities, experiments, observations, group discussions, and presentations as processes that run parallel. Students will go through these stages simultaneously.

Meanwhile, after the learning process takes place, a revision of the initial design is made. The stage of differentiation involves an initial discussion process with students prior to the demonstration (*Figure 7*). This stage provides students with an introduction to understand the

concept of trigonometric ratios in relation to the prior knowledge they have already constructed. This stage is considered crucial because linking it to the previously acquired knowledge is necessary for students to comprehend the fundamental concepts of trigonometric ratios.

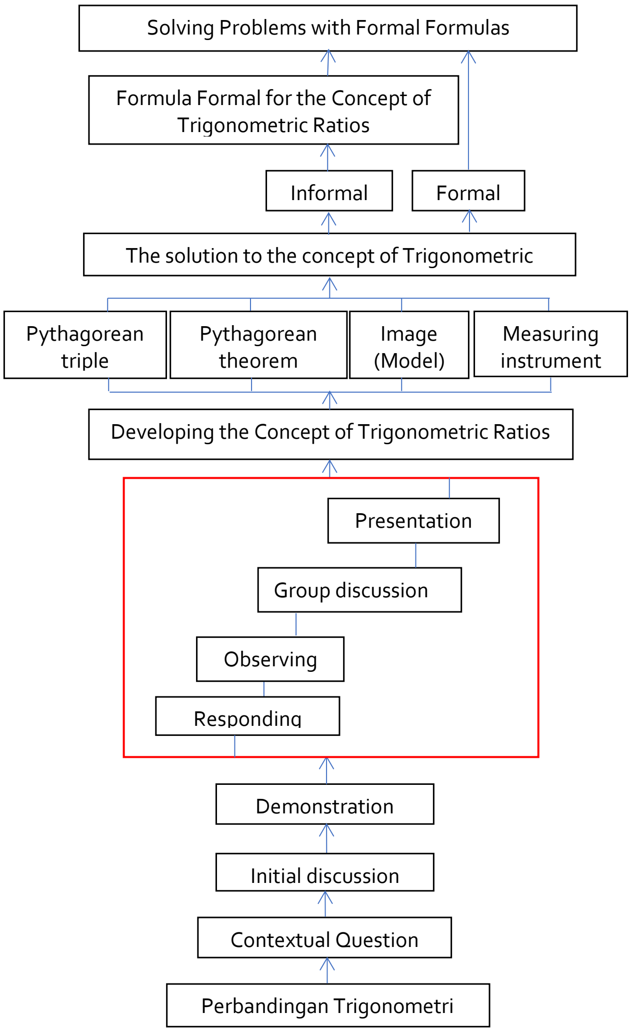


Before the learning process begins

Figure 6. Flowchart of LIT on the topic of Trigonometric Ratios before the lesson

The next difference is in the "responding" stage, which students go through after conducting a demonstration and before the "observing" stage. The act of "responding" is the first step for students to participate actively in the learning process. The students will provide their responses or feedback to the demonstration presented. Their knowledge will guide and stimulate the students to continue the learning process via the "observing" stage.

Before the learning process begins, the steps "activity/experiment," "observe," "group discussion," and "presentation" have the same level of suitabilityThere is no text provided. However, the post-learning phases in these four things exhibit distinct variations. The "responding" stage initiates the steps of learning. Next, the process begins with the "observation" stage, followed by the "group discussion" step, and concludes with the "presentation" step. The students go through the four steps of the learning process in different stages.



After the learning process is complete

Figure 7. Flowchart of LIT on the topic of Trigonometric Ratios after the lesson

The appropriate use of the LIT framework, as shown in Figure 7, allows students to construct an understanding of the concept of trigonometric ratios through everyday problems. They are able to identify trigonometric ratios and solve real-life issues accordingly, as a result of analyzing the HLT and conducting teaching experiments. Based on the results of the student's work using the LIT design for trigonometric comparison, the number of students capable of completing the learning activities is determined. The students are becoming accustomed to the learning process, which involves determining mathematics formally based on pre-established models. The utilization of

Pythagorean triples, the Pythagorean theorem, models (diagrams), and measuring tools can aid in achieving vertical mathematization. The students may effectively solve common problems by utilizing trigonometric ratios.

Discussion

Commented [Isti3]: In this discussion, you have just told the research facts. Let the research facts be in the Result section. In the discussion, you have not explained what new things you found. How did students respond to your LIT design? Then the discussion must be critical in the theoretical GAP. I have not found the big picture of your research either. You can describe it in the before-after research. What's new?

The findings reveal that there are still students who lack the prerequisite knowledge that should have been acquired based on the current curriculum. The lack of prerequisite knowledge is caused by external factors (Kurniasih et al., 2020; Puspita et al., 2023) such as mental unpreparedness in receiving knowledge, didactic design of teachers, and limitations in teaching materials. Teachers are still engaged in the process of teaching by providing students with rote memorization to remember trigonometric comparison formulas. Introducing the formula of trigonometric ratios without providing a geometric explanation leads to a lack of motivation for students to construct their understanding of trigonometric ratios (Andika et al., 2017; Urrutia et al., 2019). Learning methods that are teacher-centered and rely on memorization only provide knowledge about trigonometric comparisons to students for a short time. Still, this knowledge only lasts for a short time (Orhun, 2002). For example, the teacher introduces the concept of the sine of an angle in a right triangle as a ratio between the lengths of the opposite side and the hypotenuse. The idea of the sine of an angle in a right triangle as a ratio between the lengths of the opposite side and the hypotenuse can be found by rotating the right triangle to any position (Blackett & Tall, 1991).

Another important aspect for teachers is to enhance their comprehensive understanding of trigonometry so that they can connect and present different trigonometry concepts, which can construct students' knowledge of trigonometry (Rittle-Johnson & Schneider, 2014). Effective interactions between teachers and students facilitate students to actively ask questions, explain, and confirm the understanding they are developing. According to Dhungana's research, effective communication between teachers and students can significantly contribute to understanding trigonometry (Dhungana et al., 2023)

The majority of instructional materials provided to students focus on trigonometric functions (Fi, 2003). On the other hand, the material on trigonometric comparison is fundamental in understanding the material on trigonometric functions. Trigonometry has a significant role in various fields, such as in teaching geometric conjectures and proofs (Fiallo & Gutiérrez, 2017; Nurhayati, 2017). Possible errors in the learning process may arise in procedural knowledge, conceptual information, or the interplay between these two types of knowledge (Dhungana et al., 2023).

Testing and interviews with students unveiled that students require assistance comprehending the principles of trigonometric ratios as a result of their inadequate grasp of triangles (Dhungana et al., 2023). The difficulty of determining the correct position of angles and sides should have been overcome as students acquire the necessary knowledge. The process of building understanding among students is essential for determining the correct position of angles and sides.

Research has also discovered that certain students find learning more relevant, particularly in the subject of trigonometric ratios, when contextual life problems are utilized. Integrating students' everyday experiences using objects and diagrams (models) makes learning meaningful and constructed knowledge long-lasting (Dhungana et al., 2023).

The use of LIT (Local Instruction Theory) through the Realistic Mathematics Education (RME) approach enhances students' ability to construct their understanding of the concept of trigonometric ratios. Enhances students' ability is evident from the student's work, which

demonstrates outcomes that align with the learning objectives.

Implication of Research

The design of this LIT Trigonometric Comparison utilizes the syntax of the Realistic Mathematics Education (RME) approach. The genesis of this LIT Trigonometric ratio is a conjecture on the learning trajectory using the Realistic Mathematics Education (RME) approach. Therefore, the designed learning trajectory needs further development in accordance with the intended content to be achieved. For other researchers who will utilize the learning trajectory, it is recommended to master various learning steps or various methods in designing problems. By mastering problem design, one can obtain a variety of problem-solving solutions, both formally and informally.

Limitation

This research was conducted in two stages, each of which followed the steps of design research (Gravemeijer, 2004) namely: 1) developing the preliminary design, 2) conducting the teaching experiment, and 3) performing the retrospective analysis. There are two reasons for choosing only two stages in this research: firstly, limited researcher time and resources. Secondly, the material is only conducted once a year.

CONCLUSION

The outcome of the learning trajectory in this study is a Comparative Trigonometry LIT developed through the Realistic Mathematics Education (RME) approach. This theory encompasses several steps that students must go through in constructing their knowledge related to trigonometric comparisons. These steps nonetheless adhere to the syntax of the Realistic Mathematics Education (RME) approach.

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Development of A Local Instruction Theory for Trigonometric Ratios

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Abstract

Several students view the concept of trigonometric ratios as intricate. This work aims to create the Local Instruction Theory (LIT) for instructing trigonometric ratios utilizing Realistic Mathematic Education (RME). Researchers developed a Learning Instruction Theory (LIT) to aid students in constructing a robust comprehension of the fundamental principles of trigonometric ratios. They accomplished this by identifying a highly efficient learning trajectory. The study approach utilizes a specific research design. The development of the Hypothetical Learning Trajectory (HLT) is now underway to improve students' comprehension of trigonometric ratios. The development of local instruction theory for trigonometric ratios involves three stages: formulating an initial plan, carrying out teaching experiments, and doing retrospective analysis. Through qualitative analysis of teaching experiments, it has been determined that incorporating instructional materials on trigonometric ratios in LIT that align with RME identities effectively assists students in developing fundamental concepts in trigonometric ratios.

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Keywords: Local Instruction Theory; Hypothetical Learning Trajectory; Trigonometric Ratios; RME

Abstrak

Beberapa siswa menganggap materi perbandingan trigonometri cukup sulit. Penelitian ini bertujuan untuk mengembangkan Local Instruction Theory (LIT) untuk pembelajaran perbandingan trigonometri dengan menggunakan Realistic Mathematic Education (RME). Dalam upaya membantu siswa membangun konsep dasar pada materi perbandingan trigonometri, peneliti mengembangkan alur LIT dengan menemukan jalur pembelajaran yang efektif. Pencapaian tujuan penelitian menggunakan desain penelitian. Serangkaian kegiatan mengembangkan Hypothetical Learning Trajectory (HLT) sehingga siswa sekolah menengah atas (SHS) memiliki pemahaman yang lebih baik tentang perbandingan trigonometri. Pengembangan Local Instruction Theory untuk Perbandingan Trigonometri meliputi tiga tahap yaitu mengembangkan desain awal, melakukan percobaan pengajaran, dan melaksanakan analisis retrospektif. Siswa mampu membangun pemahaman tentang perbandingan trigonometri selama proses pembelajaran berlangsung. Berdasarkan analisis kualitatif eksperimen pengajaran, penggunaan bahan ajar LIT perbandingan trigonometri yang mengakomodir identitas RME, mampu memandu siswa dalam membangun konsep dasar pada materi perbandingan trigonometri.

Mathematics Clasification: ---- must be filled ----

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Metadata of Author
First author: SCOPUS ID: - optional - | ORCID ID: - mandatory - | Google Scholar : VmAlN5YAAAAJ
Co-author: SCOPUS ID: - *57196237689* - | ORCID ID: - mandatory - | Google Scholar : Kc-9bRgAAAAJ

INTRODUCTION

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You also should highlighted your research question, your research urgency.

The topic of trigonometric ratios is fundamental in the application of advanced applied sciences and pure sciences. Various fields of study require the use of trigonometry as an essential basis for applying their concepts. For example, the utilization of trigonometric ratios supports the proof of Kepler's first law (Simha, 2021). The importance of understanding trigonometric ratios lies in its role as fundamental knowledge for comprehending pre-calculus and calculus, such as algebra, geometry, and the ability to create graphs (Weber, 2005). Trigonometric comparison is utilized in calculating the width or height of various shapes, such as a bridge, household tools, and bicycles (Urrutia et al., 2019). Indeed, students see the topic of trigonometric ratios as challenging (Blackett & Tall, 1991; Weber, 2005; Yiğit Koyunkaya, 2016; Karthikeyan, 2017; Urrutia et al., 2019; Nordlander, 2022). During the initial stage of learning, students encounter difficulties in connecting triangle images to numerical relationships, as well as problems in manipulating the involved symbols (Blackett & Tall, 1991), and many students are unable to overcome these difficulties. While studying right triangles, students struggled to grasp the concept of trigonometric functions, such as sine, cosine, and tangent, which are defined as ratios (Weber, 2005a, 2008). According to Yiğit (2016), students exhibit a lack of flexibility when solving trigonometric comparison problems (Yiğit Koyunkaya, 2016). Students expressed that trigonometry was a subject that was highly stimulating and conceptual in comparison to other areas of mathematics (Karthikeyan, 2017). According to Sandip's research, trigonometry is a challenging subject that is disconnected from everyday life (Urrutia et al., 2019). Typically, students struggle to distinguish the front side, side view, and slanted side of a right triangle despite the fact that this distinction greatly aids their comprehension of trigonometric ratios. The topic of trigonometric comparison is considered abstract for students (Karthikeyan, 2017).

From the preliminary study and literature review conducted by the researcher on textbooks, instructional videos, analysis of students' learning difficulties, and curriculum analysis, several aspects have caught the researcher's attention. One of these aspects is the emergence of real-world contexts in trigonometry materials in textbooks. Various examples have begun to present contextual situations, but teachers still need to learn how students might construct formal concepts from the presented contextual case. Batanero dan Diaz (2012) argue that the existing textbooks and curriculum documents need to sufficiently support learning, as the presented concepts are narrow in scope . (Batanero & Díaz, 2012).

The typical pattern of learning that occurs in the delivery of contextual mathematics is the initial introduction and presentation of example problems, followed by the completion of practice exercises using general formulas. The typical pattern of learning still represents the teaching culture in carrying out the learning process. According to Weber (2005), mathematics learning in schools still follows the conventional sequence of explanation, example provision, and problem-solving practice. The learning process via direct application of general formulas can prevent students from developing their mathematical concepts independently (Weber, 2005b). Their thinking patterns need to be trained, resulting in a lack of familiarity with how these formulas are derived.

The application of real-life models to formal mathematical concepts has yet to be documented in supporting learning materials. The teacher must possess the ability to elucidate the models

generated by students during the learning process, which can subsequently facilitate the comprehension of formal mathematics. Turmudi (2018) states that students should have a strong desire in the learning process to independently discover concepts, provided that teachers provide facilities, teaching materials, and learning resources (Turmudi, 2012).

According to Sandip's research, trigonometry is a challenging subject that is disconnected from everyday life (Adhikari & Subedi, 2021; Asomah et al., 2022; Dhungana et al., 2023). To bridge this gap, Realistic Mathematics Education serves as an alternative to teaching trigonometric ratios. Realistic Mathematics Education (RME) is a teaching and learning method in math education that follows the philosophy of Hans Freudenthal, who sees math as a human endeavor. (Freudenthal, 1973; Gravemeijer & Terwel, 2000; Treffers, 1987; Van den Heuvel-Panhuizen & Drijvers, 2020).

Realistic Mathematics Education is characterized by students experiencing a concrete learning process that is aligned with real-life situations and contextualized according to their everyday lives (Yuanita et al., 2018). Students undergo an informal process of constructing their understanding based on what they see, observe, and comprehend about their surrounding environment, be it at home, school, or wherever they learn (Meika, 2018).

The three primary ideas of Realistic Mathematics Education are guided reinvention and progressive mathematizing, didactical phenomenology, and self-developed models. (Gravemeijer, 1994). During the teaching and learning process, these three principles are elaborated into five characteristics of RME, namely: 1) Building and solidifying; 2) degree and frameworks; 3) contemplation and specific task; 4) societal environment and communication; and 5) organizing and intertwining (Gravemeijer, 1994; Jzn & de, 1987). The significance of the Realistic Mathematics Education approach resides in its ability to empower students to develop concepts through interactive teaching and learning processes. The teachers has a crucial role in enabling students to explore concepts using both administrative and motivational methods (Meika, 2018).

The mistakes made by previous students should not be repeated in the following years. Penyediaan materi pembelajaran adalah salah satu solusi yang tepat untuk memulihkan keadaan menjadi lebih baik. The focus of learning will be further narrowed down if the instructional materials used are appropriate. In this teaching material, steps are needed that can facilitate students' learning paths. According to Gravemeijer, to improve mathematics education, it is necessary to implement an instructional design that facilitates learning and enables students to progress from their current level of thinking to a more sophisticated level of mathematical thinking (Gravemeijer, 2004).

The Local Instruction Theory establishes the learning process by incorporating a learning trajectory. Local Instruction Theory (LIT) is a comprehensive theory that includes both the process of acquiring knowledge on a specific topic and the methods used to facilitate that learning (Liljekvist et al., 2017). (Gravemeijer, 2004) states that Local Instruction Theory is an exact theory that can assist students in developing mathematical thinking on a particular topic or material through detailed and gradual instructional mediumResearchers will develop a systematic learning path, also known as a learning trajectory, within the process of teaching and learning. This design is the one Gravemeijer mentioned.

LIT is used to describe the appropriate learning route or learning activities. LIT not only offers how learning takes place but also facilitates a range of specific learning activities that may work. The existence of LIT provides a basis for teachers to consider adaptations that may be needed in implementing new approaches. In addition, LIT offers a framework in which teachers can develop hypotheses about learning trajectories. LIT is tailored to the specific circumstances of the particular

class and teacher. Teachers need to build HLT (Hypothetical Learning Trajectories) that are suitable for their own situations while using LIT as a foundational reference (Gravemeijer, 2004).

Gravemeijer proposes a research design for the development of local instruction theory. The research design consists of three phases: 1) preliminary design (developing initial design), 2) teaching experiment (doing teaching experiment), and 3) retrospective analysis (performing retrospective analysis) (Gravemeijer, 2004). Preliminary design is the outcome of an initial learning plan aimed at implementing ideas about the development of instructional materials for LIT (Meika, 2018). This design is expected to support students in the construction of mathematical thinking from an informal form to a formal one. The construction of mathematical thinking is achieved through a guided reinvention process facilitated by mathematizing. Mathematizing refers to several ways of managing learning activities to demonstrate mathematical characteristics, such as generality, certainty, accuracy, and agility (Loc & Hao, 2016). The purpose of the teaching experiment is to evaluate the effectiveness of the instructional materials for LIT. Retrospective analysis is employed to examine the complete dataset of the teaching experiment outcomes.

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HLT is applied in every learning activity and has a crucial role in designing learning activities for students. The execution of learning is indissociable from the learning trajectory, encompassing a deliberate progression of learning materials and idea maps that students traverse during the learning process (Prahmana, 2016). Next, in the subsequent stage, the trigonometric ratios LIT is produced.

According to the given definition, it is crucial to create and design instructional resources that can reduce students' challenges in developing cognitive processes through Realistic Mathematics Education in order to build comprehension of trigonometric concepts. The desired teaching materials are obtained through the LIT design. The LIT design is used to address the difficulties experienced by students and to enhance their mathematical thinking process, particularly their understanding of trigonometric ratios through Realistic Mathematics Education.

METHOD

Commented [User4]: Why do you use design research? How do you organize the participant of your research? How do you maintain the validation of your research? Explain after the chart.

A design research method based on Gravemeijer's approach is the research methodology used. This research method is focused on local instruction theory, or LIT, which was developed by researchers in collaboration with teachers to achieve better learning quality (Gravemeijer & van Eerde, 2009). A study paradigm known as "design research" tries to create a series of tasks and comprehend the workings of the learning process. This study aims to develop and construct a Local Instruction Theory (LIT) on the subject of trigonometric ratios, with a focus on Realistic Mathematics Education (RME).

The research implementation is carried out in two stages, with each stage referring to the steps of design research (Gravemeijer, 2004) namely: The three main stages of the project include 1) formulating the initial concept, 2) executing the educational experiment, and 3) carrying out a retrospective study. The subject of the study during the teaching experiment phase consisted of 25 tenth-grade students from SMAN 7, Bandung. An analysis was conducted on the activities performed in the initial phase, and subsequently, recommendations were given to enhance the activities in the second phase. The findings from the data analysis of the second phase provide the final recommendations of this research.

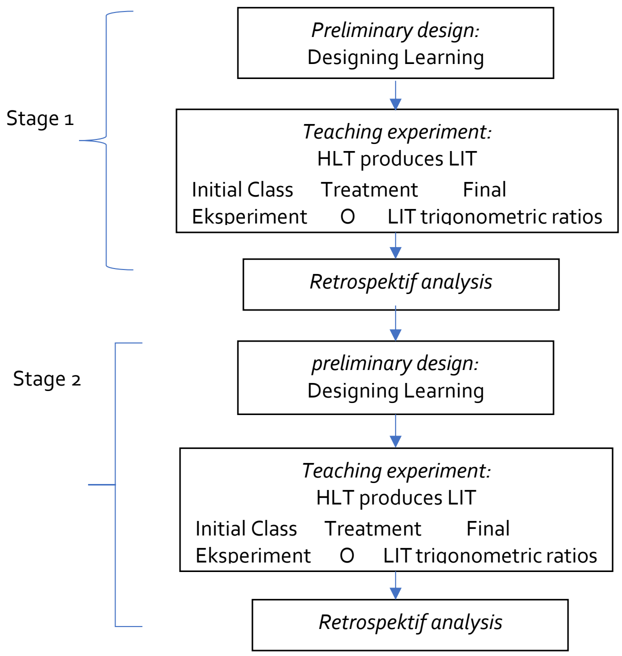


Figure 1. Research Design

RESULTS AND DISCUSSION

Results

The research is carried out in three stages: 1) formulating the fundamental design, 2) executing the teaching experiment, and 3) implementing the retrospective analysis.

Elaborating on the Initial Design (*preliminary design*)

During this phase, the researcher assembles the educational materials and finalizes the instructional content. To meet the study aims, various actions are conducted, including (1) identifying the research subjects based on their diverse cognitive abilities. The researchers selected students from class X of SMAN 7 as appropriate research subjects; (2) examining and analyzing the curriculum to identify the compatibility of trigonometric comparison material with the curriculum currently being used in the school; (3) analyzing trigonometric comparison material to obtain an overview that "the achievement of Phase E-learning based on the derived geometry domain becomes the learning objective," in accordance with the research objectives. Next, the researcher developed a teaching material design in the form of a descriptive outline of trigonometric comparisons, also known as "Local Instruction Theory Descriptive Teaching Material (UBA-LIT)". The Realistic Mathematics Education (RME) technique enhances the learning process by employing diverse learning materials, including teaching modules and research tools. The study instrument is designed to incorporate the specific attributes of Realistic Mathematics Education (RME) and the

curriculum used by the school. Additionally, the researchers analyze and assess the learning assistance tools. The result of the evaluation is referred to as the prototype. In the next stage, the research conducts an expert review, in which four experts validate the prototype based on its structure, content, and language.

Once the expert review procedure is finished, students who have been given trigonometric comparison information are tested on the prototype to get data on the readability and comprehension of the instructional material. The researchers engage in communication with the students during the testing process to anticipate any corrections or feedback regarding the instructional materials being tested by the students. The students write their input or suggestions on the prepared sheet. Students provide support for the discovery of formal concepts related to UBA-LIT through their comments. The researchers made improvements to the UBA-LIT after receiving feedback. After conducting expert validity testing and gathering comments from students, it was ultimately determined that the prototype design of the instructional material product generated by the researcher is valid at this level. The updated iteration of the prototype is referred to as the second prototype. Following that, the second prototype underwent testing in teaching experiment 1, including a total of 30 students from SMAN 7 Kota Bandung.

From a design research standpoint, the primary objective of the initial design is to develop a Local Instruction Theory (LIT) that can be visually portrayed as a portrayal of the process of acquiring and improving knowledge through teaching experiments (Gravemeijer & Cobb, 2006). During the preliminary study, the researcher designed the learning process activities based on a literature review. This set of educational exercises covers the beliefs held by students while they are building knowledge and the methods that will be used, which will be presented as an initial challenge, including advanced thinking skills. The contents of HLT are subject to change. The activities in HLT can be adapted and customized based on the circumstances and parameters of the ongoing educational experiment. HLT has a curriculum that students will adhere to when studying trigonometric ratios. The HLT curriculum comprises learning objectives, activity descriptions, and student thought conjectures that occur during the learning process. During the teaching experiment, HLT functions as a reference for teachers and researchers to determine the teaching emphasis, observations, and interviews. Moreover, HLT functions as a point of reference for the examination or analysis of a dataset in the context of a teaching experiment. Hypotethical Learning Trajectory (HLT) used in the teaching experiment is displayed in Table 1.

Tabel 1. HLT illustration for learning the concept of trigonometry comparisons

Activity	Learning Objectives	Activity Description	Conjecture
Students Demonstrate the activity of identifying by body height.	The students are able to identify the front, side, and diagonal positions by referring to specific angles of a right-angled	- The teacher instructed three students to engage in a practical exercise involving the formation of a right-angled	- Some students may be able to illustrate a sketch of a right triangle from the demonstration activity. - Some students may quickly recognize the

	triangle.	triangle using a table base, a measuring tape, and their height. - Each group carefully observes the demonstration and discusses the position of the corner, front, side, and slant.	positions of the corner, front side, side side, and hypotenuse.
Shows high shadow intensity activity	The students can articulate the trigonometric ratios (sin θ, cos θ, and tan θ) by employing the notion of ratios within a right triangle.	- The teacher instructs the students to face away from the light until a shadow appears in front of the standing student. The sketch formed is a right-angled triangle. - Students can determine the front side, side view, and diagonal side of a right triangle sketch based on the determining angle. Students will be *guided* in *rediscovering the concept* of trigonometric ratios.	- Some students may use tables as a tool to solve problems. - Certain students may employ finger manipulation methods to solve difficulties. - Some students may utilize measuring instruments to solve problems. - Some students may use the number parity method to solve problems. - Certain students may possess the ability to represent this activity in a visual manner. They initiated the construction of a model and engaged in horizontal and vertical mathematization in order to uncover the notion of trigonometric ratios through the shown examples.

			- - Some students may be able to relate the demonstration process to the concept of trigonometric ratios.
Problem-Solving	Students are able to solve real-life problems related to right triangles using trigonometric ratios	Students use trigonometric ratios to solve real-life problems by performing calculations.	- Certain students possess the ability to perceive the relationship between sine and cosine values in mathematical calculations and are capable of streamlining the calculation process. - Some students may be able to remember or repeat knowledge they have previously acquired to solve problems. - Some students may use tables, finger methods, odd-even tricks, or measuring tools to solve problems. - Some students may directly apply trigonometric ratio concepts.

Teaching Experiment

The primary objective of a teaching experiment is to systematically evaluate and enhance original ideas regarding Local Instruction Theory (LIT) while also striving to gain a more profound comprehension of the underlying mechanisms and functioning of LIT (Gravemeijer & Cobb, 2006).

Activity 1

The teacher requested three students to perform a demonstration using the provided tables/chairs and ropes.

The teacher requested one of the students to fasten the rope to the base of the table or chair provided. A student holds the end of the neat rope and moves back 1 meter from the base of the table/chair. After that, the student who is holding the end of the rapia rope shifts his grip to the end of the head. Other students are asked to draw sketches of the shapes that are occurring. The other students observed the demonstration requested by the teacher, depicted the requested sketch, and identified the positions of the angles, front side, side view, and oblique side of the formed triangle sketch. All students strive to comprehend the right-angled triangle sketch and identify the positions

of angles, front side, side view, and hypotenuse on the sketch. The teacher provides a comprehensive understanding through the following activities.

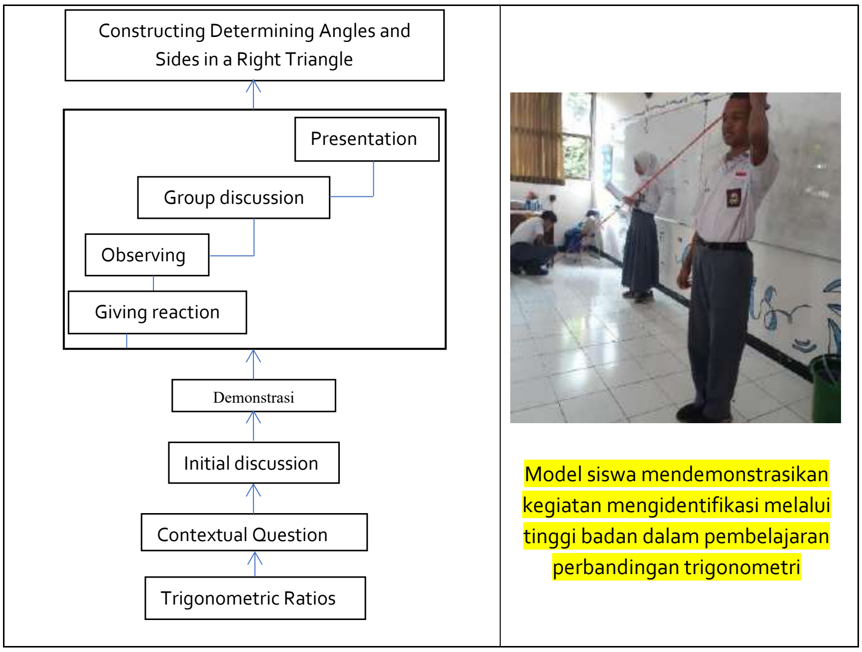


Figure 2. Demonstration of the flow of LIT and height in learning comparative trigonometry

Activity 2

The teacher instructs the class to define the trigonometric ratios (sin θ, cos θ, and tan θ) with a demonstration involving the height of a shadow. This activity guides students in formally discovering the concept of trigonometric comparisons. The students were able to perform this activity proficiently after acquiring the idea of identifying angles and sides in the previous activity. The presented issues are contextual and aligned with the student's level of understanding. The LIT design is structured from the simplest to the most complex in order to provide a general solution. In this activity, the teacher instructs the students to face away from the light source so that a shadow appears in front of the standing students. The results of the student's practice are represented in the form of an image or sketch of a right-angled triangle. This sketch facilitates students in identifying the positions of angles, front side, side view, and inclined side of a right triangle based on the predetermined angle. Creating a sketch of a right triangle and determining angles helps students mathematize vertically and find formal solutions to define trigonometric ratios.

Figure 3 illustrates various outcomes of students' process variations in discovering formal mathematics from the concept of trigonometric ratios. The students' work begins with a demonstration of a contextual problem, allowing them to find solutions using various methods based on their prior knowledge. During the learning process, students are guided to discover formal mathematics, which is a process of mathematization based on the presented problems

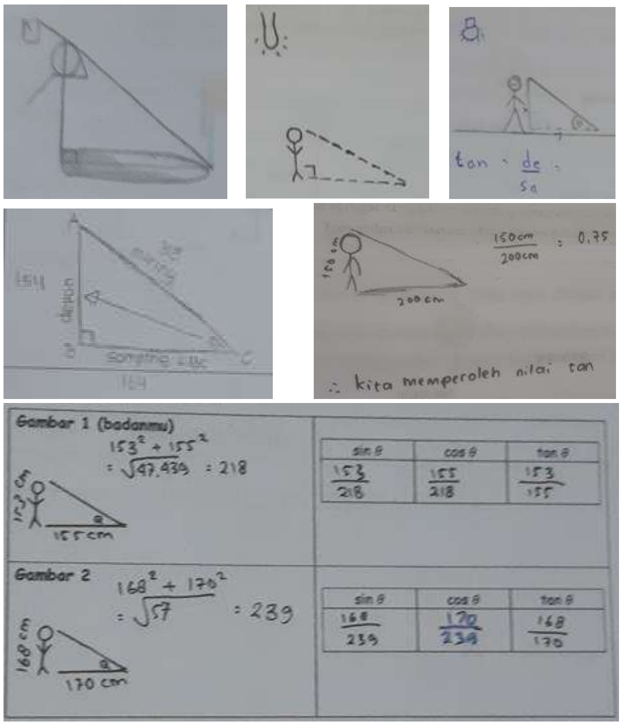


Figure 3. The vertical mathematical process of students in finding formal solutions to trigonometric ratios

Activity 3

Activity 3 begins with the teacher presenting problems based on everyday life situations. Everyday scenarios were employed to enable students to apply the notion of trigonometric ratios in practical circumstances. After the activity is finished, the teacher conducts a test as a final evaluation. One of the trigonometric comparison problems presented is as follows:

> "Perhatikan gambar tempat tidur berikut. What are the geometric shapes that occur between the stairs, bed, and floor? Sketch a model. (1) *Does the shape formed constitute a right triangle? If so, mark one corner with a right angle*. (2) *If the angle between the stairs and the floor is unknown, and the distance from the bed to the stairs is 10 cm. Place the obtained data into the image. Subsequently, calculate the distance from the floor to the end of the staircase, which forms a right-angled triangle, using the tangent ratio. (*$\tan 37^0 = 0{,}75$)"

Figure 4. Image of a bed as a model for the use of trigonometric ratios

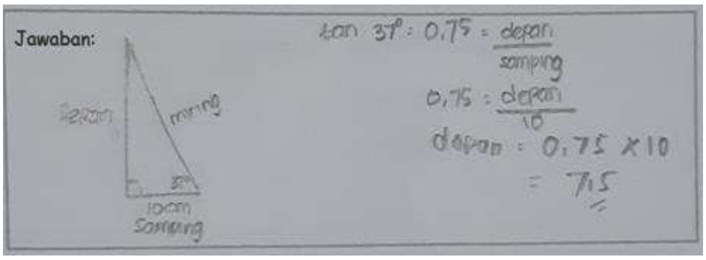


Figure 5. One of the student's responses was to solve the story problem

Figure 5 is an example of a student's response using the concept of trigonometric ratios to solve a word problem. The student solves the problem using the formal solution of trigonometric ratios that was previously constructed.

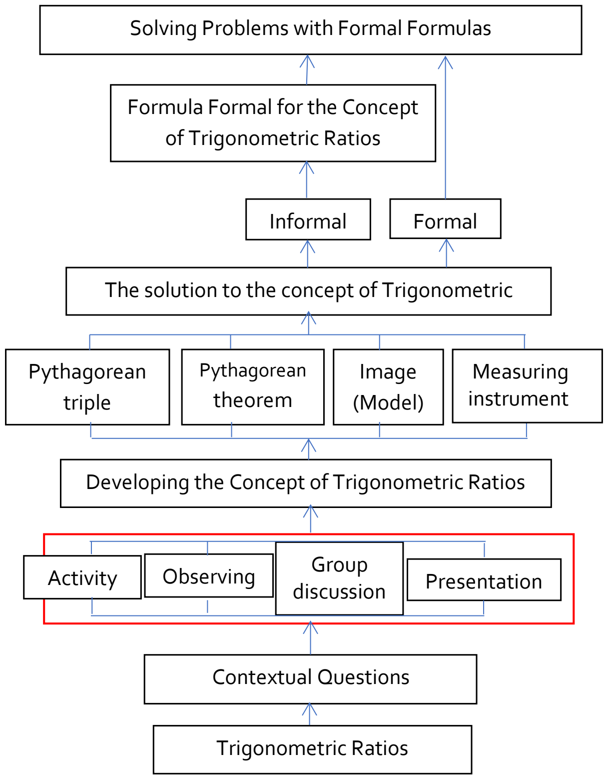
Analisis Retrospektif

Researchers employ Hypothetical Learning Trajectory (HLT) as a first guide and reference when assessing a dataset during the data analysis stage of a teaching experiment. The HLT created is compared to the classroom learning process. The analysis includes descriptions of anticipated occurrences during the learning process and examples of behaviors that will occur among students. The conclusions drawn from the analysis will answer the research questions. A good design is not the main objective of the study but rather the reason for the study's success, which is the main objective (Gravemeijer & Cobb, 2006).

The many activities carried out during the learning process are represented in the flowchart of LIT before the learning process on the topic of trigonometric ratios. The flowchart is presented in Figure 6. The image shows the differences in stages or steps during the learning process. The difference is evident in the red-colored box. Before the learning process begins, the design aims to facilitate activities, experiments, observations, group discussions, and presentations as processes that run parallel. Students will go through these stages simultaneously.

Meanwhile, after the learning process takes place, a revision of the initial design is made. The stage of differentiation involves an initial discussion process with students prior to the demonstration (*Figure 7*). This stage provides students with an introduction to understand the

concept of trigonometric ratios in relation to the prior knowledge they have already constructed. This stage is considered crucial because linking it to the previously acquired knowledge is necessary for students to comprehend the fundamental concepts of trigonometric ratios.

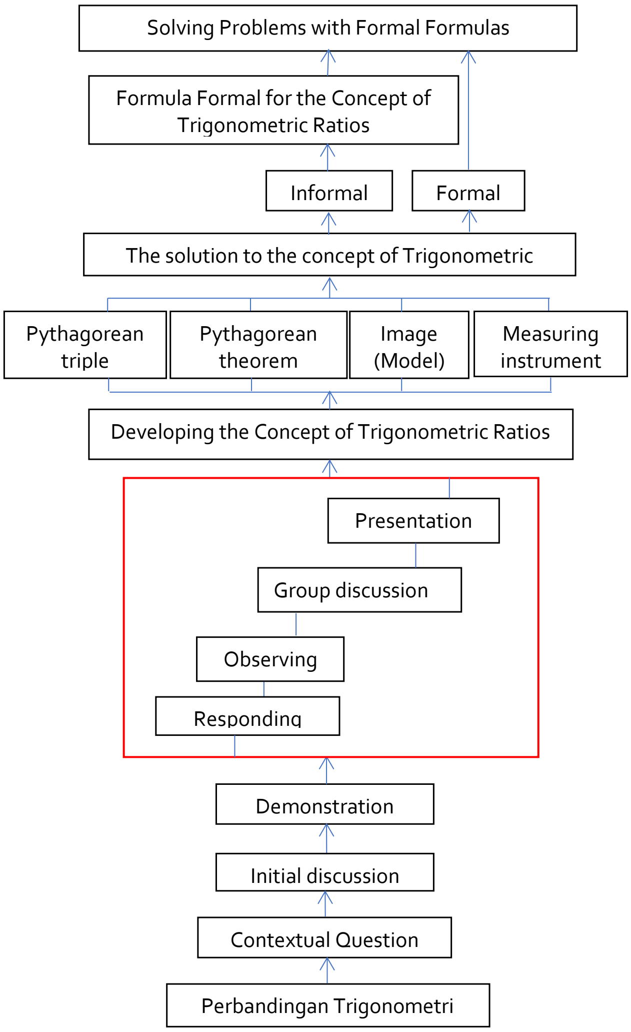


Before the learning process begins

Figure 6. Flowchart of LIT on the topic of Trigonometric Ratios before the lesson

The next difference is in the "responding" stage, which students go through after conducting a demonstration and before the "observing" stage. The act of "responding" is the first step for students to participate actively in the learning process. The students will provide their responses or feedback to the demonstration presented. Their knowledge will guide and stimulate the students to continue the learning process via the "observing" stage.

Before the learning process begins, the steps "activity/experiment," "observe," "group discussion," and "presentation" have the same level of suitabilityThere is no text provided. However, the post-learning phases in these four things exhibit distinct variations. The "responding" stage initiates the steps of learning. Next, the process begins with the "observation" stage, followed by the "group discussion" step, and concludes with the "presentation" step. The students go through the four steps of the learning process in different stages.



After the learning process is complete

Figure 7. Flowchart of LIT on the topic of Trigonometric Ratios after the lesson

The appropriate use of the LIT framework, as shown in Figure 7, allows students to construct an understanding of the concept of trigonometric ratios through everyday problems. They are able to identify trigonometric ratios and solve real-life issues accordingly, as a result of analyzing the HLT and conducting teaching experiments. Based on the results of the student's work using the LIT design for trigonometric comparison, the number of students capable of completing the learning activities is determined. The students are becoming accustomed to the learning process, which involves determining mathematics formally based on pre-established models. The utilization of

Pythagorean triples, the Pythagorean theorem, models (diagrams), and measuring tools can aid in achieving vertical mathematization. The students may effectively solve common problems by utilizing trigonometric ratios.

Discussion

The findings reveal that there are still students who lack the prerequisite knowledge that should have been acquired based on the current curriculum. The lack of prerequisite knowledge is caused by external factors (Kurniasih et al., 2020; Puspita et al., 2023) such as mental unpreparedness in receiving knowledge, didactic design of teachers, and limitations in teaching materials. Teachers are still engaged in the process of teaching by providing students with rote memorization to remember trigonometric comparison formulas. Introducing the formula of trigonometric ratios without providing a geometric explanation leads to a lack of motivation for students to construct their understanding of trigonometric ratios (Andika et al., 2017; Urrutia et al., 2019). Learning methods that are teacher-centered and rely on memorization only provide knowledge about trigonometric comparisons to students for a short time. Still, this knowledge only lasts for a short time (Orhun, 2002). For example, the teacher introduces the concept of the sine of an angle in a right triangle as a ratio between the lengths of the opposite side and the hypotenuse. The idea of the sine of an angle in a right triangle as a ratio between the lengths of the opposite side and the hypotenuse can be found by rotating the right triangle to any position (Blackett & Tall, 1991).

Another important aspect for teachers is to enhance their comprehensive understanding of trigonometry so that they can connect and present different trigonometry concepts, which can construct students' knowledge of trigonometry (Rittle-Johnson & Schneider, 2014). Effective interactions between teachers and students facilitate students to actively ask questions, explain, and confirm the understanding they are developing. According to Dhungana's research, effective communication between teachers and students can significantly contribute to understanding trigonometry (Dhungana et al., 2023)

The majority of instructional materials provided to students focus on trigonometric functions (Fi, 2003). On the other hand, the material on trigonometric comparison is fundamental in understanding the material on trigonometric functions. Trigonometry has a significant role in various fields, such as in teaching geometric conjectures and proofs (Fiallo & Gutiérrez, 2017; Nurhayati, 2017). Possible errors in the learning process may arise in procedural knowledge, conceptual information, or the interplay between these two types of knowledge (Dhungana et al., 2023).

Testing and interviews with students unveiled that students require assistance comprehending the principles of trigonometric ratios as a result of their inadequate grasp of triangles (Dhungana et al., 2023). The difficulty of determining the correct position of angles and sides should have been overcome as students acquire the necessary knowledge. The process of building understanding among students is essential for determining the correct position of angles and sides.

Research has also discovered that certain students find learning more relevant, particularly in the subject of trigonometric ratios, when contextual life problems are utilized. Integrating students' everyday experiences using objects and diagrams (models) makes learning meaningful and constructed knowledge long-lasting (Dhungana et al., 2023).

The use of LIT (Local Instruction Theory) through the Realistic Mathematics Education (RME) approach enhances students' ability to construct their understanding of the concept of trigonometric ratios. Enhances students' ability is evident from the student's work, which

Commented [User5]: Clarity and Focus:
- Some parts of the discussion are repetitive (e.g., the explanation of sine ratios). A more concise presentation would improve readability.
- The transition between problems and proposed solutions could be more seamless.

Details on LIT and RME:
- While the potential of LIT and RME is highlighted, the specifics of how these approaches were implemented could be expanded upon. For example:
  - What specific real-life problems or models were used?
  - How did students respond during the teaching experiment?

Addressing Limitations:
- The discussion mentions resource and time constraints as limitations but does not suggest how future research might address these issues.

Student Diversity:
- The research could explore variations in student experiences based on factors such as prior knowledge, learning styles, or socio-economic backgrounds.

demonstrates outcomes that align with the learning objectives.

Implication of Research

The design of this LIT Trigonometric Comparison utilizes the syntax of the Realistic Mathematics Education (RME) approach. The genesis of this LIT Trigonometric ratio is a conjecture on the learning trajectory using the Realistic Mathematics Education (RME) approach. Therefore, the designed learning trajectory needs further development in accordance with the intended content to be achieved. For other researchers who will utilize the learning trajectory, it is recommended to master various learning steps or various methods in designing problems. By mastering problem design, one can obtain a variety of problem-solving solutions, both formally and informally.

Limitation

This research was conducted in two stages, each of which followed the steps of design research (Gravemeijer, 2004) namely: 1) developing the preliminary design, 2) conducting the teaching experiment, and 3) performing the retrospective analysis. There are two reasons for choosing only two stages in this research: firstly, limited researcher time and resources. Secondly, the material is only conducted once a year.

CONCLUSION

The outcome of the learning trajectory in this study is a Comparative Trigonometry LIT developed through the Realistic Mathematics Education (RME) approach. This theory encompasses several steps that students must go through in constructing their knowledge related to trigonometric comparisons. These steps nonetheless adhere to the syntax of the Realistic Mathematics Education (RME) approach.

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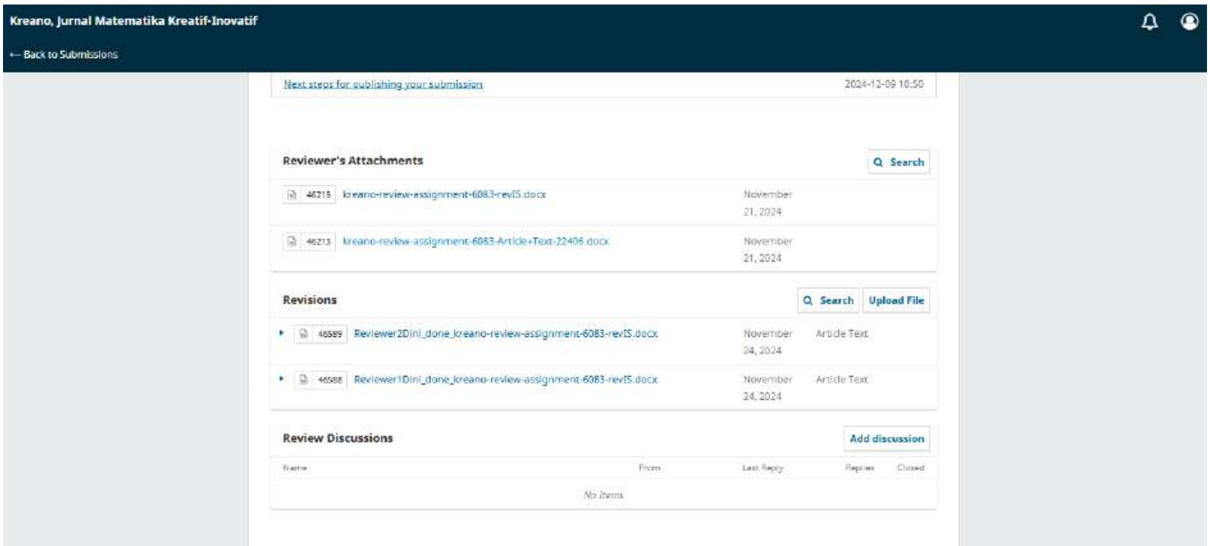
3. Bukti Konfirmasi Submit Revisi Pertama, Respon Kepada Reviewer, dan Artikel Yang Diresubmit

(24 November 2024)



Bukti Konfirmasi Submit Revisi Pertama dan Respon kepada Reviewer pada 24 November 2024 melalui OJS https://journal.unnes.ac.id/journals/kreano/authorDashboard/submission/6083

Development of A Local Instruction Theory for Trigonometric Ratios

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Abstract

Some students find trigonometric ratios challenging. Research on the application of Local Instruction Theory (LIT) in trigonometry is limited, particularly in secondary school understanding of trigonometric ratios. This study aims to develop an LIT for trigonometric ratios using Realistic Mathematics Education (RME). The researcher designed a learning pathway to help students grasp the fundamental concepts of trigonometric ratios. The study employs a research design methodology, developing a Hypothetical Learning Trajectory (HLT) to improve students' understanding. The development of LIT for Trigonometric Ratios follows three stages: initial design, teaching experiments, and retrospective analysis. Students demonstrated the ability to understand trigonometric ratios through the learning process. The findings suggest that the use of LIT-based instructional materials, incorporating RME principles, significantly enhances students' conceptual understanding of trigonometric ratios in high school.

Keywords: Local Instruction Theory; Hypothetical Learning Trajectory; Trigonometric Ratios; RME

Abstrak

Beberapa siswa menganggap materi perbandingan trigonometri cukup sulit. Penelitian tentang penerapan Local Instruction Theory (LIT) dalam pembelajaran trigonometri masih sangat terbatas, khususnya terkait dengan pemahaman rasio trigonometri di sekolah menengah. Penelitian ini bertujuan untuk mengembangkan Local Instruction Theory (LIT) perbandingan trigonometri dengan menggunakan Realistic Mathematic Education (RME). Dalam upaya membantu siswa membangun konsep dasar pada materi perbandingan trigonometri, peneliti mengembangkan alur LIT dengan menemukan jalur pembelajaran yang efektif. Pencapaian tujuan penelitian menggunakan desain penelitian. Serangkaian kegiatan mengembangkan Hypothetical Learning Trajectory (HLT) sehingga siswa sekolah menengah atas (SMA) memiliki pemahaman yang lebih baik tentang perbandingan trigonometri. Pengembangan Local Instruction Theory untuk Perbandingan Trigonometri meliputi tiga tahap yaitu mengembangkan desain awal, melakukan percobaan pengajaran, dan melaksanakan analisis retrospektif. Siswa mampu membangun pemahaman tentang perbandingan trigonometri selama proses pembelajaran berlangsung. Berdasarkan analisis kualitatif eksperimen pengajaran, penelitian ini berimplikasi pada penguatan Local Instruction Theory (LIT) perbandingan trigonometri dan pengembangan bahan ajar berbasis RME.

Mathematics Clasification: ---- must be filled ----

Please check Mathematics Clasification here:

https://mathscinet.ams.org/msc/msc2010.html?t=97-XX&s=&btn=Search&ls=s

Metadata of Author

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Co-author: SCOPUS ID: - *57196237689* - | ORCID ID: - mandatory - | Google Scholar : Kc-9bRgAAAAJ

INTRODUCTION

The topic of trigonometric ratios is fundamental in the application of advanced applied sciences and pure sciences. Various fields of study require the use of trigonometry as an essential basis for applying their concepts. For example, the utilization of trigonometric ratios supports the proof of Kepler's first law (Simha, 2021). The importance of understanding trigonometric ratios lies in its role as fundamental knowledge for comprehending pre-calculus and calculus, such as algebra, geometry, and the ability to create graphs (Weber, 2005). Trigonometric ratio is utilized in calculating the width or height of various shapes, such as a bridge, household tools, and bicycles (Urrutia et al., 2019). Indeed, students see the topic of trigonometric ratios as challenging (Blackett & Tall, 1991; Weber, 2005; Yiğit Koyunkaya, 2016; Karthikeyan, 2017; Urrutia et al., 2019; Nordlander, 2022). During the initial stage of learning, students encounter difficulties in connecting triangle images to numerical relationships, as well as problems in manipulating the involved symbols (Blackett & Tall, 1991), and many students are unable to overcome these difficulties. While studying right triangles, students struggled to grasp the concept of trigonometric functions, such as sine, cosine, and tangent, which are defined as ratios (Weber, 2005a, 2008). According to Yiğit (2016), students exhibit a lack of flexibility when solving trigonometric comparison problems (Yiğit Koyunkaya, 2016). Students expressed that trigonometry was a subject that was highly stimulating and conceptual in comparison to other areas of mathematics (Karthikeyan, 2017). According to Sandip's research, trigonometry is a challenging subject that is disconnected from everyday life (Urrutia et al., 2019). Typically, students struggle to distinguish the front side, side view, and slanted side of a right triangle despite the fact that this distinction greatly aids their comprehension of trigonometric ratios. The topic of trigonometric ratio is considered abstract for students (Karthikeyan, 2017).

From the preliminary study and literature review conducted by the researcher on textbooks, instructional videos, analysis of students' learning difficulties, and curriculum analysis, several aspects have caught the researcher's attention. One of these aspects is the emergence of real-world contexts in trigonometry materials in textbooks. Various examples have begun to present contextual situations, but teachers still need to learn how students might construct formal concepts from the presented contextual case. Batanero dan Diaz (2012) argue that the existing textbooks and curriculum documents need to sufficiently support learning, as the presented concepts are narrow in scope *(Batanero & Díaz, 2012)*.

The typical pattern of learning that occurs in the delivery of contextual mathematics is the initial introduction and presentation of example problems, followed by the completion of practice exercises using general formulas. The typical pattern of learning still represents the teaching culture in carrying out the learning process. According to Weber (2005), mathematics learning in schools still follows the conventional sequence of explanation, example provision, and problem-solving practice. The learning process via direct application of general formulas can prevent students from developing their mathematical concepts independently (Weber, 2005b). Their thinking patterns need to be trained, resulting in a lack of familiarity with how these formulas are derived.

The application of real-life models to formal mathematical concepts has yet to be documented in supporting learning materials. The teacher must possess the ability to elucidate the models generated by students during the learning process, which can subsequently facilitate the comprehension of formal mathematics. Turmudi (2018) states that students should have a strong

Commented [Isti1]: You said that LIT is very related to teachers. In the introduction, I could not find, examples of LIT implementation by teachers and problems of its implementation where? How about LIT in mathematics? examples with mathematical materials need to be revealed in the introduction so that mathematics education becomes clearer. Why do teachers need to understand LIT? You need to describe this to form a research GAP in this article.

Commented [A2R1]: Thank you for your valuable feedback. You are correct that further clarification is needed regarding the implementation of Local Instruction Theory (LIT) by teachers, as well as its relevance in mathematics education. I have expanded on these points in the revised version of the manuscript.

In response to your comment, I have included specific examples of LIT implementation in mathematics classrooms, as well as the challenges teachers face when applying LIT. This can be found in the introduction, where I discuss both the potential and the difficulties teachers encounter in incorporating LIT within their teaching practices, especially in mathematics education (Gravemeijer, 2004; Van Eerde, 2009). For example, teachers may struggle to align LIT with traditional teaching methods or face challenges related to resource constraints when implementing more interactive, context-based instruction (Rittle-Johnson & Schneider, 2014).

Additionally, I have elaborated on why teachers need to understand LIT. LIT provides teachers with a framework to help students develop deeper conceptual understanding through real-world contexts and student-centered learning activities (Akker et al., 2006). Understanding LIT allows teachers to design instructional materials that foster critical thinking and problem-solving skills, which are crucial for mathematics education (Meika, 2018).

desire in the learning process to independently discover concepts, provided that teachers provide facilities, teaching materials, and learning resources (Turmudi, 2012).

According to Sandip's research, trigonometry is a challenging subject that is disconnected from everyday life *(Adhikari & Subedi, 2021; Asomah et al., 2022; Dhungana et al., 2023).* To bridge this gap, Realistic Mathematics Education serves as an alternative to teaching trigonometric ratios. Realistic Mathematics Education (RME) is a teaching and learning method in math education that follows the philosophy of Hans Freudenthal, who sees math as a human endeavor. *(Freudenthal, 1973; Gravemeijer & Terwel, 2000; Treffers, 1987; Van den Heuvel-Panhuizen & Drijvers, 2020).*

Realistic Mathematics Education is characterized by students experiencing a concrete learning process that is aligned with real-life situations and contextualized according to their everyday lives (Yuanita et al., 2018). Students undergo an informal process of constructing their understanding based on what they see, observe, and comprehend about their surrounding environment, be it at home, school, or wherever they learn (Meika, 2018).

The three primary ideas of Realistic Mathematics Education are guided reinvention and progressive mathematizing, didactical phenomenology, and self-developed models. (Gravemeijer, 1994). During the teaching and learning process, these three principles are elaborated into five characteristics of RME, namely: 1) Building and solidifying; 2) degree and frameworks; 3) contemplation and specific task; 4) societal environment and communication; and 5) organizing and intertwining (Gravemeijer, 1994; Jzn & de, 1987). The significance of the Realistic Mathematics Education approach resides in its ability to empower students to develop concepts through interactive teaching and learning processes. The teachers has a crucial role in enabling students to explore concepts using both administrative and motivational methods (Meika, 2018).

The mistakes made by previous students should not be repeated in the following years. The provision of instructional materials is an effective solution for improving the situation and facilitating recovery. The focus of learning will be further narrowed down if the instructional materials used are appropriate. In this teaching material, steps are needed that can facilitate students' learning paths. According to Gravemeijer, to improve mathematics education, it is necessary to implement an instructional design that facilitates learning and enables students to progress from their current level of thinking to a more sophisticated level of mathematical thinking (Gravemeijer, 2004).

Local Instruction Theory (LIT) is a comprehensive theory that includes both the process of acquiring knowledge on a specific topic and the methods used to facilitate that learning (Liljekvist et al., 2017). LIT was developed in the 1990s as part of an ongoing research initiative into the adaptation of instructional strategies within mathematics education. LIT has been primarily applied in various educational contexts, particularly in mathematics classrooms across several countries, including the Netherlands, where the theory was initially developed. It is also relevant to the Indonesian educational context, as demonstrated in our study. Gravemeijer states that Local Instruction Theory is an exact theory that can assist students in developing mathematical thinking on a particular topic or material through detailed and gradual instructional medium (Gravemeijer, 2004). Researchers will develop a systematic learning path, also known as a learning trajectory, within the process of teaching and learning. This design is the one Gravemeijer mentioned.

LIT is used to describe the appropriate learning route or learning activities. LIT not only offers how learning takes place but also facilitates a range of specific learning activities that may work. The existence of LIT provides a basis for teachers to consider adaptations that may be needed in implementing new approaches. In addition, LIT offers a framework in which teachers can develop hypotheses about learning trajectories. LIT is tailored to the specific circumstances of the particular

class and teacher. Teachers need to build HLT (Hypothetical Learning Trajectories) that are suitable for their own situations while using LIT as a foundational reference (Gravemeijer, 2004).

Understanding and applying (LIT) is essential for teachers as it serves as a bridge between theoretical frameworks and practical classroom instruction. LIT provides a structured approach to creating Hypothetical Learning Trajectories (HLTs), which help teachers anticipate and guide students' learning processes while adapting to their evolving needs. This process enables educators to align instructional activities with students' thinking and learning paths, ensuring a deeper and more meaningful understanding of mathematical concepts.

For example, guided reinvention and mathematization are central aspects of LIT. These strategies empower students to develop informal problem-solving methods that can be refined into formal mathematical reasoning, fostering their ability to connect classroom learning with real-world applications. Teachers' proficiency in LIT also facilitates the identification and addressing of misconceptions, promoting continuous conceptual development (Gravemeijer & Doorman, 1999; Van Den Heuvel-Panhuizen, 2002).

Furthermore, professional development programs incorporating LIT principles have demonstrated that teachers equipped with this knowledge are better prepared to design effective and flexible instructional sequences tailored to their students' needs. These programs emphasize the importance of using LIT to design activities that encourage reasoning, discussion, and reflection, which are key for fostering higher-order thinking skills (Gravemeijer, 2004).

The implementation of Local Instruction Theory (LIT) by teachers is often done through a contextual activity-based approach to help students understand mathematical concepts. For example, in teaching the concept of combination with the RME approach, teachers can use demonstrations such as handshakes or color mixing to connect abstract concepts to real-world situations, which have been shown to be effective in improving student understanding. However, teachers face challenges in adjusting learning designs to meet the needs of heterogeneous students, especially in aligning the Hypothetical Learning Trajectory (HLT) with classroom dynamics. Teachers have difficulty in designing learning activities that are contextual enough and engage students at a deeper level of engagement (Meika et al., 2019).

Based on preliminary research conducted by the researcher, no teacher at this school has ever implemented *Local Instruction Theory* (LIT) in teaching, particularly in the topic of trigonometric ratios. The lack of training and understanding of how to apply LIT to the curriculum is the main reason why teachers have not utilized this approach. As Gravemeijer notes, although LIT has proven effective in certain educational contexts, its implementation requires a deep understanding of its principles and methods. Teachers often face challenges when trying to apply innovative teaching strategies like LIT due to limited opportunities for professional development and curriculum constraints (Gravemeijer & Doorman, 1999). Furthermore, adapting LIT to specific topics, such as trigonometric ratios, can be a barrier if teachers do not have the necessary tools and support. Therefore, the lack of LIT implementation in this school makes it a relevant area for further research development.

HLT is developed for each learning activity and plays a crucial role in designing student learning tasks. The implementation of teaching is closely tied to the learning trajectory, which includes a planned sequence of learning content and the conceptual map students navigate during the learning process (Prahmana, 2016). In the subsequent phase, a Local Instruction Theory (LIT) for trigonometric ratios is developed.

Based on the explanation above, there is a need for the development and design of instructional materials that can minimize students' difficulties in developing thinking processes through Realistic Mathematics Education (RME) to construct an understanding of trigonometry content. These instructional materials are expected to be obtained through the design of LIT. This approach aims to address several challenges faced by students and to foster the development of their mathematical thinking, particularly in understanding trigonometric ratios through Realistic Mathematics Education.

According to the given definition, it is crucial to create and design instructional resources that can reduce students' challenges in developing cognitive processes through Realistic Mathematics Education in order to build comprehension of trigonometric concepts. The desired teaching materials are obtained through the LIT design. The LIT design is used to address the difficulties experienced by students and to enhance their mathematical thinking process, particularly their understanding of trigonometric ratios through Realistic Mathematics Education.

METHOD

In the book *Educational Design Research* by Koeno Gravemeijer, the conclusion highlights the importance of design research in bridging theory and practice within education. Design research allows for the development of innovative and contextually relevant learning tools that can be adapted and tested in real classroom environments. One of its key strengths is the ability to generate new theories based on real-world learning experiences, emphasizing the interaction between students, teachers, and learning materials (Akker et al., 2006).

Moreover, design research provides a framework for continuous reflection and improvement of teaching practices and learning strategies through iterative cycles of development, implementation, and evaluation. This ensures that the educational solutions developed are both effective and adaptable, ultimately enhancing the overall quality of education. Through this process, design research contributes to creating educational solutions that are both grounded in theory and practical in diverse educational settings.

A design research method based on Gravemeijer's approach is the research methodology used. This research method is focused on local instruction theory, or LIT, which was developed by researchers in collaboration with teachers to achieve better learning quality (Gravemeijer & van Eerde, 2009). A study paradigm known as "design research" tries to create a series of tasks and comprehend the workings of the learning process. This study aims to develop and construct a Local Instruction Theory (LIT) on the subject of trigonometric ratios, with a focus on Realistic Mathematics Education (RME).

Gravemeijer proposes a research design for the development of local instruction theory. The research design consists of three phases: 1) preliminary design (developing initial design), 2) teaching experiment (doing teaching experiment), and 3) retrospective analysis (performing retrospective analysis) (Gravemeijer, 2004). Preliminary design is the outcome of an initial learning plan aimed at implementing ideas about the development of instructional materials for LIT (Meika, 2018). This design is expected to support students in the construction of mathematical thinking from an informal form to a formal one. The construction of mathematical thinking is achieved through a guided reinvention process facilitated by mathematizing. Mathematizing refers to several ways of managing learning activities to demonstrate mathematical characteristics, such as generality, certainty, accuracy, and agility (Loc & Hao, 2016). The purpose of the teaching experiment is to evaluate the

Commented [Isti3]: I still haven't got any information, why design research? why are the subjects students of SMA 7 Bandung, what's special about it? are there any teachers who have practiced LIT? academic reasons need to be conveyed in this section. How are the research data collected? how is it analyzed? how is the conclusion drawn? you must describe all of this in this section.

Commented [A4R3]: Thank you for your valuable feedback. I have reviewed the manuscript and clarified the points raised in your comment. The requested information regarding the rationale for using design research, the selection of SMA 7 Bandung as the study site, and other aspects of the study methodology have been explicitly addressed in the revised manuscript. Here is a brief summary of how I have incorporated this feedback:

1.**Why Design Research?**
The choice of design research as the methodology was explained in the introduction, where it is noted that design research bridges theory and practice in education. It allows for the iterative development of educational interventions that are tested and refined in real classroom settings. This process ensures that the educational tools developed are contextually relevant and practically applicable (Akker et al., 2006). The use of design research allows for continuous feedback and improvement, which is why it was deemed the most suitable methodology for this study.

2.**Why the Students of SMA 7 Bandung?**
The rationale for selecting SMA 7 Bandung is provided in the manuscript. This school was chosen because it is representative of typical senior high schools in Indonesia that adhere to the national curriculum. The students are already familiar with the curriculum content, specifically trigonometric ratios, making it an appropriate context for implementing and evaluating the Local Instruction Theory (LIT) approach. This allows the study to focus on how LIT can enhance learning outcomes in a real-world classroom environment.

3.**Have Teachers Practiced LIT?**
The manuscript now includes a discussion of the fact that the teachers at SMA 7 Bandung have not previously implemented LIT, primarily due to a lack of professional development opportunities and resources. This is a critical point in the study, as it highlights the need for teacher training in LIT and other innovative teaching methods. Further information on this has been included in the "Limitation" section, where recommendations for addressing this gap in future research are provided.

4.**How Was Data Collected?**
Details on data collection methods are now provided in the "Method" section. Data were collected through a ...

Commented [User5]: This part better put it on method.

Commented [A6R5]: I have moved the section you referred to into the "Method" section as per your recommendation. The revised manuscript now reflects this adjustment to improve the clarity and structure of the paper. I believe this will enhance the overall organization and flow of the research explanation.

effectiveness of the instructional materials for LIT. Retrospective analysis is employed to examine the complete dataset of the teaching experiment outcomes.

HLT is applied in every learning activity and has a crucial role in designing learning activities for students. The execution of learning is indissociable from the learning trajectory, encompassing a deliberate progression of learning materials and idea maps that students traverse during the learning process (Prahmana, 2016). Next, in the subsequent stage, the trigonometric ratios LIT is produced.

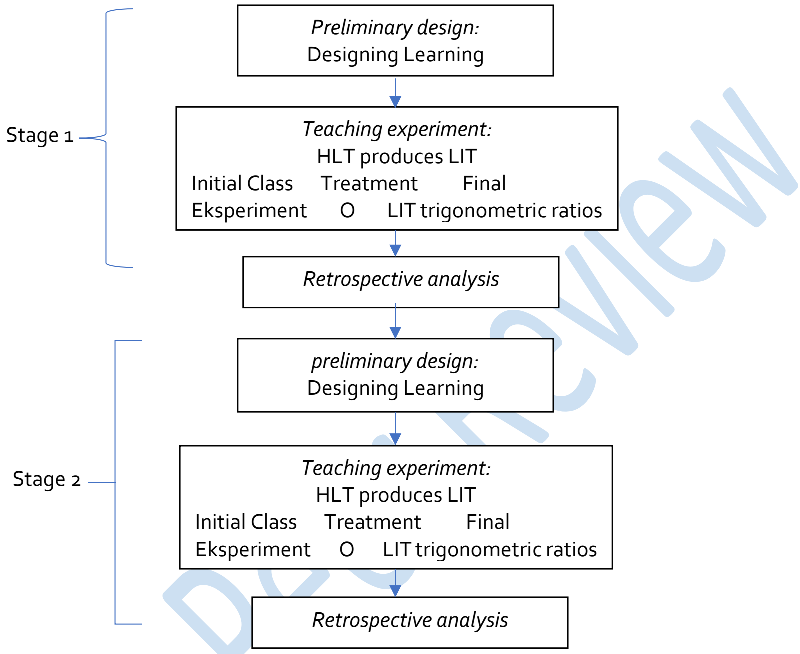


Figure 1. Research Design

The research implementation is carried out in two stages, with each stage referring to the steps of design research (Gravemeijer, 2004) namely: The three main stages of the project include 1) formulating the initial concept, 2) executing the educational experiment, and 3) carrying out a retrospective study. During the teaching experiment phase, this study involved 30 tenth-grade students from SMAN 7 Bandung. The selection of Grade 10 at SMAN 7 Bandung was based on the rationale that this class had already engaged with the national curriculum currently implemented in Indonesia, where trigonometric ratios constitute a key topic. Additionally, the school was chosen because it represents senior high schools that consistently adhere to the implementation of the national curriculum. This selection provided an opportunity to explore the application of trigonometric ratios within a context relevant to Indonesia's existing educational framework.

An analysis was conducted on the activities performed in the initial phase, and subsequently, recommendations were given to enhance the activities in the second phase. The findings from the data analysis of the second phase provide the final recommendations of this research.

Preliminary findings indicate that no teachers at this school have implemented Local Instruction Theory (LIT) in their teaching practices, particularly in the context of trigonometric ratios.

The primary reasons for this include limited training opportunities and a lack of understanding regarding LIT's application in the classroom. Gravemeijer (1999) highlights that while LIT has proven effective in various educational contexts, its implementation requires a comprehensive understanding of its principles and methodologies. Teachers often face challenges in adopting innovative teaching strategies like LIT due to insufficient professional development opportunities and curriculum constraints. Additionally, adapting LIT to specific topics, such as trigonometric ratios, poses further challenges if educators lack adequate tools and support. Consequently, the absence of LIT's application for this topic at the school underscores the relevance and potential for further research and development in this area.

Data for this research was collected using multiple methods to gather comprehensive insights. First, written data were obtained from test results, tasks such as UBA-LIT and LKPD, and observation sheets during the teaching experiments. Second, documentation in the form of video recordings and photographs were used to observe classroom activities, student interactions, and the process of mathematization. Additionally, direct observation was conducted by the researcher, who also acted as the teacher, noting student responses during the learning process. Interviews with selected students were carried out to gather deeper insights into their experiences with UBA-LIT and LKPD. The data were analyzed qualitatively by comparing the observations and video recordings with the initial teaching design (HLT). Internal validity was ensured through data triangulation from various sources, while reliability was strengthened by testing the instruments and conducting cross-interpretation with experts. The conclusions were drawn from the retrospective analysis of the collected data, explaining how students grasped the concept of trigonometric ratios within the learning process using LIT.

RESULTS AND DISCUSSION

Results

The research is carried out in three stages: 1) formulating the fundamental design, 2) executing the teaching experiment, and 3) implementing the retrospective analysis.

Elaborating on the Initial Design (*preliminary design*)

During this phase, the researcher assembles the educational materials and finalizes the instructional content. To meet the study aims, various actions are conducted, including (1) identifying the research subjects based on their diverse cognitive abilities. The researchers selected students from class X of SMAN 7 as appropriate research subjects; (2) examining and analyzing the curriculum to identify the compatibility of trigonometric comparison material with the curriculum currently being used in the school; (3) analyzing trigonometric comparison material to obtain an overview that "the achievement of Phase E-learning based on the derived geometry domain becomes the learning objective," in accordance with the research objectives. Next, the researcher developed a teaching material design in the form of a descriptive outline of trigonometric comparisons, also known as "Local Instruction Theory Descriptive Teaching Material (UBA-LIT)". The Realistic Mathematics Education (RME) technique enhances the learning process by employing diverse learning materials, including teaching modules and research tools. The study instrument is designed to incorporate the specific attributes of Realistic Mathematics Education (RME) and the curriculum used by the school. Additionally, the researchers analyze and assess the learning assistance tools. The result of the evaluation is referred to as the prototype. In the next stage, the research conducts an expert review, in which four experts validate the prototype based on its structure, content, and language.

Once the expert review procedure is finished, students who have been given trigonometric comparison information are tested on the prototype to get data on the readability and comprehension of the instructional material. The researchers engage in communication with the students during the testing process to anticipate any corrections or feedback regarding the instructional materials being tested by the students. The students write their input or suggestions on the prepared sheet. Students provide support for the discovery of formal concepts related to UBA-LIT through their comments. The researchers made improvements to the UBA-LIT after receiving feedback. After conducting expert validity testing and gathering comments from students, it was ultimately determined that the prototype design of the instructional material product generated by the researcher is valid at this level. The updated iteration of the prototype is referred to as the second prototype. Following that, the second prototype underwent testing in teaching experiment 1, including a total of 30 students from SMAN 7 Kota Bandung.

From a design research standpoint, the primary objective of the initial design is to develop a Local Instruction Theory (LIT) that can be visually portrayed as a portrayal of the process of acquiring and improving knowledge through teaching experiments (Gravemeijer & Cobb, 2006). During the preliminary study, the researcher designed the learning process activities based on a literature review. This set of educational exercises covers the beliefs held by students while they are building knowledge and the methods that will be used, which will be presented as an initial challenge, including advanced thinking skills. The contents of HLT are subject to change. The activities in HLT can be adapted and customized based on the circumstances and parameters of the ongoing educational experiment. HLT has a curriculum that students will adhere to when studying trigonometric ratios. The HLT curriculum comprises learning objectives, activity descriptions, and student thought conjectures that occur during the learning process. During the teaching experiment, HLT functions as a reference for teachers and researchers to determine the teaching emphasis, observations, and interviews. Moreover, HLT functions as a point of reference for the examination or analysis of a dataset in the context of a teaching experiment. Hypothetical Learning Trajectory (HLT) used in the teaching experiment is displayed in Table 1.

Table 1. HLT illustration for learning the concept of trigonometry ratios

Activity	Learning Objectives	Activity Description	Conjecture
Students Demonstrate the activity of identifying by body height.	The students are able to identify the front, side, and diagonal positions by referring to specific angles of a right-angled triangle.	- The teacher instructed three students to engage in a practical exercise involving the formation of a right-angled triangle using a table base, a	- Some students may be able to illustrate a sketch of a right triangle from the demonstration activity. - Some students may quickly recognize the positions of the corner, front side, side side, and

		measuring tape, and their height. - Each group carefully observes the demonstration and discusses the position of the corner, front, side, and slant.	hypotenuse.
Shows high shadow intensity activity	The students can articulate the trigonometric ratios (sin θ, cos θ, and tan θ) by employing the notion of ratios within a right triangle.	- The teacher instructs the students to face away from the light until a shadow appears in front of the standing student. The sketch formed is a right-angled triangle. - Students can determine the front side, side view, and diagonal side of a right triangle sketch based on the determining angle. Students will be *guided* in *rediscovering the concept* of trigonometric ratios.	- Some students may use tables as a tool to solve problems. - Certain students may employ finger manipulation methods to solve difficulties. - Some students may utilize measuring instruments to solve problems. - Some students may use the number parity method to solve problems. - Certain students may possess the ability to represent this activity in a visual manner. They initiated the construction of a model and engaged in horizontal and vertical mathematization in order to uncover the notion of trigonometric ratios through the shown examples. -

			- Some students may be able to relate the demonstration process to the concept of trigonometric ratios.
Problem-Solving	Students are able to solve real-life problems related to right triangles using trigonometric ratios	Students use trigonometric ratios to solve real-life problems by performing calculations.	- Certain students possess the ability to perceive the relationship between sine and cosine values in mathematical calculations and are capable of streamlining the calculation process. - Some students may be able to remember or repeat knowledge they have previously acquired to solve problems. - Some students may use tables, finger methods, odd-even tricks, or measuring tools to solve problems. - Some students may directly apply trigonometric ratio concepts.

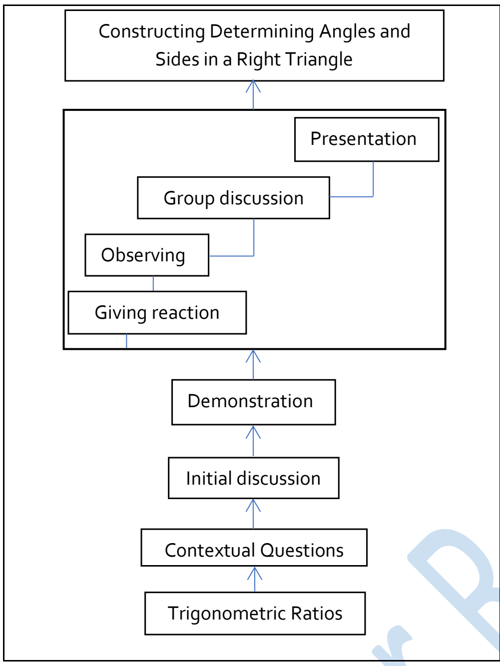
Teaching Experiment

The primary objective of a teaching experiment is to systematically evaluate and enhance original ideas regarding Local Instruction Theory (LIT) while also striving to gain a more profound comprehension of the underlying mechanisms and functioning of LIT (Gravemeijer & Cobb, 2006).

Activity 1

The teacher requested three students to perform a demonstration using the provided tables/chairs and ropes.

The teacher requested one of the students to fasten the rope to the base of the table or chair provided. A student holds the end of the neat rope and moves back 1 meter from the base of the table/chair. After that, the student who is holding the end of the rapia rope shifts his grip to the end of the head. Other students are asked to draw sketches of the shapes that are occurring. The other students observed the demonstration requested by the teacher, depicted the requested sketch, and identified the positions of the angles, front side, side view, and oblique side of the formed triangle sketch. All students strive to comprehend the right-angled triangle sketch and identify the positions of angles, front side, side view, and hypotenuse on the sketch. The teacher provides a comprehensive understanding through the following activities.



The student model illustrated the activity of identification by using height as part of the learning process for understanding trigonometric ratios.

Figure 2. Demonstration of the flow of LIT and height in learning comparative trigonometry

Activity 2

The teacher instructs the class to define the trigonometric ratios (sin θ, cos θ, and tan θ) with a demonstration involving the height of a shadow. This activity guides students in formally discovering the concept of trigonometric comparisons. The students were able to perform this activity proficiently after acquiring the idea of identifying angles and sides in the previous activity. The presented issues are contextual and aligned with the student's level of understanding. The LIT design is structured from the simplest to the most complex in order to provide a general solution. In this activity, the teacher instructs the students to face away from the light source so that a shadow appears in front of the standing students. The results of the student's practice are represented in the form of an image or sketch of a right-angled triangle. This sketch facilitates students in identifying the positions of angles, front side, side view, and inclined side of a right triangle based on the predetermined angle. Creating a sketch of a right triangle and determining angles helps students mathematize vertically and find formal solutions to define trigonometric ratios.

Figure 3 illustrates various outcomes of students' process variations in discovering formal mathematics from the concept of trigonometric ratios. The students' work begins with a demonstration of a contextual problem, allowing them to find solutions using various methods based on their prior knowledge. During the learning process, students are guided to discover formal mathematics, which is a process of mathematization based on the presented problems.

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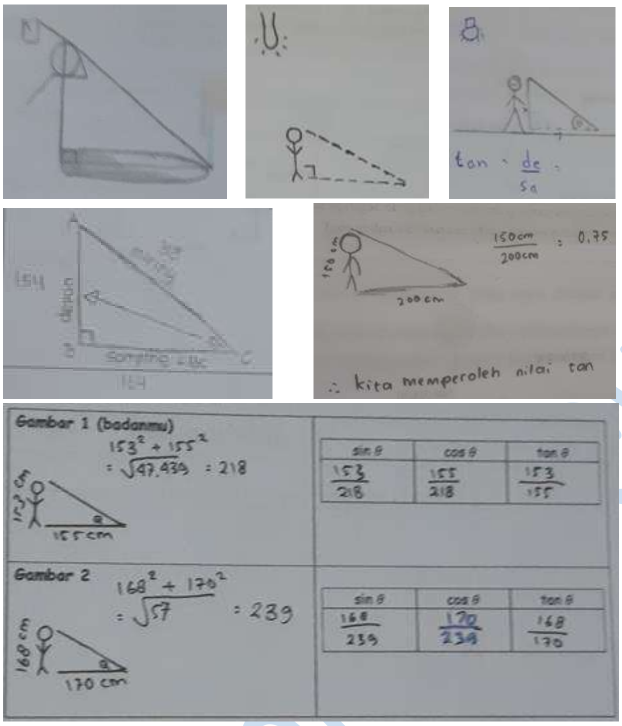


Figure 3. The vertical mathematical process of students in finding formal solutions to trigonometric ratios

Activity 3

Activity 3 begins with the teacher presenting problems based on everyday life situations. Everyday scenarios were employed to enable students to apply the notion of trigonometric ratios in practical circumstances. After the activity is finished, the teacher conducts a test as a final evaluation. One of the trigonometric comparison problems presented is as follows:

> "Refer to the following image of a bed. What are the geometric shapes that occur between the stairs, bed, and floor? Sketch a model. (1) *Does the shape formed constitute a right triangle? If so, mark one corner with a right angle*. (2) *If the angle between the stairs and the floor is unknown, and the distance from the bed to the stairs is 10 cm. Place the obtained data into the image. Subsequently, calculate the distance from the floor to the end of the staircase, which forms a right-angled triangle, using the tangent ratio.* ($\tan 37^0 = 0{,}75$)"

Figure 4. Image of a bed as a model for the use of trigonometric ratios

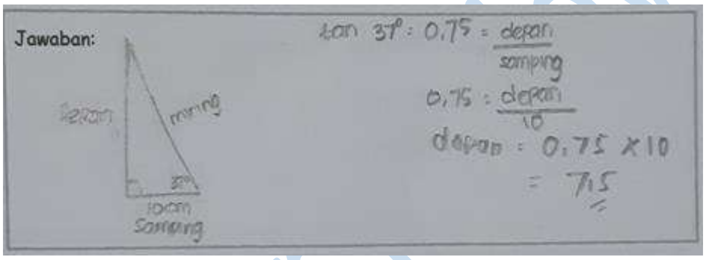


Figure 5. One of the student's responses was to solve the story problem

Figure 5 is an example of a student's response using the concept of trigonometric ratios to solve a word problem. The student solves the problem using the formal solution of trigonometric ratios that was previously constructed.

Retrospective Analysis

Researchers employ Hypothetical Learning Trajectory (HLT) as a first guide and reference when assessing a dataset during the data analysis stage of a teaching experiment. The HLT created is compared to the classroom learning process. The analysis includes descriptions of anticipated occurrences during the learning process and examples of behaviors that will occur among students. The conclusions drawn from the analysis will answer the research questions. A good design is not the main objective of the study but rather the reason for the study's success, which is the main objective (Gravemeijer & Cobb, 2006).

The many activities carried out during the learning process are represented in the flowchart of LIT before the learning process on the topic of trigonometric ratios. The flowchart is presented in Figure 6. The image shows the differences in stages or steps during the learning process. The difference is evident in the red-colored box. Before the learning process begins, the design aims to facilitate activities, experiments, observations, group discussions, and presentations as processes that run parallel. Students will go through these stages simultaneously.

Meanwhile, after the learning process takes place, a revision of the initial design is made. The stage of differentiation involves an initial discussion process with students prior to the demonstration (*Figure 7*). This stage provides students with an introduction to understand the

concept of trigonometric ratios in relation to the prior knowledge they have already constructed. This stage is considered crucial because linking it to the previously acquired knowledge is necessary for students to comprehend the fundamental concepts of trigonometric ratios.

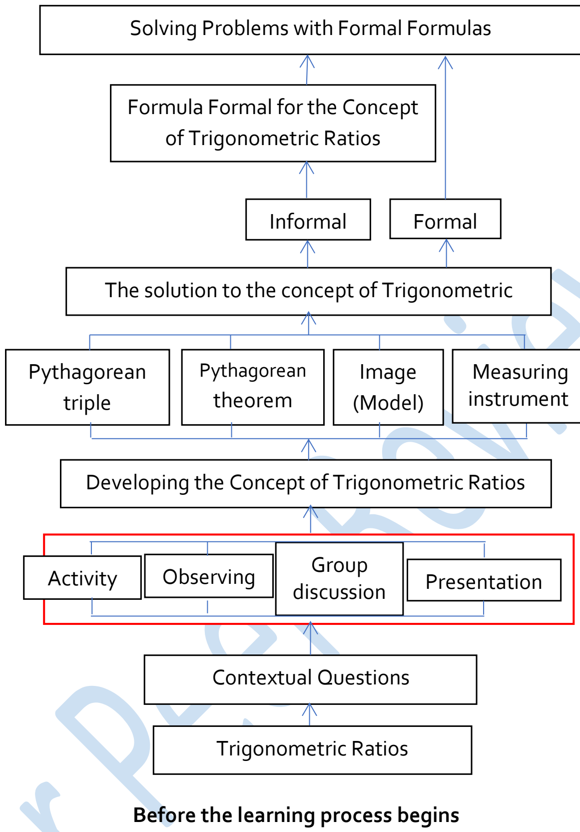
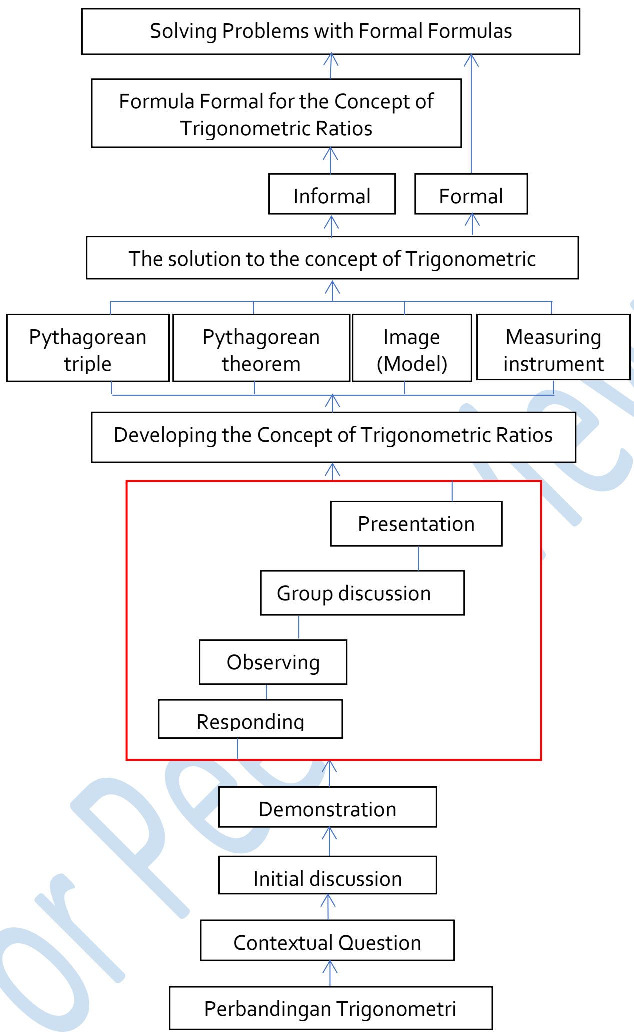


Figure 6. Flowchart of LIT on the topic of Trigonometric Ratios before the lesson

The next difference is in the "responding" stage, which students go through after conducting a demonstration and before the "observing" stage. The act of "responding" is the first step for students to participate actively in the learning process. The students will provide their responses or feedback to the demonstration presented. Their knowledge will guide and stimulate the students to continue the learning process via the "observing" stage.

Before the learning process begins, the steps "activity/experiment," "observe," "group discussion," and "presentation" have the same level of suitabilityThere is no text provided. However, the post-learning phases in these four things exhibit distinct variations. The "responding" stage initiates the steps of learning. Next, the process begins with the "observation" stage, followed by the "group discussion" step, and concludes with the "presentation" step. The students go through the four steps of the learning process in different stages.



After the learning process is complete

Figure 7. Flowchart of LIT on the topic of Trigonometric Ratios after the lesson

The appropriate use of the LIT framework, as shown in Figure 7, allows students to construct an understanding of the concept of trigonometric ratios through everyday problems. They are able to identify trigonometric ratios and solve real-life issues accordingly, as a result of analyzing the HLT and conducting teaching experiments. Based on the results of the student's work using the LIT design for trigonometric comparison, the number of students capable of completing the learning activities is determined. The students are becoming accustomed to the learning process, which involves determining mathematics formally based on pre-established models. The utilization of

Pythagorean triples, the Pythagorean theorem, models (diagrams), and measuring tools can aid in achieving vertical mathematization. The students may effectively solve common problems by utilizing trigonometric ratios.

Some students found the Local Instruction Theory (LIT) approach challenging as it shifted their understanding from merely applying formulas to understanding the deeper concepts. For example, one student expressed initial difficulty with LIT's emphasis on understanding the relationships between concepts rather than just following procedural steps. However, after grasping the underlying logic, the student felt satisfied with being able to understand the reasoning behind trigonometric ratios, rather than just memorizing formulas.

On the other hand, some students struggled to adapt to this new teaching design, especially those who were accustomed to more traditional, structured methods. A few students felt overwhelmed by the responsibility of independent learning and actively solving problems. This feedback suggests that while LIT can provide a deeper and more engaging learning experience, its effectiveness depends heavily on how well students are supported in transitioning to this new method.

Additionally, student feedback highlighted the benefits of the collaborative learning aspect of LIT. Students appreciated the group discussions and the opportunity to explore different approaches to problem-solving, which provided a more shared learning experience. However, some also criticized the method, feeling that they lacked the foundational knowledge to fully engage in the collaborative or reflective aspects of LIT, emphasizing the need for careful planning when implementing this method. These student responses offer valuable insights for refining the design and application of LIT in teaching.

The findings indicate that some students lack the prerequisite knowledge they should have acquired, which is influenced by factors such as mental unpreparedness, teacher didactics, and inadequate teaching materials (Kurniasih et al., 2020; Puspita et al., 2023). In particular, the emphasis on rote memorization of formulas, such as trigonometric ratios, without a geometric explanation, leads to a lack of motivation and understanding among students (Andika et al., 2017; Urrutia et al., 2019). Teacher-centered methods that rely solely on memorization often result in short-term retention of knowledge (Orhun, 2002). To overcome this, teachers need a comprehensive understanding of trigonometry to connect various concepts, enhancing students' learning (Andiani et al., 2020, 2024; Rittle-Johnson & Schneider, 2014). Additionally, incorporating real-life contexts into lessons, such as through models and diagrams, makes learning more relevant and aids in the long-term retention of knowledge (Dhungana et al., 2023). The use of Local Instruction Theory (LIT) and Realistic Mathematics Education (RME) approaches has been shown to improve students' ability to construct their understanding of trigonometric ratios, as evidenced by their improved learning outcomes.

Discussion

The introduction of Local Instruction Theory (LIT) in this study revealed significant improvements in students' conceptual understanding of trigonometric ratios. Unlike traditional teacher-centered approaches, LIT facilitated deeper connections between abstract mathematical concepts and real-life applications. This was evident in the students' increased ability to relate geometric properties to trigonometric functions.

During the teaching experiment, students exhibited greater engagement with the material, particularly when real-life problems were introduced. Many students showed improved motivation

Commented [Isti7]: In this discussion, you have just told the research facts. Let the research facts be in the Result section. In the discussion, you have not explained what new things you found. How did students respond to your LIT design? Then the discussion must be critical in the theoretical GAP. I have not found the big picture of your research either. You can describe it in the before-after research. What's new?

Commented [A8R7]: Thank you for your detailed feedback and insightful comments. We have carefully addressed your concerns as follows:

1. **Separation of Results and Discussion Sections:** We have reviewed and ensured that the research facts are clearly presented in the *Results* section. The *Discussion* section now focuses on interpreting these findings, particularly highlighting the implications, theoretical contributions, and practical significance of the study.
2. **New Contributions and Theoretical Gap:** In the revised *Discussion*, we have explicitly stated the novelty of the findings, particularly regarding the application of Local Instruction Theory (LIT) in teaching trigonometric ratios. This research bridges a theoretical gap by demonstrating how LIT can enhance conceptual understanding through real-life applications and geometric reasoning, which has not been extensively explored in previous studies. This contribution is highlighted in paragraph 3 of the *Discussion*.
3. **Student Responses to LIT Design:** We have elaborated on how students responded to the LIT design. Specifically, we described their improved engagement and motivation when working with real-life problems, as well as their ability to connect abstract formulas with practical applications. These details are provided in paragraph 2 of the *Discussion*.
4. **Big Picture and Before-After Comparison:** To address the lack of a "big picture," we have added a before-and-after comparison to showcase the impact of LIT on students' learning outcomes. Before the intervention, students struggled with rote memorization and had limited understanding of geometric foundations. After the implementation, they demonstrated improved problem-solving abilities and better retention, as noted in paragraph 4 of the *Discussion*.
5. **Critical Analysis of the Findings:** The revised *Discussion* also includes a critical reflection on how the findings align with and extend existing literature. For instance, we discuss how this research confirms and expands upon prior studies (e.g., Dhungana et al., 2023) while offering new insights into trigonometric education through the lens of LIT.

and understanding, especially those who had struggled with rote memorization in previous lessons. Their ability to visualize trigonometric ratios in practical contexts, such as through diagramming angles and distances in real-world scenarios, was significantly enhanced.

Previous studies on Local Instruction Theory have largely focused on its implementation in general mathematical contexts, but this study fills a gap by specifically applying LIT to the teaching of trigonometric ratios. By incorporating real-world contexts and geometrical reasoning, this research extends the application of LIT to improve both conceptual understanding and long-term retention in mathematics education.

Before the intervention, students struggled with memorizing trigonometric formulas and showed limited understanding of their geometric foundations. After the intervention, students were able to connect these formulas with real-world contexts, as evidenced by their improved problem-solving abilities and retention rates.

Implication of Research

The design of this Local Instruction Theory (LIT) for trigonometric ratios incorporates the syntax of the Realistic Mathematics Education (RME) approach. The foundation of this LIT is a conjecture based on the learning trajectory using RME. Consequently, the designed learning trajectory needs to be further developed to align with the specific content objectives. Researchers intending to implement a learning trajectory are advised to master various instructional strategies or approaches for designing problems. By mastering problem design, a wide array of solutions both formal and informal can be found. Additionally, exploring variations in students' experiences, influenced by factors such as prior knowledge, learning styles, or socio-economic backgrounds, could provide alternative perspectives when applying LIT.

Limitation

This study was conducted in two phases, each of which aligns with the steps outlined in design research (Gravemeijer, 2004), namely: 1) developing the preliminary design, 2) conducting the teaching experiment, and 3) performing retrospective analysis. There are two reasons for selecting only two phases in this research. First, the researcher's time and resources were limited. Second, the material in question is only implemented once a year. To address the limitations of time and resources, the researcher recommends extending the duration of the teaching experiments and incorporating digital resources. These approaches would enhance the efficiency and effectiveness of both time and resource utilization in the study. The extended timeframe allows for more comprehensive data collection, while digital resources provide flexible, accessible learning tools that can alleviate constraints related to physical materials (Means et al., 2014). Furthermore, leveraging technology enables more dynamic and scalable teaching practices, supporting diverse learning needs (Çelik & Baturay, 2024).

CONCLUSION

The outcome of the learning trajectory in this study is a Comparative Trigonometry LIT developed through the Realistic Mathematics Education (RME) approach. This theory encompasses several steps that students must go through in constructing their knowledge related to trigonometric comparisons. These steps nonetheless adhere to the syntax of the Realistic Mathematics Education (RME) approach.

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Development of A Local Instruction Theory for Trigonometric Ratios

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Abstract

Some students find trigonometric ratios challenging. Research on the application of Local Instruction Theory (LIT) in trigonometry is limited, particularly in secondary school understanding of trigonometric ratios. This study aims to develop an LIT for trigonometric ratios using Realistic Mathematics Education (RME). The researcher designed a learning pathway to help students grasp the fundamental concepts of trigonometric ratios. The study employs a research design methodology, developing a Hypothetical Learning Trajectory (HLT) to improve students' understanding. The development of LIT for Trigonometric Ratios follows three stages: initial design, teaching experiments, and retrospective analysis. Students demonstrated the ability to understand trigonometric ratios through the learning process. The findings suggest that the use of LIT-based instructional materials, incorporating RME principles, significantly enhances students' conceptual understanding of trigonometric ratios in high school.

Keywords: Local Instruction Theory; Hypothetical Learning Trajectory; Trigonometric Ratios; RME

Abstrak

Beberapa siswa menganggap materi perbandingan trigonometri cukup sulit. Penelitian tentang penerapan Local Instruction Theory (LIT) dalam pembelajaran trigonometri masih sangat terbatas, khususnya terkait dengan pemahaman rasio trigonometri di sekolah menengah. Penelitian ini bertujuan untuk mengembangkan Local Instruction Theory (LIT) perbandingan trigonometri dengan menggunakan Realistic Mathematic Education (RME). Dalam upaya membantu siswa membangun konsep dasar pada materi perbandingan trigonometri, peneliti mengembangkan alur LIT dengan menemukan jalur pembelajaran yang efektif. Pencapaian tujuan penelitian menggunakan desain penelitian. Serangkaian kegiatan mengembangkan Hypothetical Learning Trajectory (HLT) sehingga siswa sekolah menengah atas (SMA) memiliki pemahaman yang lebih baik tentang perbandingan trigonometri. Pengembangan Local Instruction Theory untuk Perbandingan Trigonometri meliputi tiga tahap yaitu mengembangkan desain awal, melakukan percobaan pengajaran, dan melaksanakan analisis retrospektif. Siswa mampu membangun pemahaman tentang perbandingan trigonometri selama proses pembelajaran berlangsung. Berdasarkan analisis kualitatif eksperimen pengajaran, penelitian ini berimplikasi pada penguatan Local Instruction Theory (LIT) perbandingan trigonometri dan pengembangan bahan ajar berbasis RME.

Mathematics Clasification: ---- must be filled ----

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Metadata of Author
First author: SCOPUS ID: - optional - | ORCID ID: - mandatory - | Google Scholar : VmAlN5YAAAAJ
Co-author: SCOPUS ID: - *57196237689* - | ORCID ID: - mandatory - | Google Scholar : Kc-9bRgAAAAJ

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Commented [A2R1]: Regarding your suggestion to demonstrate the research gap in the abstract, we understand the importance of clearly identifying the novelty and motivation behind the study at the outset.

We have revised the first paragraph of the abstract to explicitly state the research gap. Specifically, we have included a sentence highlighting the limited research addressing local instruction theory of trigonometric ratios, which is the basis of our investigation.

The revised section in the abstract now reads as follows:
Some students find trigonometric ratios challenging. Research on the application of Local Instruction Theory (LIT) in trigonometry is limited, particularly in secondary school understanding of trigonometric ratios.

In response to your request to include the implications of our findings, we have added a concluding sentence to the abstract:
The findings suggest that the use of LIT-based instructional materials, incorporating RME principles, significantly enhances students' conceptual understanding of trigonometric ratios in high school.

INTRODUCTION

The topic of trigonometric ratios is fundamental in the application of advanced applied sciences and pure sciences. Various fields of study require the use of trigonometry as an essential basis for applying their concepts. For example, the utilization of trigonometric ratios supports the proof of Kepler's first law (Simha, 2021). The importance of understanding trigonometric ratios lies in its role as fundamental knowledge for comprehending pre-calculus and calculus, such as algebra, geometry, and the ability to create graphs (Weber, 2005). Trigonometric comparison is utilized in calculating the width or height of various shapes, such as a bridge, household tools, and bicycles (Urrutia et al., 2019). Indeed, students see the topic of trigonometric ratios as challenging *(Blackett & Tall, 1991; Weber, 2005; Yiğit Koyunkaya, 2016; Karthikeyan, 2017; Urrutia et al., 2019; Nordlander, 2022).* During the initial stage of learning, students encounter difficulties in connecting triangle images to numerical relationships, as well as problems in manipulating the involved symbols *(Blackett & Tall, 1991)*, and many students are unable to overcome these difficulties. While studying right triangles, students struggled to grasp the concept of trigonometric functions, such as sine, cosine, and tangent, which are defined as ratios (Weber, 2005a, 2008). According to Yiğit (2016), students exhibit a lack of flexibility when solving trigonometric comparison problems (Yiğit Koyunkaya, 2016). Students expressed that trigonometry was a subject that was highly stimulating and conceptual in comparison to other areas of mathematics (Karthikeyan, 2017). According to Sandip's research, trigonometry is a challenging subject that is disconnected from everyday life (Urrutia et al., 2019). Typically, students struggle to distinguish the front side, side view, and slanted side of a right triangle despite the fact that this distinction greatly aids their comprehension of trigonometric ratios. The topic of trigonometric comparison is considered abstract for students (Karthikeyan, 2017).

From the preliminary study and literature review conducted by the researcher on textbooks, instructional videos, analysis of students' learning difficulties, and curriculum analysis, several aspects have caught the researcher's attention. One of these aspects is the emergence of real-world contexts in trigonometry materials in textbooks. Various examples have begun to present contextual situations, but teachers still need to learn how students might construct formal concepts from the presented contextual case. Batanero dan Diaz (2012) argue that the existing textbooks and curriculum documents need to sufficiently support learning, as the presented concepts are narrow in scope *(Batanero & Díaz, 2012)*.

The typical pattern of learning that occurs in the delivery of contextual mathematics is the initial introduction and presentation of example problems, followed by the completion of practice exercises using general formulas. The typical pattern of learning still represents the teaching culture in carrying out the learning process. According to Weber (2005), mathematics learning in schools still follows the conventional sequence of explanation, example provision, and problem-solving practice. The learning process via direct application of general formulas can prevent students from developing their mathematical concepts independently (Weber, 2005b). Their thinking patterns need to be trained, resulting in a lack of familiarity with how these formulas are derived.

The application of real-life models to formal mathematical concepts has yet to be documented in supporting learning materials. The teacher must possess the ability to elucidate the models generated by students during the learning process, which can subsequently facilitate the comprehension of formal mathematics. Turmudi (2018) states that students should have a strong

Commented [User3]: Some part need to be moved to Method.
Author, your keyword is LIT. so, in introduction you should explain what, who, when, why, where, and how LIT implemented. I can't find it.
You also should highlighted your research question, your research urgency.

Commented [A4R3]: Some parts that were mentioned have been moved to the Method section.
Thank you for your feedback. We have addressed your request by providing more specific information on Local Instruction Theory (LIT) in the introduction.

1. **What is LIT?** Local Instruction Theory (LIT) is an educational framework that integrates knowledge acquisition processes with instructional methods designed to facilitate learning, particularly through the creation of learning trajectories (Liljekvist et al., 2017).
2. **Who developed it?** LIT was developed by **Gravemeijer** in the **1990s** as part of research focused on adapting instructional strategies to better support student learning in mathematics (Gravemeijer, 2004).
3. **When was it developed?** LIT was developed in the **1990s**, initially in the context of mathematics education research in the Netherlands (Gravemeijer, 2004).
4. **Why is LIT important?** LIT is crucial because it helps educators design Hypothetical Learning Trajectories (HLTs) that align with students' cognitive development, ensuring that instructional activities adapt to students' evolving learning needs (Gravemeijer, 2004).
5. **Where is it applied?** LIT has been applied in various educational contexts globally, including mathematics classrooms in the Netherlands, where it was initially developed, and more recently in the Indonesian context, as discussed in our study (Liljekvist et al., 2017).
6. **How is LIT applied?** LIT is applied through contextual activity-based learning, where students develop informal problem-solving methods that are then formalized into mathematical reasoning. Teachers use LIT to design HLTs tailored to their specific class needs (Gravemeijer, 2004).

These points are addressed in the introduction, where we clarify LIT's theoretical foundation and its application in mathematics education.

desire in the learning process to independently discover concepts, provided that teachers provide facilities, teaching materials, and learning resources (Turmudi, 2012).

According to Sandip's research, trigonometry is a challenging subject that is disconnected from everyday life (Adhikari & Subedi, 2021; Asomah et al., 2022; Dhungana et al., 2023). To bridge this gap, Realistic Mathematics Education serves as an alternative to teaching trigonometric ratios. Realistic Mathematics Education (RME) is a teaching and learning method in math education that follows the philosophy of Hans Freudenthal, who sees math as a human endeavor. (Freudenthal, 1973; Gravemeijer & Terwel, 2000; Treffers, 1987; Van den Heuvel-Panhuizen & Drijvers, 2020).

Realistic Mathematics Education is characterized by students experiencing a concrete learning process that is aligned with real-life situations and contextualized according to their everyday lives (Yuanita et al., 2018). Students undergo an informal process of constructing their understanding based on what they see, observe, and comprehend about their surrounding environment, be it at home, school, or wherever they learn (Meika, 2018).

The three primary ideas of Realistic Mathematics Education are guided reinvention and progressive mathematizing, didactical phenomenology, and self-developed models. (Gravemeijer, 1994). During the teaching and learning process, these three principles are elaborated into five characteristics of RME, namely: 1) Building and solidifying; 2) degree and frameworks; 3) contemplation and specific task; 4) societal environment and communication; and 5) organizing and intertwining (Gravemeijer, 1994; Jzn & de, 1987). The significance of the Realistic Mathematics Education approach resides in its ability to empower students to develop concepts through interactive teaching and learning processes. The teachers has a crucial role in enabling students to explore concepts using both administrative and motivational methods (Meika, 2018).

The mistakes made by previous students should not be repeated in the following years. The provision of instructional materials is an effective solution for improving the situation and facilitating recovery. The focus of learning will be further narrowed down if the instructional materials used are appropriate. In this teaching material, steps are needed that can facilitate students' learning paths. According to Gravemeijer, to improve mathematics education, it is necessary to implement an instructional design that facilitates learning and enables students to progress from their current level of thinking to a more sophisticated level of mathematical thinking (Gravemeijer, 2004).

Local Instruction Theory (LIT) is a comprehensive theory that includes both the process of acquiring knowledge on a specific topic and the methods used to facilitate that learning (Liljekvist et al., 2017). LIT was developed in the 1990s as part of an ongoing research initiative into the adaptation of instructional strategies within mathematics education. LIT has been primarily applied in various educational contexts, particularly in mathematics classrooms across several countries, including the Netherlands, where the theory was initially developed. It is also relevant to the Indonesian educational context, as demonstrated in our study. Gravemeijer states that Local Instruction Theory is an exact theory that can assist students in developing mathematical thinking on a particular topic or material through detailed and gradual instructional medium (Gravemeijer, 2004). Researchers will develop a systematic learning path, also known as a learning trajectory, within the process of teaching and learning. This design is the one Gravemeijer mentioned.

LIT is used to describe the appropriate learning route or learning activities. LIT not only offers how learning takes place but also facilitates a range of specific learning activities that may work. The existence of LIT provides a basis for teachers to consider adaptations that may be needed in implementing new approaches. In addition, LIT offers a framework in which teachers can develop hypotheses about learning trajectories. LIT is tailored to the specific circumstances of the particular

class and teacher. Teachers need to build HLT (Hypothetical Learning Trajectories) that are suitable for their own situations while using LIT as a foundational reference (Gravemeijer, 2004).

Understanding and applying (LIT) is essential for teachers as it serves as a bridge between theoretical frameworks and practical classroom instruction. LIT provides a structured approach to creating Hypothetical Learning Trajectories (HLTs), which help teachers anticipate and guide students' learning processes while adapting to their evolving needs. This process enables educators to align instructional activities with students' thinking and learning paths, ensuring a deeper and more meaningful understanding of mathematical concepts.

For example, guided reinvention and mathematization are central aspects of LIT. These strategies empower students to develop informal problem-solving methods that can be refined into formal mathematical reasoning, fostering their ability to connect classroom learning with real-world applications. Teachers' proficiency in LIT also facilitates the identification and addressing of misconceptions, promoting continuous conceptual development (Gravemeijer & Doorman, 1999; Van Den Heuvel-Panhuizen, 2002).

Furthermore, professional development programs incorporating LIT principles have demonstrated that teachers equipped with this knowledge are better prepared to design effective and flexible instructional sequences tailored to their students' needs. These programs emphasize the importance of using LIT to design activities that encourage reasoning, discussion, and reflection, which are key for fostering higher-order thinking skills (Gravemeijer, 2004).

The implementation of Local Instruction Theory (LIT) by teachers is often done through a contextual activity-based approach to help students understand mathematical concepts. For example, in teaching the concept of combination with the RME approach, teachers can use demonstrations such as handshakes or color mixing to connect abstract concepts to real-world situations, which have been shown to be effective in improving student understanding. However, teachers face challenges in adjusting learning designs to meet the needs of heterogeneous students, especially in aligning the Hypothetical Learning Trajectory (HLT) with classroom dynamics. Teachers have difficulty in designing learning activities that are contextual enough and engage students at a deeper level of engagement (Meika et al., 2019).

Based on preliminary research conducted by the researcher, no teacher at this school has ever implemented *Local Instruction Theory* (LIT) in teaching, particularly in the topic of trigonometric ratios. The lack of training and understanding of how to apply LIT to the curriculum is the main reason why teachers have not utilized this approach. As Gravemeijer notes, although LIT has proven effective in certain educational contexts, its implementation requires a deep understanding of its principles and methods. Teachers often face challenges when trying to apply innovative teaching strategies like LIT due to limited opportunities for professional development and curriculum constraints (Gravemeijer & Doorman, 1999). Furthermore, adapting LIT to specific topics, such as trigonometric ratios, can be a barrier if teachers do not have the necessary tools and support. Therefore, the lack of LIT implementation in this school makes it a relevant area for further research development.

HLT is developed for each learning activity and plays a crucial role in designing student learning tasks. The implementation of teaching is closely tied to the learning trajectory, which includes a planned sequence of learning content and the conceptual map students navigate during the learning process (Prahmana, 2016). In the subsequent phase, a Local Instruction Theory (LIT) for trigonometric ratios is developed.

Based on the explanation above, there is a need for the development and design of instructional materials that can minimize students' difficulties in developing thinking processes through Realistic Mathematics Education (RME) to construct an understanding of trigonometry content. These instructional materials are expected to be obtained through the design of LIT. This approach aims to address several challenges faced by students and to foster the development of their mathematical thinking, particularly in understanding trigonometric ratios through Realistic Mathematics Education.

According to the given definition, it is crucial to create and design instructional resources that can reduce students' challenges in developing cognitive processes through Realistic Mathematics Education in order to build comprehension of trigonometric concepts. The desired teaching materials are obtained through the LIT design. The LIT design is used to address the difficulties experienced by students and to enhance their mathematical thinking process, particularly their understanding of trigonometric ratios through Realistic Mathematics Education.

METHOD

In the book *Educational Design Research* by Koeno Gravemeijer, the conclusion highlights the importance of design research in bridging theory and practice within education. Design research allows for the development of innovative and contextually relevant learning tools that can be adapted and tested in real classroom environments. One of its key strengths is the ability to generate new theories based on real-world learning experiences, emphasizing the interaction between students, teachers, and learning materials (Akker et al., 2006).

Moreover, design research provides a framework for continuous reflection and improvement of teaching practices and learning strategies through iterative cycles of development, implementation, and evaluation. This ensures that the educational solutions developed are both effective and adaptable, ultimately enhancing the overall quality of education. Through this process, design research contributes to creating educational solutions that are both grounded in theory and practical in diverse educational settings.

A design research method based on Gravemeijer's approach is the research methodology used. This research method is focused on local instruction theory, or LIT, which was developed by researchers in collaboration with teachers to achieve better learning quality (Gravemeijer & van Eerde, 2009). A study paradigm known as "design research" tries to create a series of tasks and comprehend the workings of the learning process. This study aims to develop and construct a Local Instruction Theory (LIT) on the subject of trigonometric ratios, with a focus on Realistic Mathematics Education (RME).

Gravemeijer proposes a research design for the development of local instruction theory. The research design consists of three phases: 1) preliminary design (developing initial design), 2) teaching experiment (doing teaching experiment), and 3) retrospective analysis (performing retrospective analysis) (Gravemeijer, 2004). Preliminary design is the outcome of an initial learning plan aimed at implementing ideas about the development of instructional materials for LIT (Meika, 2018). This design is expected to support students in the construction of mathematical thinking from an informal form to a formal one. The construction of mathematical thinking is achieved through a guided reinvention process facilitated by mathematizing. Mathematizing refers to several ways of managing learning activities to demonstrate mathematical characteristics, such as generality, certainty, accuracy, and agility (Loc & Hao, 2016). The purpose of the teaching experiment is to evaluate the

Commented [User5]: Why do you use design research? How do you organize the participant of your research? How do you maintain the validation of your research? Explain after the chart.

Commented [A6R5]: Thank you for your questions. I have provided detailed explanations for each of the points raised in the methods section of the manuscript. Specifically:

1. **Why design research is used**: I discussed the advantages of design research in bridging theory and practice, as well as its capacity to generate new insights from real-world educational settings (Gravemeijer, 2004; Akker et al., 2006).
2. **How participants are organized**: In the methods section, I outlined the rationale for selecting 30 students from SMAN 7 Bandung and the specific context of the curriculum and grade level they were part of.
3. **How the validation of the research is maintained**: I described how multiple methods (observations, interviews, tests) were used for data collection, along with ensuring internal validity through data triangulation and testing instruments.

For further clarification, please refer to the detailed descriptions in the methods section.

Commented [User7]: This part better put it on method.

Commented [A8R7]: Thank you for your helpful suggestion. I have moved the section you referred to into the "Method" section as per your recommendation. The revised manuscript now reflects this adjustment to improve the clarity and structure of the paper. I believe this will enhance the overall organization and flow of the research explanation.

effectiveness of the instructional materials for LIT. Retrospective analysis is employed to examine the complete dataset of the teaching experiment outcomes.

HLT is applied in every learning activity and has a crucial role in designing learning activities for students. The execution of learning is indissociable from the learning trajectory, encompassing a deliberate progression of learning materials and idea maps that students traverse during the learning process (Prahmana, 2016). Next, in the subsequent stage, the trigonometric ratios LIT is produced.

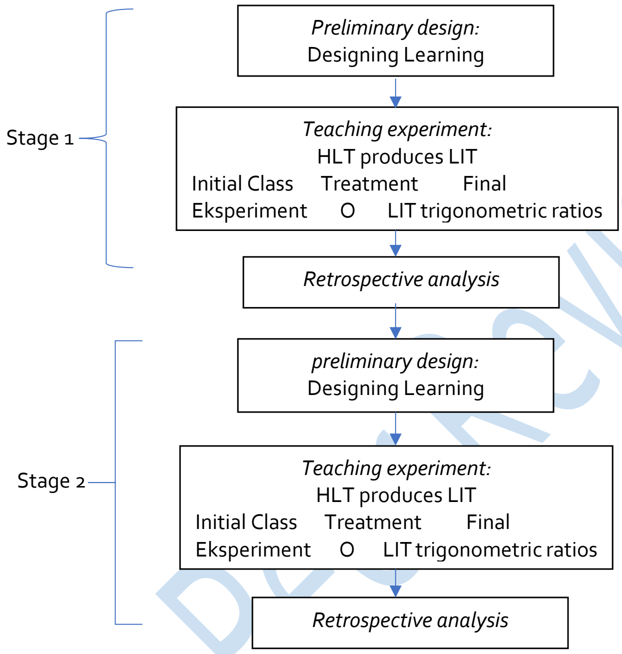


Figure 1. Research Design

The research implementation is carried out in two stages, with each stage referring to the steps of design research (Gravemeijer, 2004) namely: The three main stages of the project include 1) formulating the initial concept, 2) executing the educational experiment, and 3) carrying out a retrospective study. During the teaching experiment phase, this study involved 30 tenth-grade students from SMAN 7 Bandung. The selection of Grade 10 at SMAN 7 Bandung was based on the rationale that this class had already engaged with the national curriculum currently implemented in Indonesia, where trigonometric ratios constitute a key topic. Additionally, the school was chosen because it represents senior high schools that consistently adhere to the implementation of the national curriculum. This selection provided an opportunity to explore the application of trigonometric ratios within a context relevant to Indonesia's existing educational framework.

An analysis was conducted on the activities performed in the initial phase, and subsequently, recommendations were given to enhance the activities in the second phase. The findings from the data analysis of the second phase provide the final recommendations of this research.

Preliminary findings indicate that no teachers at this school have implemented Local Instruction Theory (LIT) in their teaching practices, particularly in the context of trigonometric ratios.

The primary reasons for this include limited training opportunities and a lack of understanding regarding LIT's application in the classroom. Gravemeijer (1999) highlights that while LIT has proven effective in various educational contexts, its implementation requires a comprehensive understanding of its principles and methodologies. Teachers often face challenges in adopting innovative teaching strategies like LIT due to insufficient professional development opportunities and curriculum constraints. Additionally, adapting LIT to specific topics, such as trigonometric ratios, poses further challenges if educators lack adequate tools and support. Consequently, the absence of LIT's application for this topic at the school underscores the relevance and potential for further research and development in this area.

Data for this research was collected using multiple methods to gather comprehensive insights. First, written data were obtained from test results, tasks such as UBA-LIT and LKPD, and observation sheets during the teaching experiments. Second, documentation in the form of video recordings and photographs were used to observe classroom activities, student interactions, and the process of mathematization. Additionally, direct observation was conducted by the researcher, who also acted as the teacher, noting student responses during the learning process. Interviews with selected students were carried out to gather deeper insights into their experiences with UBA-LIT and LKPD. The data were analyzed qualitatively by comparing the observations and video recordings with the initial teaching design (HLT). Internal validity was ensured through data triangulation from various sources, while reliability was strengthened by testing the instruments and conducting cross-interpretation with experts. The conclusions were drawn from the retrospective analysis of the collected data, explaining how students grasped the concept of trigonometric ratios within the learning process using LIT.

RESULTS AND DISCUSSION

Results

The research is carried out in three stages: 1) formulating the fundamental design, 2) executing the teaching experiment, and 3) implementing the retrospective analysis.

Elaborating on the Initial Design (*preliminary design*)

During this phase, the researcher assembles the educational materials and finalizes the instructional content. To meet the study aims, various actions are conducted, including (1) identifying the research subjects based on their diverse cognitive abilities. The researchers selected students from class X of SMAN 7 as appropriate research subjects; (2) examining and analyzing the curriculum to identify the compatibility of trigonometric comparison material with the curriculum currently being used in the school; (3) analyzing trigonometric comparison material to obtain an overview that "the achievement of Phase E-learning based on the derived geometry domain becomes the learning objective," in accordance with the research objectives. Next, the researcher developed a teaching material design in the form of a descriptive outline of trigonometric comparisons, also known as "Local Instruction Theory Descriptive Teaching Material (UBA-LIT)". The Realistic Mathematics Education (RME) technique enhances the learning process by employing diverse learning materials, including teaching modules and research tools. The study instrument is designed to incorporate the specific attributes of Realistic Mathematics Education (RME) and the curriculum used by the school. Additionally, the researchers analyze and assess the learning assistance tools. The result of the evaluation is referred to as the prototype. In the next stage, the research conducts an expert review, in which four experts validate the prototype based on its structure, content, and language.

Once the expert review procedure is finished, students who have been given trigonometric comparison information are tested on the prototype to get data on the readability and comprehension of the instructional material. The researchers engage in communication with the students during the testing process to anticipate any corrections or feedback regarding the instructional materials being tested by the students. The students write their input or suggestions on the prepared sheet. Students provide support for the discovery of formal concepts related to UBA-LIT through their comments. The researchers made improvements to the UBA-LIT after receiving feedback. After conducting expert validity testing and gathering comments from students, it was ultimately determined that the prototype design of the instructional material product generated by the researcher is valid at this level. The updated iteration of the prototype is referred to as the second prototype. Following that, the second prototype underwent testing in teaching experiment 1, including a total of 30 students from SMAN 7 Kota Bandung.

From a design research standpoint, the primary objective of the initial design is to develop a Local Instruction Theory (LIT) that can be visually portrayed as a portrayal of the process of acquiring and improving knowledge through teaching experiments (Gravemeijer & Cobb, 2006). During the preliminary study, the researcher designed the learning process activities based on a literature review. This set of educational exercises covers the beliefs held by students while they are building knowledge and the methods that will be used, which will be presented as an initial challenge, including advanced thinking skills. The contents of HLT are subject to change. The activities in HLT can be adapted and customized based on the circumstances and parameters of the ongoing educational experiment. HLT has a curriculum that students will adhere to when studying trigonometric ratios. The HLT curriculum comprises learning objectives, activity descriptions, and student thought conjectures that occur during the learning process. During the teaching experiment, HLT functions as a reference for teachers and researchers to determine the teaching emphasis, observations, and interviews. Moreover, HLT functions as a point of reference for the examination or analysis of a dataset in the context of a teaching experiment. Hypothetical Learning Trajectory (HLT) used in the teaching experiment is displayed in Table 1.

Table 1. HLT illustration for learning the concept of trigonometry comparisons

Activity	Learning Objectives	Activity Description	Conjecture
Students Demonstrate the activity of identifying by body height.	The students are able to identify the front, side, and diagonal positions by referring to specific angles of a right-angled triangle.	- The teacher instructed three students to engage in a practical exercise involving the formation of a right-angled triangle using a	- Some students may be able to illustrate a sketch of a right triangle from the demonstration activity. - Some students may quickly recognize the positions of the corner,

		table base, a measuring tape, and their height. - Each group carefully observes the demonstration and discusses the position of the corner, front, side, and slant.	front side, side side, and hypotenuse.
Shows high shadow intensity activity	The students can articulate the trigonometric ratios (sin θ, cos θ, and tan θ) by employing the notion of ratios within a right triangle.	- The teacher instructs the students to face away from the light until a shadow appears in front of the standing student. The sketch formed is a right-angled triangle. - Students can determine the front side, side view, and diagonal side of a right triangle sketch based on the determining angle. Students will be *guided* in *rediscovering the concept* of trigonometric ratios.	- Some students may use tables as a tool to solve problems. - Certain students may employ finger manipulation methods to solve difficulties. - Some students may utilize measuring instruments to solve problems. - Some students may use the number parity method to solve problems. - Certain students may possess the ability to represent this activity in a visual manner. They initiated the construction of a model and engaged in horizontal and vertical mathematization in order to uncover the notion of trigonometric ratios through the shown examples. -

			- Some students may be able to relate the demonstration process to the concept of trigonometric ratios.
Problem-Solving	Students are able to solve real-life problems related to right triangles using trigonometric ratios	Students use trigonometric ratios to solve real-life problems by performing calculations.	- Certain students possess the ability to perceive the relationship between sine and cosine values in mathematical calculations and are capable of streamlining the calculation process. - Some students may be able to remember or repeat knowledge they have previously acquired to solve problems. - Some students may use tables, finger methods, odd-even tricks, or measuring tools to solve problems. - Some students may directly apply trigonometric ratio concepts.

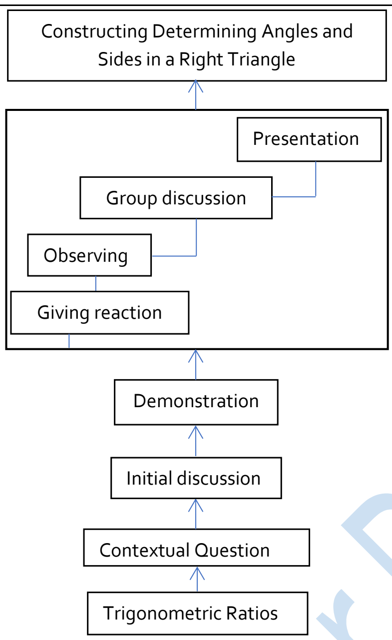
Teaching Experiment

The primary objective of a teaching experiment is to systematically evaluate and enhance original ideas regarding Local Instruction Theory (LIT) while also striving to gain a more profound comprehension of the underlying mechanisms and functioning of LIT (Gravemeijer & Cobb, 2006).

Activity 1

The teacher requested three students to perform a demonstration using the provided tables/chairs and ropes.

The teacher requested one of the students to fasten the rope to the base of the table or chair provided. A student holds the end of the neat rope and moves back 1 meter from the base of the table/chair. After that, the student who is holding the end of the rapia rope shifts his grip to the end of the head. Other students are asked to draw sketches of the shapes that are occurring. The other students observed the demonstration requested by the teacher, depicted the requested sketch, and identified the positions of the angles, front side, side view, and oblique side of the formed triangle sketch. All students strive to comprehend the right-angled triangle sketch and identify the positions of angles, front side, side view, and hypotenuse on the sketch. The teacher provides a comprehensive understanding through the following activities.



The student model illustrated the activity of identification by using height as part of the learning process for understanding trigonometric ratios.

Figure 2. Demonstration of the flow of LIT and height in learning comparative trigonometry

Activity 2

The teacher instructs the class to define the trigonometric ratios (sin θ, cos θ, and tan θ) with a demonstration involving the height of a shadow. This activity guides students in formally discovering the concept of trigonometric comparisons. The students were able to perform this activity proficiently after acquiring the idea of identifying angles and sides in the previous activity. The presented issues are contextual and aligned with the student's level of understanding. The LIT design is structured from the simplest to the most complex in order to provide a general solution. In this activity, the teacher instructs the students to face away from the light source so that a shadow appears in front of the standing students. The results of the student's practice are represented in the form of an image or sketch of a right-angled triangle. This sketch facilitates students in identifying the positions of angles, front side, side view, and inclined side of a right triangle based on the predetermined angle. Creating a sketch of a right triangle and determining angles helps students mathematize vertically and find formal solutions to define trigonometric ratios.

Figure 3 illustrates various outcomes of students' process variations in discovering formal mathematics from the concept of trigonometric ratios. The students' work begins with a demonstration of a contextual problem, allowing them to find solutions using various methods based on their prior knowledge. During the learning process, students are guided to discover formal mathematics, which is a process of mathematization based on the presented problems.

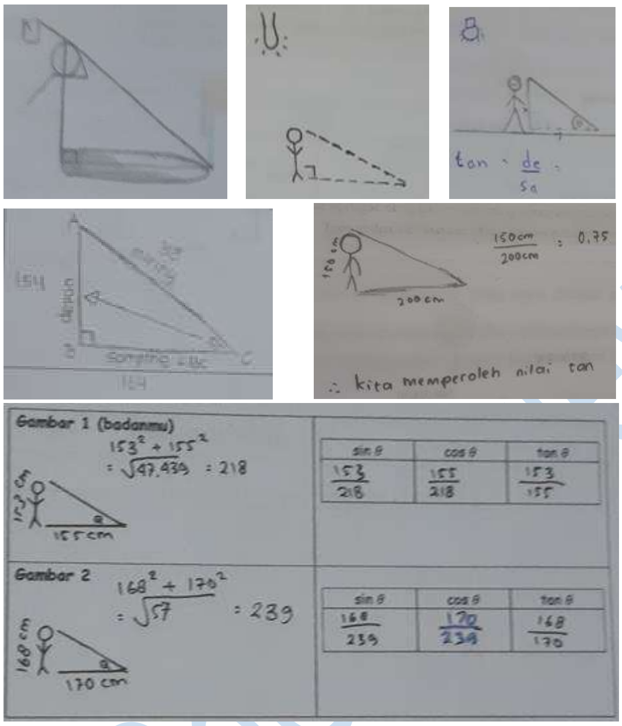


Figure 3. The vertical mathematical process of students in finding formal solutions to trigonometric ratios

Activity 3

Activity 3 begins with the teacher presenting problems based on everyday life situations. Everyday scenarios were employed to enable students to apply the notion of trigonometric ratios in practical circumstances. After the activity is finished, the teacher conducts a test as a final evaluation. One of the trigonometric comparison problems presented is as follows:

> "Refer to the following image of a bed. What are the geometric shapes that occur between the stairs, bed, and floor? Sketch a model. (1) *Does the shape formed constitute a right triangle? If so, mark one corner with a right angle*. (2) *If the angle between the stairs and the floor is unknown, and the distance from the bed to the stairs is 10 cm. Place the obtained data into the image. Subsequently, calculate the distance from the floor to the end of the staircase, which forms a right-angled triangle, using the tangent ratio.* ($\tan 37^0 = 0{,}75$)"

Figure 4. Image of a bed as a model for the use of trigonometric ratios

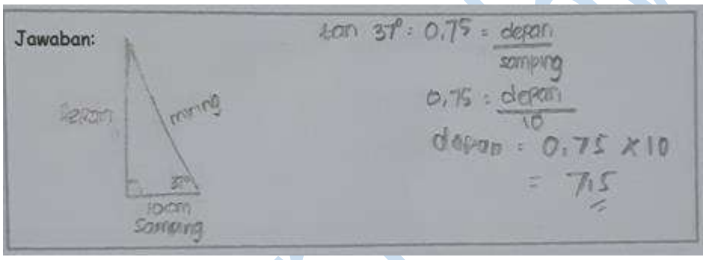


Figure 5. One of the student's responses was to solve the story problem

Figure 5 is an example of a student's response using the concept of trigonometric ratios to solve a word problem. The student solves the problem using the formal solution of trigonometric ratios that was previously constructed.

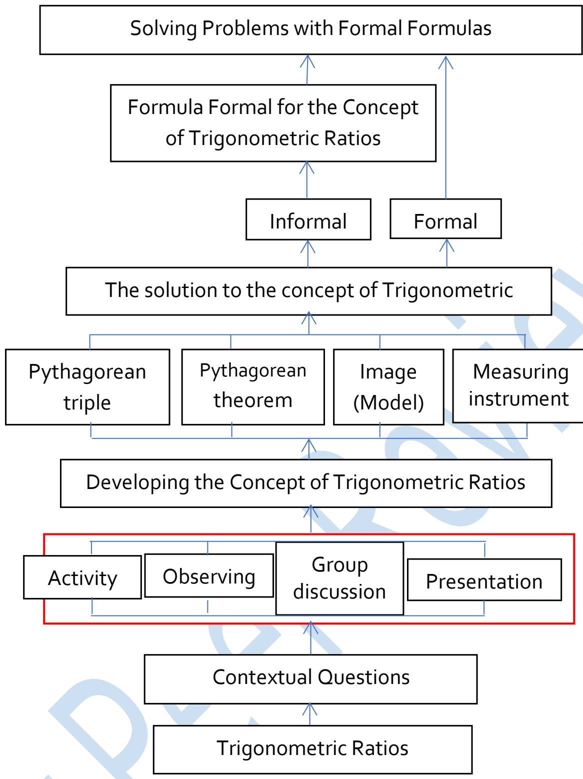
Retrospective Analysis

Researchers employ Hypothetical Learning Trajectory (HLT) as a first guide and reference when assessing a dataset during the data analysis stage of a teaching experiment. The HLT created is compared to the classroom learning process. The analysis includes descriptions of anticipated occurrences during the learning process and examples of behaviors that will occur among students. The conclusions drawn from the analysis will answer the research questions. A good design is not the main objective of the study but rather the reason for the study's success, which is the main objective (Gravemeijer & Cobb, 2006).

The many activities carried out during the learning process are represented in the flowchart of LIT before the learning process on the topic of trigonometric ratios. The flowchart is presented in Figure 6. The image shows the differences in stages or steps during the learning process. The difference is evident in the red-colored box. Before the learning process begins, the design aims to facilitate activities, experiments, observations, group discussions, and presentations as processes that run parallel. Students will go through these stages simultaneously.

Meanwhile, after the learning process takes place, a revision of the initial design is made. The stage of differentiation involves an initial discussion process with students prior to the demonstration (*Figure 7*). This stage provides students with an introduction to understand the

concept of trigonometric ratios in relation to the prior knowledge they have already constructed. This stage is considered crucial because linking it to the previously acquired knowledge is necessary for students to comprehend the fundamental concepts of trigonometric ratios.



Before the learning process begins

Figure 6. Flowchart of LIT on the topic of Trigonometric Ratios before the lesson

The next difference is in the "responding" stage, which students go through after conducting a demonstration and before the "observing" stage. The act of "responding" is the first step for students to participate actively in the learning process. The students will provide their responses or feedback to the demonstration presented. Their knowledge will guide and stimulate the students to continue the learning process via the "observing" stage.

Before the learning process begins, the steps "activity/experiment," "observe," "group discussion," and "presentation" have the same level of suitabilityThere is no text provided. However, the post-learning phases in these four things exhibit distinct variations. The "responding" stage initiates the steps of learning. Next, the process begins with the "observation" stage, followed by the "group discussion" step, and concludes with the "presentation" step. The students go through the four steps of the learning process in different stages.

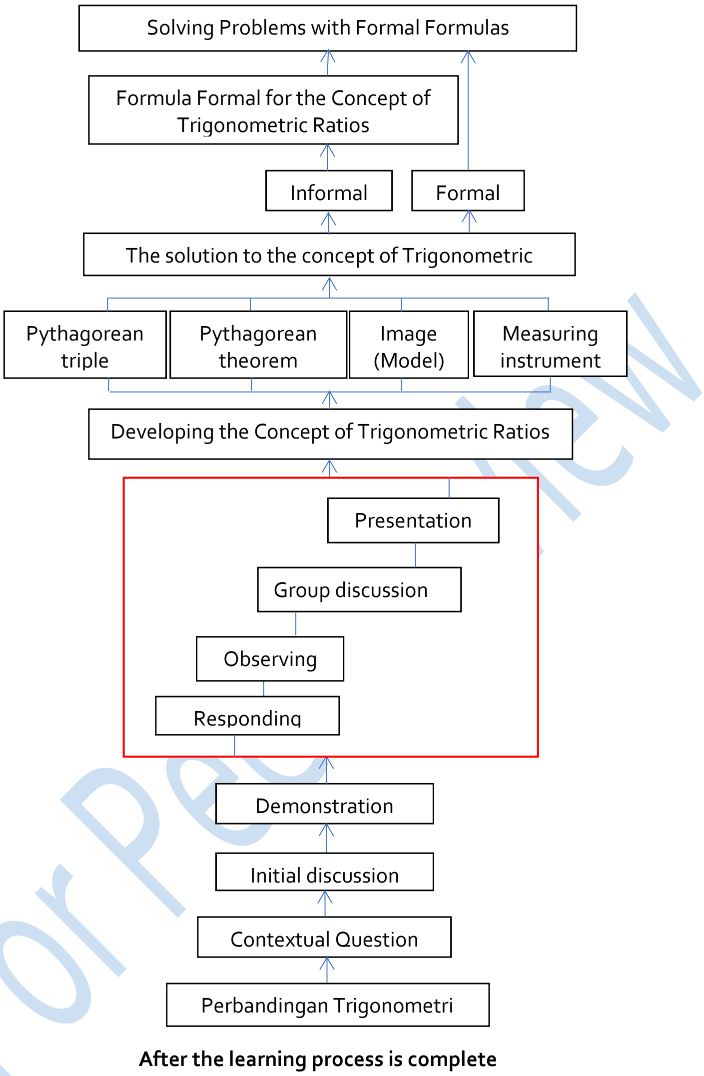


Figure 7. Flowchart of LIT on the topic of Trigonometric Ratios after the lesson

The appropriate use of the LIT framework, as shown in Figure 7, allows students to construct an understanding of the concept of trigonometric ratios through everyday problems. They are able to identify trigonometric ratios and solve real-life issues accordingly, as a result of analyzing the HLT and conducting teaching experiments. Based on the results of the student's work using the LIT design for trigonometric comparison, the number of students capable of completing the learning activities is determined. The students are becoming accustomed to the learning process, which involves determining mathematics formally based on pre-established models. The utilization of

Pythagorean triples, the Pythagorean theorem, models (diagrams), and measuring tools can aid in achieving vertical mathematization. The students may effectively solve common problems by utilizing trigonometric ratios.

Some students found the Local Instruction Theory (LIT) approach challenging as it shifted their understanding from merely applying formulas to understanding the deeper concepts. For example, one student expressed initial difficulty with LIT's emphasis on understanding the relationships between concepts rather than just following procedural steps. However, after grasping the underlying logic, the student felt satisfied with being able to understand the reasoning behind trigonometric ratios, rather than just memorizing formulas.

On the other hand, some students struggled to adapt to this new teaching design, especially those who were accustomed to more traditional, structured methods. A few students felt overwhelmed by the responsibility of independent learning and actively solving problems. This feedback suggests that while LIT can provide a deeper and more engaging learning experience, its effectiveness depends heavily on how well students are supported in transitioning to this new method.

Additionally, student feedback highlighted the benefits of the collaborative learning aspect of LIT. Students appreciated the group discussions and the opportunity to explore different approaches to problem-solving, which provided a more shared learning experience. However, some also criticized the method, feeling that they lacked the foundational knowledge to fully engage in the collaborative or reflective aspects of LIT, emphasizing the need for careful planning when implementing this method. These student responses offer valuable insights for refining the design and application of LIT in teaching.

The findings indicate that some students lack the prerequisite knowledge they should have acquired, which is influenced by factors such as mental unpreparedness, teacher didactics, and inadequate teaching materials (Kurniasih et al., 2020; Puspita et al., 2023). In particular, the emphasis on rote memorization of formulas, such as trigonometric ratios, without a geometric explanation, leads to a lack of motivation and understanding among students (Andika et al., 2017; Urrutia et al., 2019). Teacher-centered methods that rely solely on memorization often result in short-term retention of knowledge (Orhun, 2002). To overcome this, teachers need a comprehensive understanding of trigonometry to connect various concepts, enhancing students' learning (Andiani et al., 2020, 2024; Rittle-Johnson & Schneider, 2014). Additionally, incorporating real-life contexts into lessons, such as through models and diagrams, makes learning more relevant and aids in the long-term retention of knowledge (Dhungana et al., 2023). The use of Local Instruction Theory (LIT) and Realistic Mathematics Education (RME) approaches has been shown to improve students' ability to construct their understanding of trigonometric ratios, as evidenced by their improved learning outcomes.

Discussion

The introduction of Local Instruction Theory (LIT) in this study revealed significant improvements in students' conceptual understanding of trigonometric ratios. Unlike traditional teacher-centered approaches, LIT facilitated deeper connections between abstract mathematical concepts and real-life applications. This was evident in the students' increased ability to relate geometric properties to trigonometric functions.

During the teaching experiment, students exhibited greater engagement with the material, particularly when real-life problems were introduced. Many students showed improved motivation

Commented [User9]: Clarity and Focus:

- Some parts of the discussion are repetitive (e.g., the explanation of sine ratios). A more concise presentation would improve readability.
- The transition between problems and proposed solutions could be more seamless.

Details on LIT and RME:

- While the potential of LIT and RME is highlighted, the specifics of how these approaches were implemented could be expanded upon. For example:
  - What specific real-life problems or models were used?
  - How did students respond during the teaching experiment?

Addressing Limitations:

- The discussion mentions resource and time constraints as limitations but does not suggest how future research might address these issues.

Student Diversity:

- The research could explore variations in student experiences based on factors such as prior knowledge, learning styles, or socio-economic backgrounds.

Commented [A10R9]: Clarity and Focus:

- **Repetitiveness:** Thank you for your suggestion regarding the repetitiveness in the discussion, specifically around the explanation of sine ratios. I have revised this section to make it more concise and eliminate unnecessary repetition. I believe this streamlined approach improves the readability of the manuscript and strengthens the overall presentation of the findings.
- **Transition between Problems and Solutions:** I appreciate your comment on improving the transition between the problems and proposed solutions. I have edited this section to make the connections more seamless, ensuring a smoother flow from the identified issues to the suggested approaches. This should enhance the overall coherence of the discussion.

Details on LIT and RME:

- **Real-Life Problems and Models:** Thank you for emphasizing the need to elaborate on the application of Local Instruction Theory (LIT) and Realistic Mathematics Education (RME). In response, I have further developed the discussion in the introduction (page 4, paragraph 4), specifically detailing the real-life problems and models employed during the teaching experiment. This section now describes how these models were integrated into the lesson plans and the ways in which they contributed to engaging students with the learning content. The integration of real-world contexts is essential for improving the relevancy of mathematical concepts and fostering deeper student understanding (Gravemeijer & van Eerde, 2009; Freudenthal, 1991).
- **Student Responses during the Teaching Experiment:** Additionally, I have expanded upon the students' responses during the teaching experiment in the retrospective analysis section (page 16, paragraphs 1-3). This addition provides a more thorough account of how students interacted with the teaching materials, ...

and understanding, especially those who had struggled with rote memorization in previous lessons. Their ability to visualize trigonometric ratios in practical contexts, such as through diagramming angles and distances in real-world scenarios, was significantly enhanced.

Previous studies on Local Instruction Theory have largely focused on its implementation in general mathematical contexts, but this study fills a gap by specifically applying LIT to the teaching of trigonometric ratios. By incorporating real-world contexts and geometrical reasoning, this research extends the application of LIT to improve both conceptual understanding and long-term retention in mathematics education.

Before the intervention, students struggled with memorizing trigonometric formulas and showed limited understanding of their geometric foundations. After the intervention, students were able to connect these formulas with real-world contexts, as evidenced by their improved problem-solving abilities and retention rates.

Implication of Research

The design of this Local Instruction Theory (LIT) for trigonometric ratios incorporates the syntax of the Realistic Mathematics Education (RME) approach. The foundation of this LIT is a conjecture based on the learning trajectory using RME. Consequently, the designed learning trajectory needs to be further developed to align with the specific content objectives. Researchers intending to implement a learning trajectory are advised to master various instructional strategies or approaches for designing problems. By mastering problem design, a wide array of solutions both formal and informal can be found. Additionally, exploring variations in students' experiences, influenced by factors such as prior knowledge, learning styles, or socio-economic backgrounds, could provide alternative perspectives when applying LIT.

Limitation

This study was conducted in two phases, each of which aligns with the steps outlined in design research (Gravemeijer, 2004), namely: 1) developing the preliminary design, 2) conducting the teaching experiment, and 3) performing retrospective analysis. There are two reasons for selecting only two phases in this research. First, the researcher's time and resources were limited. Second, the material in question is only implemented once a year. To address the limitations of time and resources, the researcher recommends extending the duration of the teaching experiments and incorporating digital resources. These approaches would enhance the efficiency and effectiveness of both time and resource utilization in the study. The extended timeframe allows for more comprehensive data collection, while digital resources provide flexible, accessible learning tools that can alleviate constraints related to physical materials (Means et al., 2014). Furthermore, leveraging technology enables more dynamic and scalable teaching practices, supporting diverse learning needs (Çelik & Baturay, 2024).

CONCLUSION

The outcome of the learning trajectory in this study is a Comparative Trigonometry LIT developed through the Realistic Mathematics Education (RME) approach. This theory encompasses several steps that students must go through in constructing their knowledge related to trigonometric comparisons. These steps nonetheless adhere to the syntax of the Realistic Mathematics Education (RME) approach.

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4. Bukti Konfirmasi Artikel Accepted (26 November 2024)

[Kreano] Your submission has been accepted to Kreano, Jurnal Matematika Kreatif-Inovatif

2024-11-26 05:38

Dear Dini Andiani,

I am pleased to inform you that we have decided to accept your submission. After careful review, we found your submission, Development of A Local Instruction Theory for Trigonometric Ratios, to meet or exceed our expectations. We are excited to publish your piece in Kreano, Jurnal Matematika Kreatif-Inovatif and we thank you for choosing our journal as a venue for your work.

As an open access journal, publication of articles in Kreano, Jurnal Matematika Kreatif-Inovatif are associated with Article Processing Charges (APC). With respect to the business, we convey that the APC is charged to the author of the article for IDR [redacted] (for any additional fee, such as translate, etc please see Services and Fees).

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Proof of payment can be sent via discussion menu at the copy-editing stage.

Your submission is now forthcoming in a future issue of Kreano, Jurnal Matematika Kreatif-Inovatif and you are welcome to include it in your list of publications. We recognize the hard work that goes into every successful submission and we want to congratulate you on reaching this stage.

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You will shortly receive further instructions.

If you have any questions, please contact me from your submission dashboard.

Kind regards,

Executive Editor

5. Bukti Konfirmasi Artikel Published Online

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Next steps for publishing your submission

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Dear Dini Andiani,

I am writing from Kreano, Jurnal Matematika Kreatif-Inovatif to let you know that the editing of your submission, Development of A Local Instruction Theory for Trigonometric Ratios, is complete. Your submission will now advance to the production stage, where the final galleys will be prepared for publication. We will contact you if we need any further assistance.

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Kind regards,

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